Fondamenti di Grafica Tridimensionale

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Incremental Simp Method

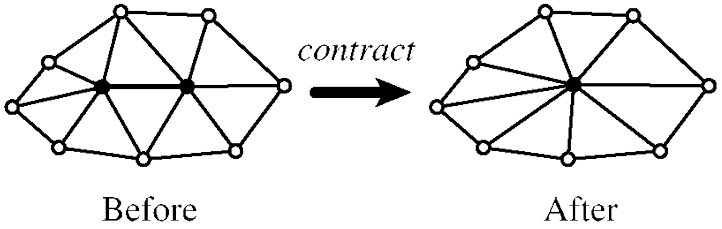
The common framework:

- **loop**
  - ***select*** the element to be deleted/collapsed;
  - ***evaluate approximation*** introduced;
  - ***update*** the mesh after deletion/collapse;

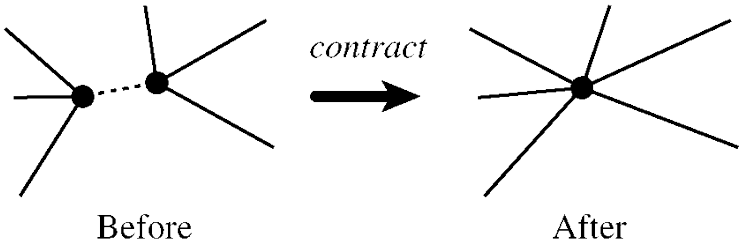
until mesh **size/precision** is satisfactory;

Simplification using Quadric Error Metrics [Garland et al. Sig'97]

- Based on incremental **edge-collapsing**



- **but** can also collapse vertex couples which are **not connected** (topology is not preserved)

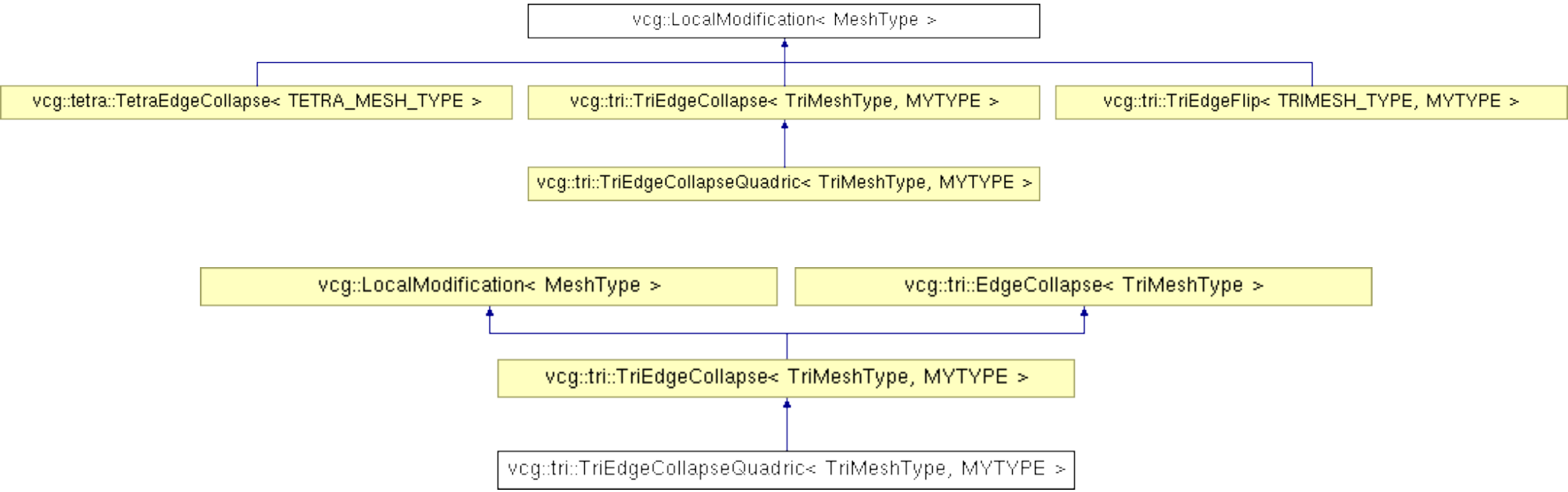


The main simplification loop

```
vcg::LocalOptimization<MyMesh> DeciSession(mesh);

DeciSession.Init<MyTriEdgeCollapse >();

DeciSession.SetTargetSimplices(FinalSize);
DeciSession.SetTimeBudget(0.5f);

while(DeciSession.DoOptimization() && mesh.fn>FinalSize)
    printf("Current Mesh size %7i heap sz %9i err %9g \r",
            mesh.fn,DeciSession.h.size(),DeciSession.currMetric);

printf("mesh  %d %d Error %g \n",
                  mesh.vn,mesh.fn,DeciSession.currMetric);
```

Classi in gioco

- LocalOptimization
  - Classe astratta per il loop di ottimizzazione
- LocalModification
  - Classe astratta per una generica operazione che modifica la mesh localmente con un certo costo
- EdgeCollapse
- TriEdgeCollapse
  - Particolare local modification
- TriEdgeCollapse Quadric


vcg::LocalModification< MeshType >
vcg::tetra::TetraEdgeCollapse< TETRA_MESH_TYPE >
vcg::tri::TriEdgeCollapse< TriMeshType, MYTYPE >
vcg::tri::TriEdgeFlip< TRIMESH_TYPE, MYTYPE >
vcg::tri::TriEdgeCollapseQuadric< TriMeshType, MYTYPE >
vcg::LocalModification< MeshType >
vcg::tri::EdgeCollapse< TriMeshType >
vcg::tri::TriEdgeCollapse< TriMeshType, MYTYPE >
vcg::tri::TriEdgeCollapseQuadric< TriMeshType, MYTYPE >

Local Modification

```
template <class MeshType> class LocalModification
{
 public:
        typedef typename LocalOptimization<MeshType>::HeapType HeapType;
        typedef typename MeshType::ScalarType ScalarType;

inline LocalModification(){};
virtual ~LocalModification(){};

virtual ModifierType IsOfType() = 0 ; /// return the type of operation
/// return true if the data have not changed since it was created
virtual bool IsUpToDate() = 0 ;
/// return true if no constraint disallow this operation to be performed (ex: change
   of topology in edge collapses)
virtual bool IsFeasible() = 0;
/// Compute the priority to be used in the heap
virtual ScalarType ComputePriority()=0;
/// Return the priority to be used in the heap (implement static priority)
virtual ScalarType Priority() const =0;
/// Perform the operation and return the variation in the number of simplicies (>0
   is refinement, <0 is simplification)
virtual void Execute(MeshType &m)=0;
/// perform initialization
static void Init(MeshType &m, HeapType&);
  virtual const char *Info(MeshType &) {return 0;}
/// Update the heap as a consequence of this operation
virtual void UpdateHeap(HeapType&)=0;
};      //end class local modification
```

Local Modification

- Classe astratta generica
  - Potrebbe essere un edge collapse
  - Uno swap
  - Un vertex deletion ecc.
- Astrarre una generica operazione di modifica locale alla mesh
  - Adatta ad essere prioritizzata
    - Deve saper dare una priorita'
  - Sapersi applicare alla mesh
  - Sapere se e' sempre valida

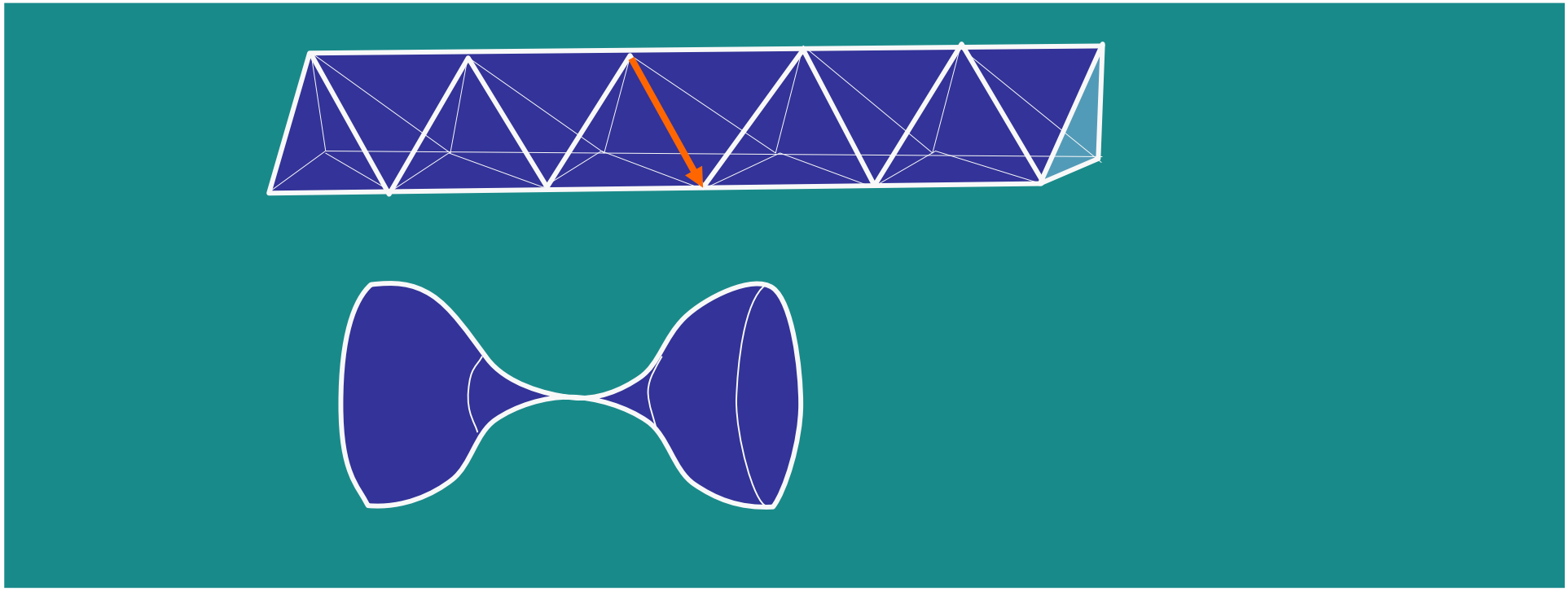
EdgeCollapse e TriEdgeCollapse

- EdgeCollapse
  - Classe astratta per rappresentare un collasso di un edge su una generica mesh
  - Non sa nulla di priorita' quadriche ecc
- TriEdgeCollapse
  - Generica local op basata su collasso
    - Sa aggiornare lo heap
    - Eseguirsi, sapere se e' valida ecc.
  - Da questa si deriva quella con le quadriche

```
template <class TRI_MESH_TYPE>
class EdgeCollapse
{
typedef typename vcg::face::VFIterator<FaceType>  VFI;
typedef typename std::vector<vcg::face::VFIterator<FaceType> >
   VFIVec;

static VFIVec & AV0(){static VFIVec av0; return av0;}
static VFIVec & AV1(){static VFIVec av1; return av1;}
static VFIVec & AV01(){static VFIVec av01; return av01;}

bool LinkConditions(EdgeType  pos);
void FindSets(EdgeType &p)bool LinkConditions(EdgeType  pos);
int DoCollapse(EdgeType & c, const Point3<ScalarType> &p);
}
```

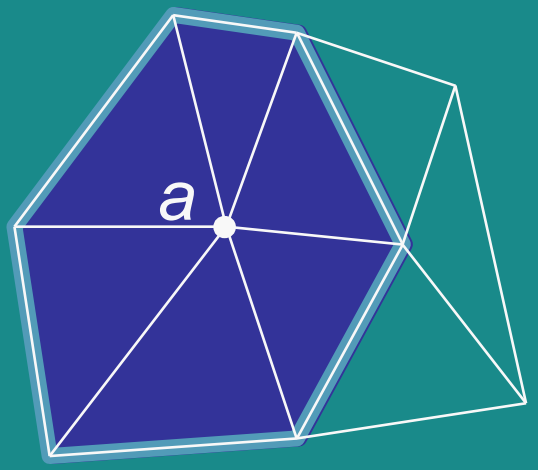
Topology Preservation

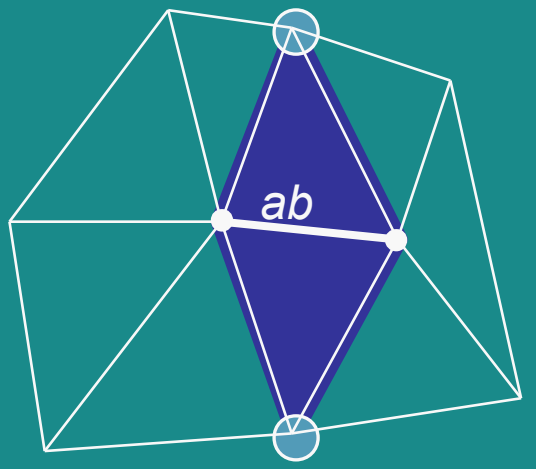
- 2-Manifold
  - A surface Σ in $\boldsymbol{R}^2$ such that any point on Σ has an open neighborhood homeomorphic to an open disc or to half an open disc in $\boldsymbol{R}^2$
- A edge collapse can create non manifold situations



Topology Preservation

- Let Σ be a 2 simplicial complex ***without boundary***
- Σ' is obtained by collapsing the edge $e = (ab)$
- Let $Lk(\sigma)$ be the set of all the faces of the co-faces of σ disjoint from σ





Σ and Σ' are homeomorphic ***iff***

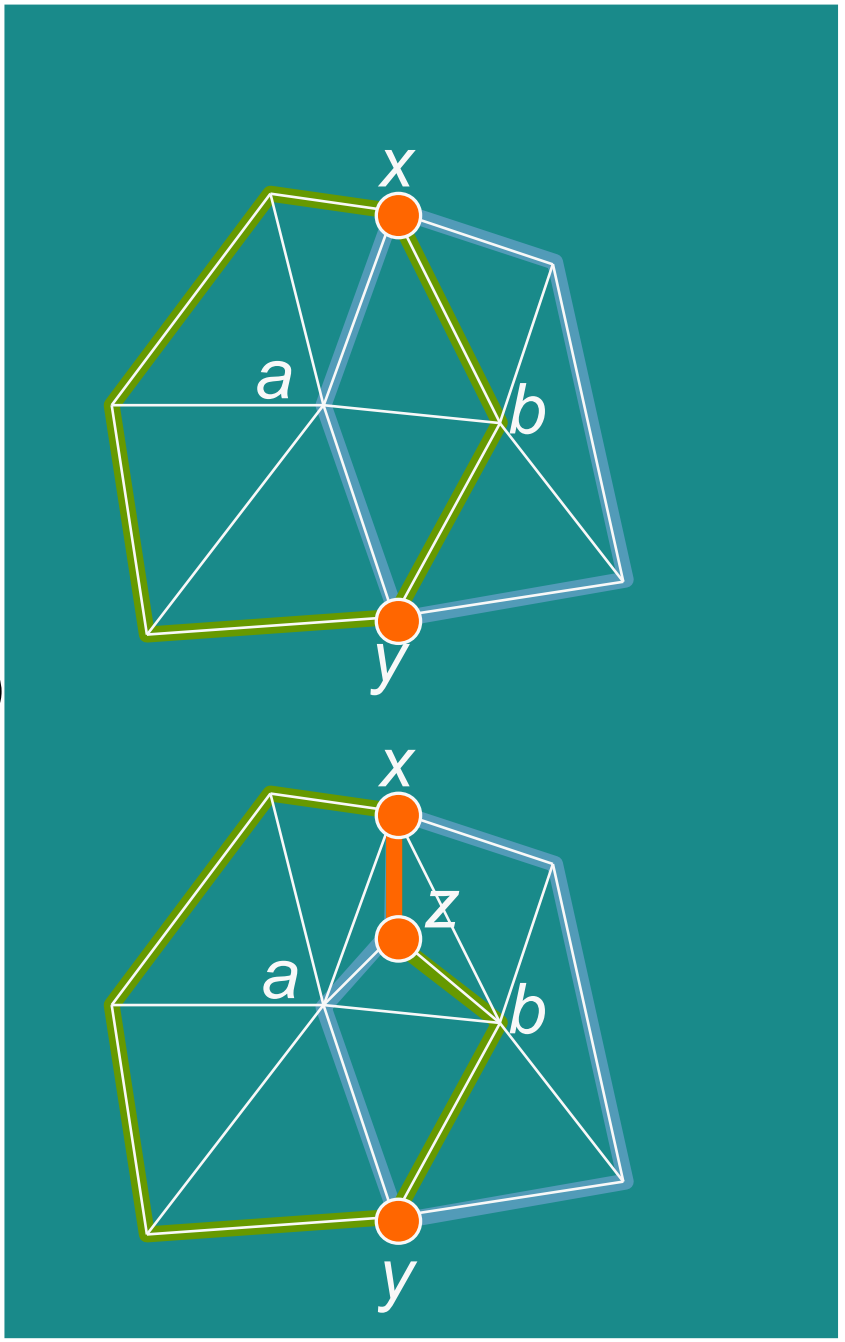
$$Lk(a) \cap Lk(b) = Lk(ab)$$

[Dey 99]

Topology Preservation

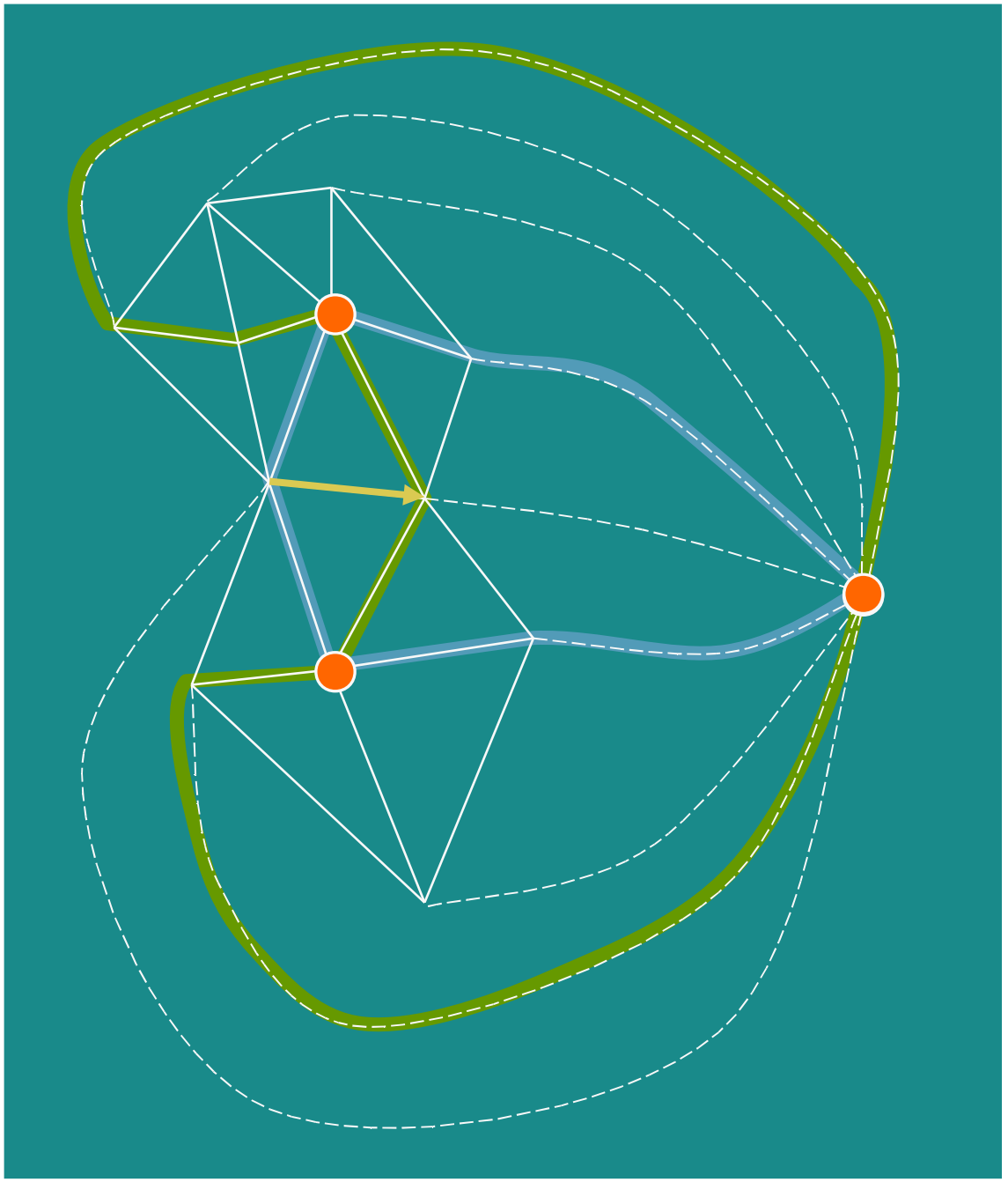
$Lk(a) \cap Lk(b) = \{x, y\} = Lk(ab)$

$Lk(a) \cap Lk(b) = \{x, y, z, zx\} \neq \{y, z\} = Lk(ab)$



Topology Preservation

- Mesh with boundary can be managed by considering a dummy vertex v_d and, for each boundary edge *e* a tetrahedron connecting e with v_d
- Think it wrapped on the surface of a sphere
-

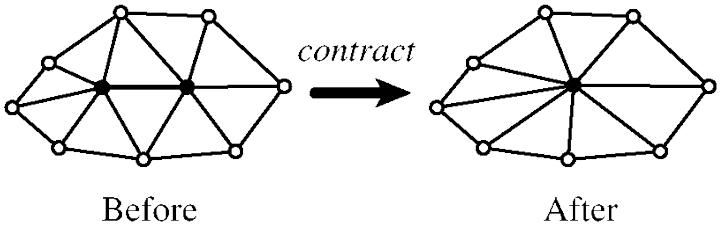


doCollapse

```
for(i=AV01().begin();i!=AV01().end();++i)
{
 FaceType  & f = *((*i).f);
 assert(f.V((*i).z) == c.V(0));
 vcg::face::VFDetach(f,((*i).z+1)%3);
 vcg::face::VFDetach(f,((*i).z+2)%3);
 f.SetD();
 n_face_del++;
}

//set Vertex Face topology
for(i=AV0().begin();i!=AV0().end();++i)
{
 (*i).f->V((*i).z) = c.V(1);                          // In tutte le facce
   incidenti in v0, si sostituisce v0 con v1
 (*i).f->VFp((*i).z) = (*i).f->V((*i).z)->VFp();
    // e appendo la lista di facce incidenti in v1 a questa faccia
(*i).f->VFi((*i).z) = (*i).f->V((*i).z)->VFi();
(*i).f->V((*i).z)->VFp() = (*i).f;
(*i).f->V((*i).z)->VFi() = (*i).z;
}

c.V(0)->SetD();
c.V(1)->P()=p;
return n_face_del;
```

Lazy heap

- Si suppone di avere uno heap con tutte le operazioni
- Estraggo da heap e aggiorno la mesh
  - tali operazioni invalidano/modificano la mesh e quindi le priorità/validità di parte delle azioni già presenti nello Heap



Lazy Heap

- Due Soluzioni
  - Link espliciti elementi mesh->heap e aggiornamento dello stesso
- Lazy update
  - Si mettono nello heap tutte le nuove operazioni con la nuova priorità
  - Quando si estrae un'op dall heap si controlla che sia sempre valida
  - Di tanto in tanto garbage collection sullo heap

Marche incrementali

- Strumento generico per marcare oggetti in una collezione con
  - C(mark elem) = O(1)
  - C(unmark elem) = O(1)
  - C(unmark All Elem) = O(1)

  - Memorizza per ogni elem un intero *mark* invece di un bit
  - Esiste una marca globale a livello della collezione di elementi

Marche incrementali

- Un oggetto è marcato se
  - elem.mark==global.mark
- Marcatura di un elem
  - elem.mark := global.mark
- Smarcatura globale
  - global.mark++
  - Spesso le marche vengono dette anche marche ***temporali*** per indicare che dicono quando un certo elem è valido

Validità collasso

- Dati
  - Ogni vertice ha una marca temporale:
    - quando e' stato modificato l'ultima volta
  - Ogni collasso (coppia di vertici) ha una marca temporale
    - quando è stato inserito nello heap
- Un collasso è valido se
  - I due vertici non sono stati cancellati
  - Il collasso e' stato messo nello heap piu recentemente della data di ultima modifica dei vertici

Quadric Error for Surfaces

- Let $\mathbf{n}^T\mathbf{v} + d = 0$ be the equation representing a plane
- The squared distance of a point $\boldsymbol{x}$ from the plane is

$$D(\mathbf{x}) = \mathbf{x}(\mathbf{n}\mathbf{n}^T)\mathbf{x} + 2d\mathbf{n}^T\mathbf{x} + d^2$$

- This distance can be represented as a quadric

$$Q = (A, \mathbf{b}, c) = (\mathbf{n}\mathbf{n}^T, d\mathbf{n}, d^2)$$

$$Q(\mathbf{x}) = \mathbf{x}A\mathbf{x} + 2\mathbf{b}^T\mathbf{x} + c$$

Quadric

- The boundary error is estimated by providing for each boundary vertex *v* a quadric Q_v representing the sum of the all the squared distances from the faces incident in *v*
  - The error of collapsing an edge e=(v,w) can be evaluated as Q_w (*v*).
    - After the collapse the quadric of *v* is updated as follow $Q_v = Q_v + Q_w$