Data structures for 3D Meshes

Paolo Cignoni
p.cignoni@isti.cnr.it
http://vcg.isti.cnr.it/~cignoni

Introduction

- Premise:
  - 3D Graphics involves the rendering of digitally represented objects
  - Rendering means reasoning about how the light interacts with the surface of the objects (mostly).
- Issue:
  - representing boundary of 3D objects

Key idea:

- Discretize the surface in a set of simple primitives
  - Simple!
  - Polygons
  - Triangles
  - Small curved elements (nurbs)
  - Even points
- We will focus only on one of them:
  - simplicial complexes

Why triangular meshes

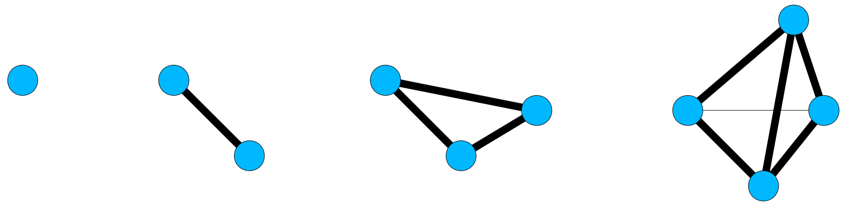
- Perche' solo e soltanto mesh triangolari?
  - In modellazione si vedono spesso mesh composte da poligoni generici...
  - Risposta teorica
    - Hanno un bel formalismo (complessi simpliciali)
    - Meno casi degeneri
      - Un triangolo e' sempre planare
      - Uniformita' se tolgo un vertice ottengo sempre un simplesso...
      - Estendibilita
      - Fixed size relations

Why triangular meshes

- Perche' solo e soltanto mesh triangolari?
  - In modellazione si vedono spesso mesh composte da poligoni generici...
  - Risposta Pratica
    - Hardware grafico basato solo su triangoli
    - Strutture dati semplici

Simplessi

- Un ***k simplesso*** è definito come la combinazione convessa di k+1 punti non linearmente dipendenti



- k è l'ordine del simplesso
- I punti si chiamano vertici

Sotto-Simplesso

- Un simplesso σ' è detto *faccia* di un simplesso σ se e' definito da un sottoinsieme dei vertici di σ
-
- Se $\sigma \neq \sigma'$ si dice che é una faccia propria

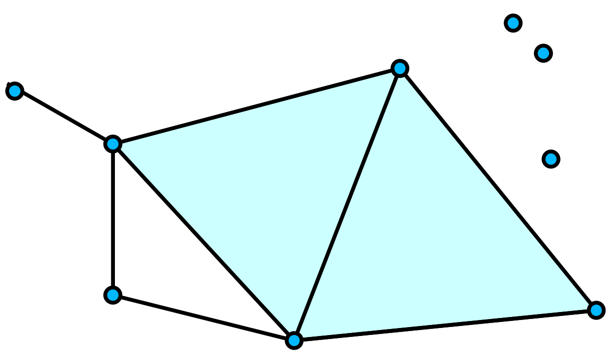
Complesso Simpliciale

- Una collezione di simplessi Σ e' un k-complesso simpliciale se:

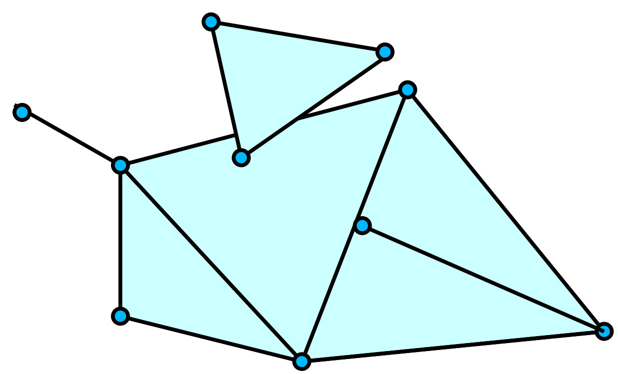
$$\forall \sigma_{1,} \sigma_2 \in \Sigma \quad \sigma_1 \cap \sigma_2 \neq \emptyset \rightarrow \sigma_1 \cap \sigma_2 \text{ is a simplex of } \Sigma$$

$$\forall \sigma \in \Sigma \text{ all the faces of } \sigma \text{ belong to } \Sigma$$

$$k \text{ is the maximum order } \forall \sigma \in \Sigma$$



OK

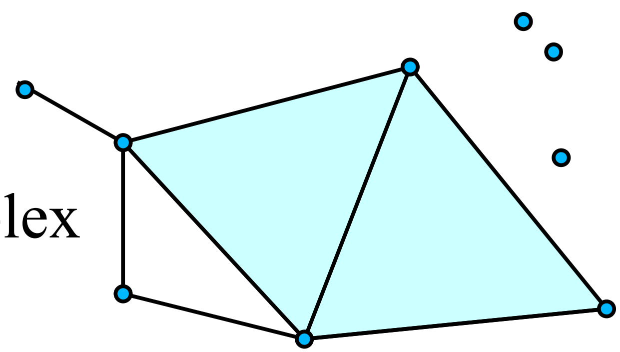


Not Ok

Complesso Simpliciale

- Un simplesso σ è massimale in un complesso simpliciale Σ se non è faccia propria di nessun altro simplesso di Σ
- Un k-complesso simpliciale Σ è massimale se tutti simplessi massimali sono di ordine k
  - In pratica non penzolano pezzi di ordine inferiore

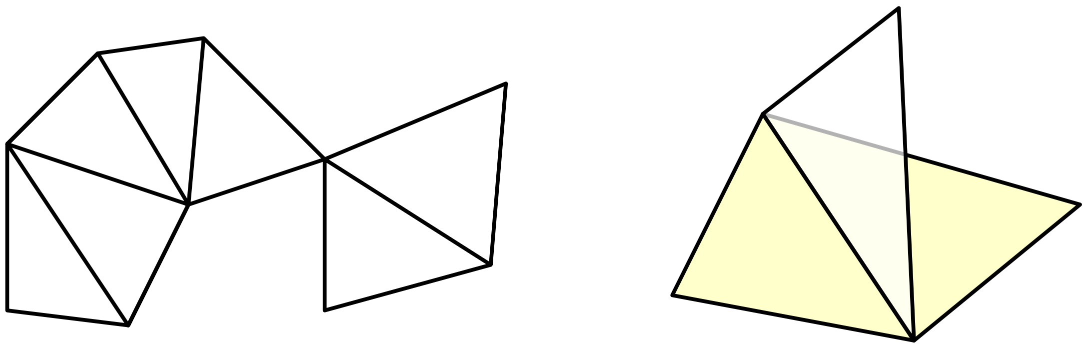
Non maximal 2-simplicial complex



2-Manifold

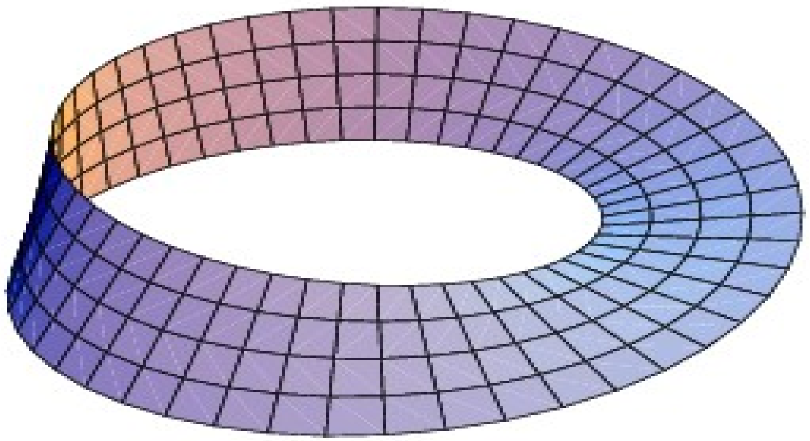
- Una superficie Σ immersa in ***R³*** tale che ogni punto su Σ ha un intorno aperto omeomorfo ad un disco aperto o a un semidisco aperto in ***R²***

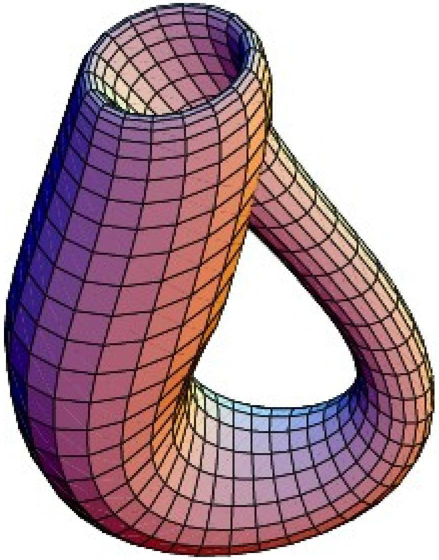
- Esempi non manifold



Orientable

- If it is possible to set a coherent normal to each point of the surface
  - Moebius strips, klein bottles, and non manifold surfaces are not orientable





Mesh

- Le classiche mesh triangolari cui siamo abituati sono 2-complessi simpliciali massimali la cui realizzazione in $\boldsymbol{R}^3$ è una superficie 2-manifold.

- Note:
  - A volte (spesso) capitano superfici non 2-manifold
  - A volte non sono orientabili
  - Che siano massimali invece lo assumiamo
    - e' facile trasformale in massimali distruttivamente...

Topology vs Geometry

- Di un complesso simpliciale e' buona norma distinguere
  - Realizzazione geometrica
    - Dove stanno effettivamente nello spazio i vertici del nostro complesso simpliciale
  - Caratterizzazione topologica
    - Come sono connessi combinatoriamente i vari elementi

Topology vs geometry 2

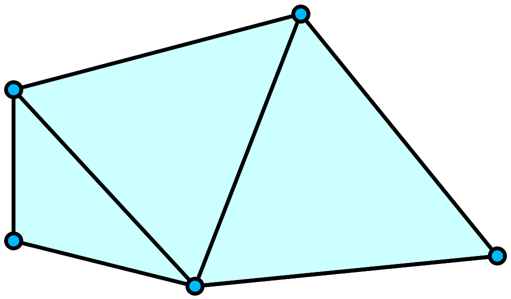
- Nota: Di uno stesso oggetto e' possibile dare rappresentazioni con eguale realizzazione geometrica ma differente caratterizzazione topologica (molto differente!) Demo kleine
- Nota: Di un oggetto si puo' dire molte cose considerandone solo la componente topologica
  - Orientabilita
  - componenti connesse
  - bordi

Cell Complexes

- Esistono anche generalizzazioni di questi concetti basate sul concetto di generici sottoinsiemi di uno spazio legati tra loro in maniera analoga ai simplicial complexes
  - Formalizzazione teorica di mesh non basate su triangoli
  - Il concetto di realizzazione geometrica e' ***molto*** piu' delicato (sono patch generiche in effetti)
  - Noi qui non ne parleremo...

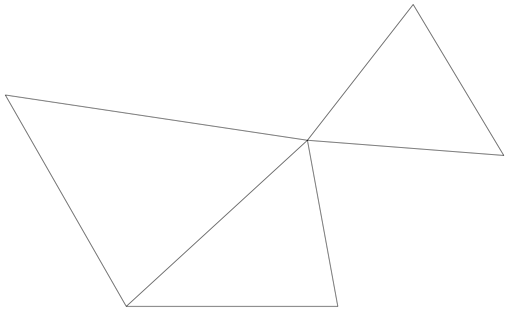
Incidenza Adiacenza

- Due simplessi σ e σ' sono incidenti se □è una faccia propria di □' o vale il viceversa.
- Due k-simplessi sono m-adiacenti (k>m) se esiste un m-simplesso che è una faccia propria di entrambi.
  - Due triangoli che condividono un edge sono 1 -adiacenti
  - Due triangoli che condividono un vertice sono 0-adiacenti



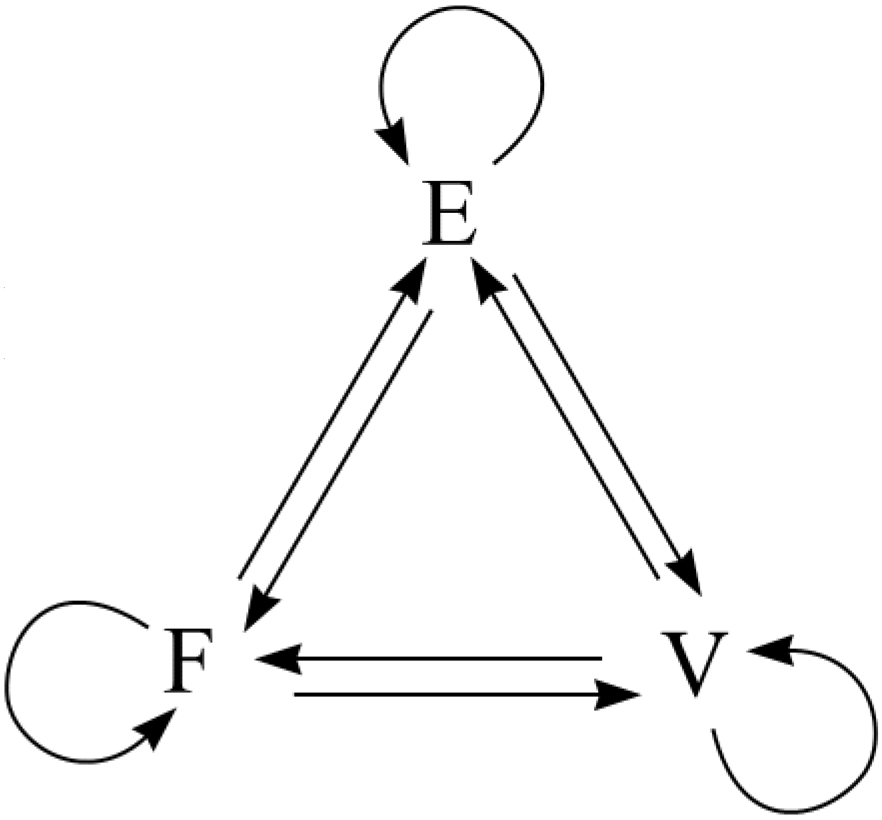
Relazioni di Adiacenza

- Per semplicità nel caso di mesh si una relazione di adiacenza con un una coppia (ordinata!) di lettere che indicano le entità coinvolte
  - FF adiacenza tra triangoli
  - FV i vertici che compongono un triangolo
  - VF i triangoli incidenti su un dato vertice



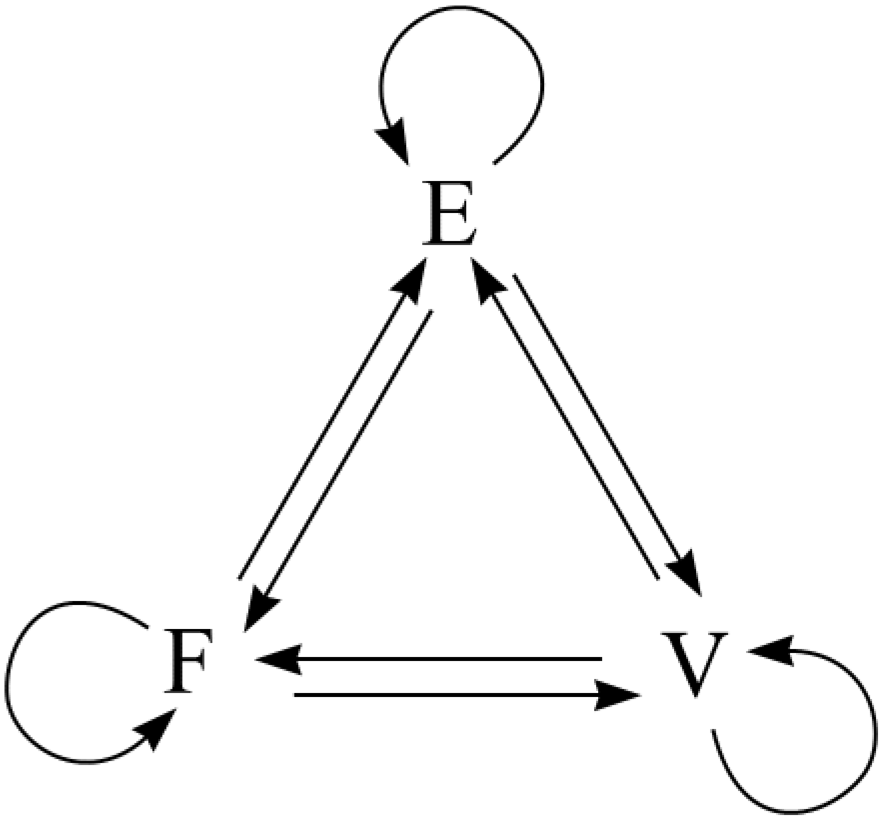
Relazioni di adiacenza

- Di tutte le possibili relazioni di adiacenza di solito vale la pena se ne considera solo un sottoinsieme (su 9 e ricavare le altre proceduralmente



Relazioni di adiacenza

- FF ~ 1-adiacenza
- EE ~ 0 adiacenza
- FE ~ sottofacce proprie di F con dim 1
- FV ~ sottofacce proprie di F con dim 0
- EV ~ sottofacce proprie di E con dim 0
- VF ~ F in Σ : V sub faccia di F
- VE ~ E in Σ : V sub faccia di E
- EF ~ F in Σ : E sub faccia di F
- VV ~ V' in Σ : Esiste E (V,V')



Partial adiacency

- Per risparmiare a volte si mantiene una informazione di adiacenza parziale
  - VF* memorizzo solo un riferimento dal vertice ad una delle facce e poi 'navigo' sulla mesh usando la FF per trovare le altre facce incidenti su V

Relazioni di adiacenza

- In un 2-complesso simpliciale immerso in R3, che sia 2 manifold
  - FV FE FF EF EV sono di cardinalità bounded (costante nel caso non abbia bordi)
    - |FV|= 3 |EV| = 2 |FE| = 3
    - |FF| <= 3
    - |EF| <= 2
  - VV VE VF EE sono di card. variabile ma in stimabile in media
    - |VV|~|VE|~|VF|~6
    - |EE|~10
    - F ~ 2V

Euler characteristic

- Invariante topologico

$$\chi = V - E + F$$
$$\chi = |\Sigma_0| - |\Sigma_1| + |\Sigma_2| - \ldots$$

- Per tutto quello omeomorfo ad una sfera

$$\chi = 2$$

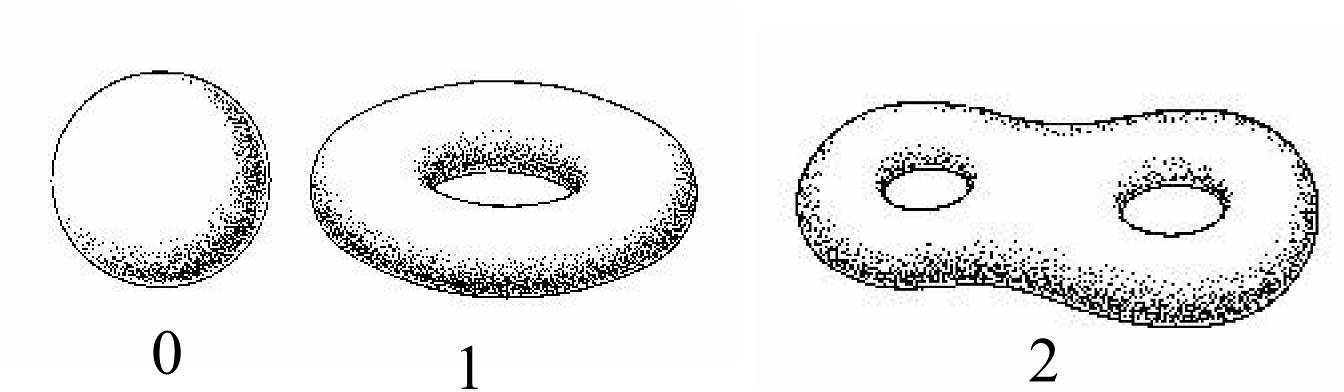
- In generale per una sup qualsiasi

$$\chi = 2 - 2g$$

- Con g genus della superficie

Genus

- Il genus di una superficie chiusa, orientabile 2manifold: e`il massimo numero di tagli lungo curve chiuse semplici che si possono fare senza rendere l'insieme sconnesso



- Per i profani numero di maniglie sulla superficie

Euler characteristic

- Se la superficie non e' chiusa

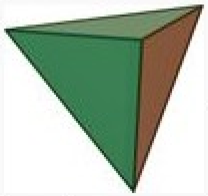
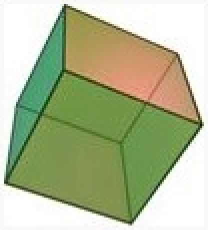
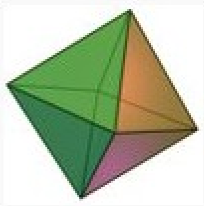
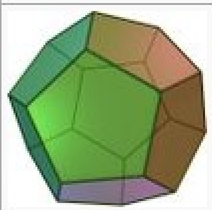
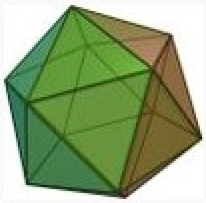
$$\chi = 2 - 2g - B$$

- Dove B e' il numero di bordi
  - (non di elementi sul bordo)

- Per multiple connected surfaces

$$\chi = 2C - 2\sum g_i$$

- Con C numero delle componenti connesse

Name	Image	*V* (vertices)	*E* (edges)	*F* (faces)	Euler characteristic: *V* − *E* + *F*
Tetrahedron		4	6	4	**2**
Hexahedron or cube		8	12	6	**2**
Octahedron		6	12	8	**2**
Dodecahedron		20	30	12	**2**
Icosahedron		12	30	20	**2**

Example data structure

- Simplest
- List of triangles:
  - For each triangle store its coords.
  -
  - 1. (3,-2,5), (3,6,2), (-6,2,4)
  - 2. (2,2,4), (0,-1,-2), (9,4,0)
  - 3. (1,2,-2), (8,8,7), (-4,-5,1)
  - 4. (-8,2,7), (-2,3,9), (1,2,-7)
- How to find any adjacency?
- Does it store FV?

Example data structure

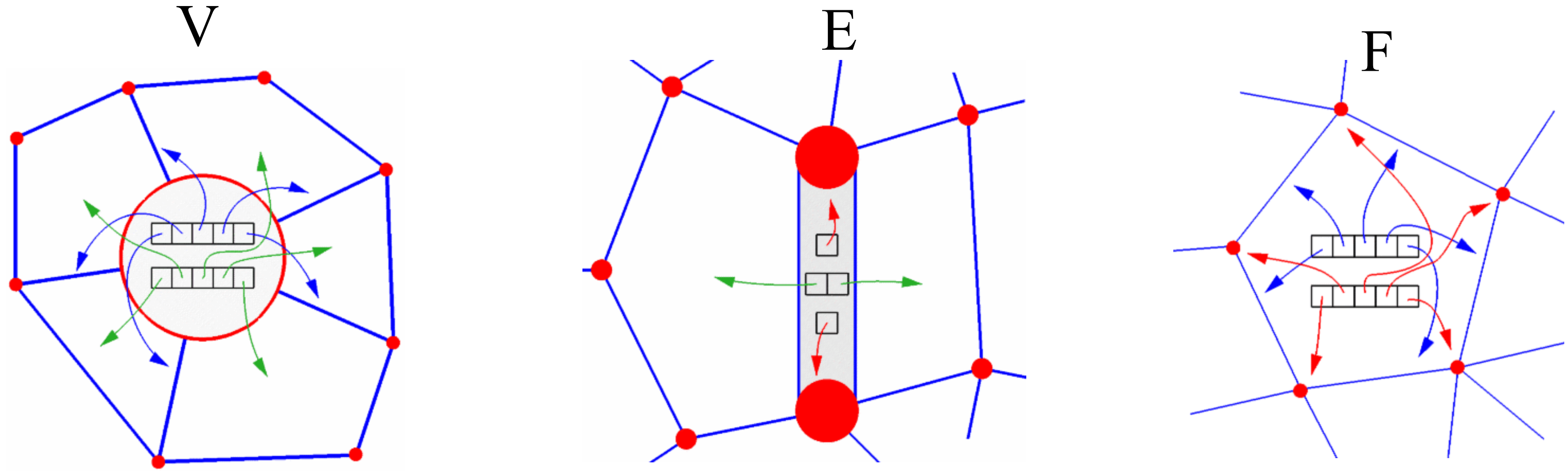
- Slightly better
- List of unique vertices with indexed faces
  - Storing the FV relation

- Vertices:
- 1. (-1.0, -1.0, -1.0)
- 2. (-1.0, -1.0, 1.0)
- 3. (-1.0, 1.0, -1.0)
- 4. (-1, 1, 1.0)
- 5. (1.0, -1.0, -1.0)
- 6. (1.0, -1.0, 1.0)
- 7. (1.0, 1.0, -1.0)
- 8. (1.0, 1.0, 1.0)

- Faces:
- 1. 1 2 4
- 2. 5 7 6
- 3. 1 5 2
- 4. 3 4 7
- 5. 1 7 5

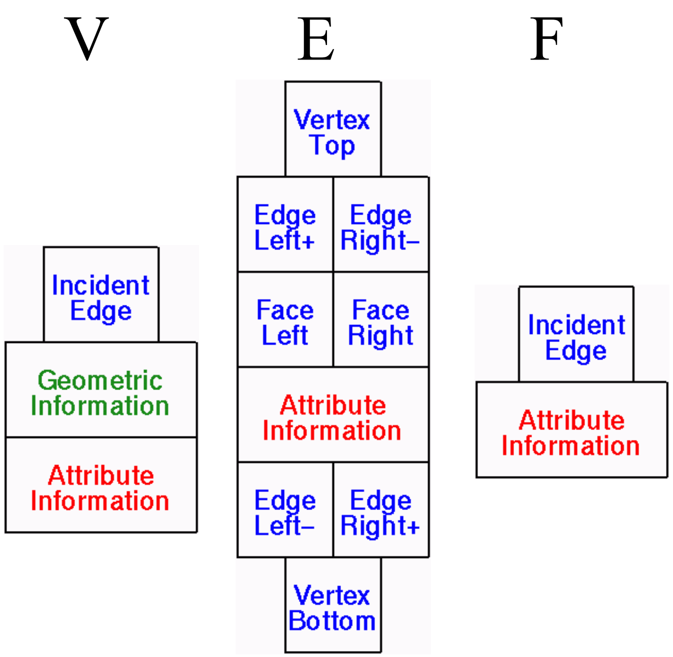
Example data structure

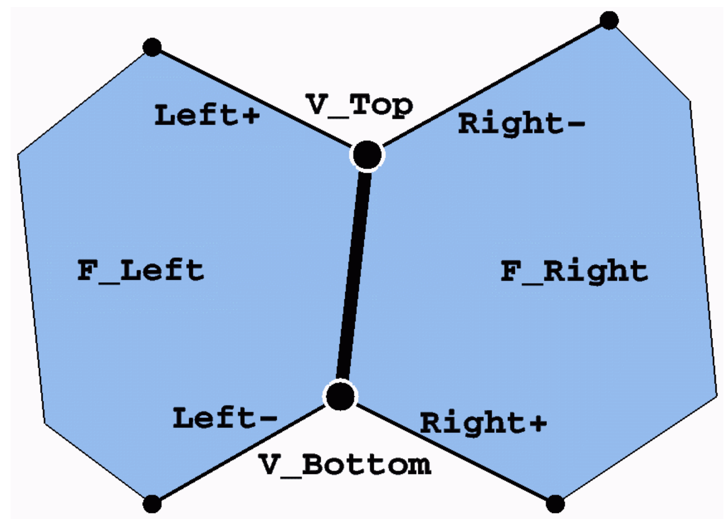
- Issue of Adjacency
  - Vertex, Edge, and Face Structures
    - Each element has list of pointers to all incident elements
    - Queries depend only on local complexity of mesh!
    - Slow! Big! Too much work to maintain!
    - Data structures do not have fixed size



Example data structure

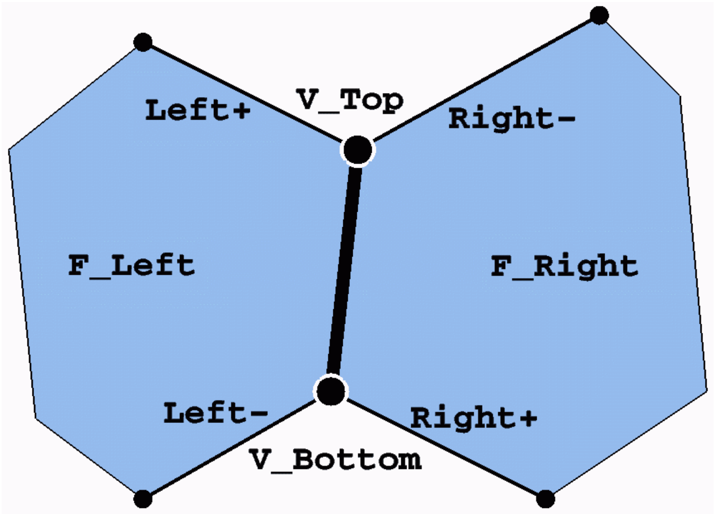
- Winged edge
  - Classical real smart structure
  - Nice for generic polygonal meshes
  - Used in many sw packages





Winged Edge

- Winged edge
  - Compact
  - All the query requires some kind of "traversal"
  - Not fitted for rendering...



What is a mesh processing algorithm?

- Finding the border
  - Using various Adjacency relations
- Filling holes
- Cutting a mesh
- Finding Corners or computing curvature
- Remove noise
- Deform the mesh
- Remesh
  - Etc etc

Designing data structures

- Data
  - What i am going to keep behind to represent all the information
    - Redundancy vs efficiency
    - Explicit vs implict
- Iterator/circulator
  - How can i access to my mesh
    - Navigating your mesh
    - What is a *position* over a mesh

The goal

- A framework to implement algorithms on Simplicial Complexes of order d=0..3 in R^n:
  - Efficient code
  - Easy to understand
  - Flexible
  - Reusable

Representing Simplicial Complexes

- A good problem.
- Meshes requires different information for different algorithms and purposes
  - Topology
  - Different geometric informations
  - Additional datas

- Templated solutions.
  - Generic algorithms on generic meshes

Vertex

- What is a vertex?
  - position in n-space (almost always)
  - normal
  - color
  - quality
  - quadric
  - connectivity information (topology)
  - ...
- One may want any combination of attributes

Vertex (cntd)

- How to do it?
  - every user derives an empty VertexBase class to implement the vertex type
    - annoying cut and paste of code
    - need to agree upon interface (name of access functions)
    - potentially memory consuming (memory padding)
  - multiple inheritance: not well supported in old compilers. It could be done now, but still dangerous sometimes...

vertex (cntd)

- Classical MetaProgramming approach
  - Build a compile time a linear derivation chain from a set of attributes
  - All the desired attributes are passed as template class to the vertex:

```
typedef VertexSimp1< Vertex0,EdgeProto,
        vcg::vert::VFAdj,  vcg::vert::Normal3f,  vcg::vert::Color4b> MyVertex;
```

- The template parameters can be passed in any order
- More elegant
- Much more complex implementation
- Better typed: ex. the normal and the position have different type even if their structure is the very same

Complexes

- PointSet
- EdgeMesh
- TriMesh
- TetraMesh
- As for simplices, they could be:

```
template <int order,....> Complex{...};
```

- but they are not (at the moment)

Complex (ex: TriMesh)

- A complex is just a collection of simplices

```
template < class VertContainerType, class FaceContainerType >
class TriMesh{
        public:

        /// Set of vertices
        VertContainer vert;
        /// Real number of vertices
        int vn;
        /// Set of faces
        FaceContainer face;
        /// Real number of faces
        int fn;
        ...
        };
```

- Max and min order simplices are the only ones explicitly kept

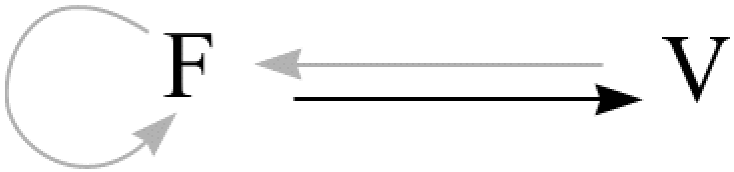
containers

- Usually vectors
  - Most of the code works also with other generic containers
- Lazy deletion strategy
  - Object that have to be deleted are just marked and purged away later...
    - SetD() IsD() function over simplces
- No edges.
  - Only maximal complexes are usually kept
  - It could be discussed...

Complex

- Topological relations stored optionally inside the simplexes
- Each Simplex knows its own geometric realization

  (a face contains pointers instead of indexes)

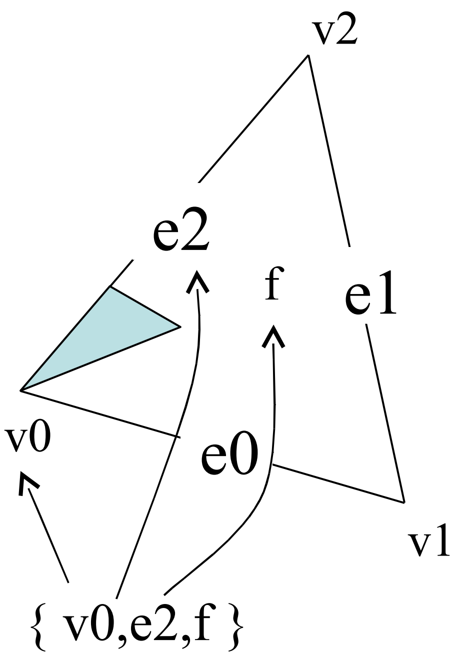


Surfing a the mesh (I)

- All based on the concept of `pos` (`pos`ition)
- a `pos` is a d-tuple:

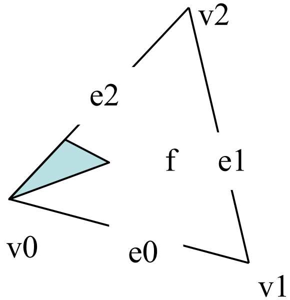
$$p = \{ s_0,\ldots,s_d \}$$

such that each s_i is a reference to a d-simplex For triangle meshes is a triple of references to
vertex, edge, face



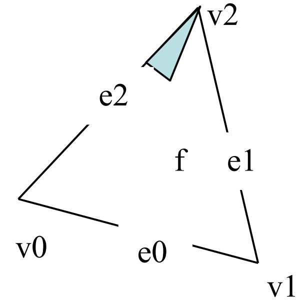
Pos

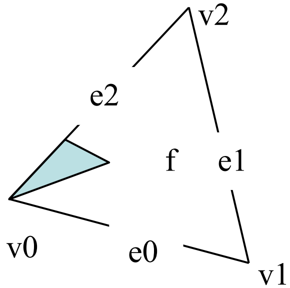
- any component of a pos can be changed only into another value to obtain another pos



p.FlipV()

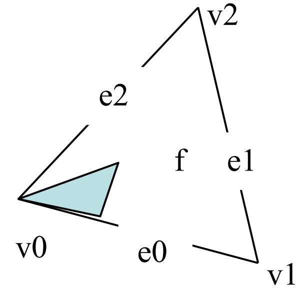
{v0,e2,f} → {v2,e2,f}

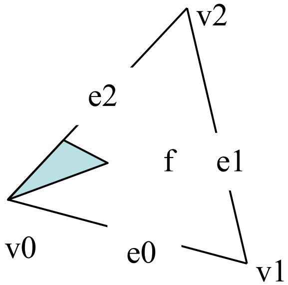




p.FlipE()

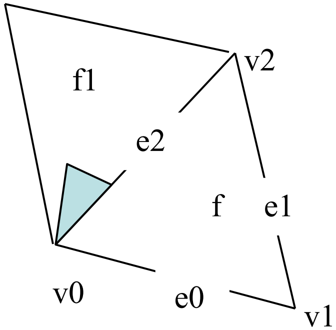
{v0,e2,f} → {v0,e0,f}





p.FlipF()

{v0,e2,f} → {v2,e2,f1}

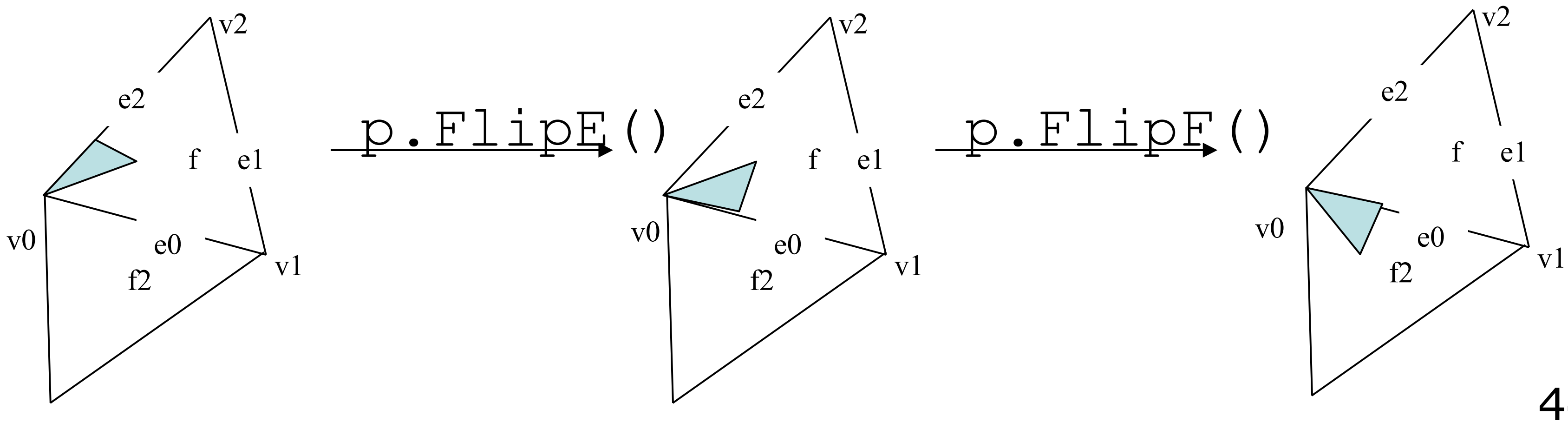


Pos

- Example: running over the faces around a vertex

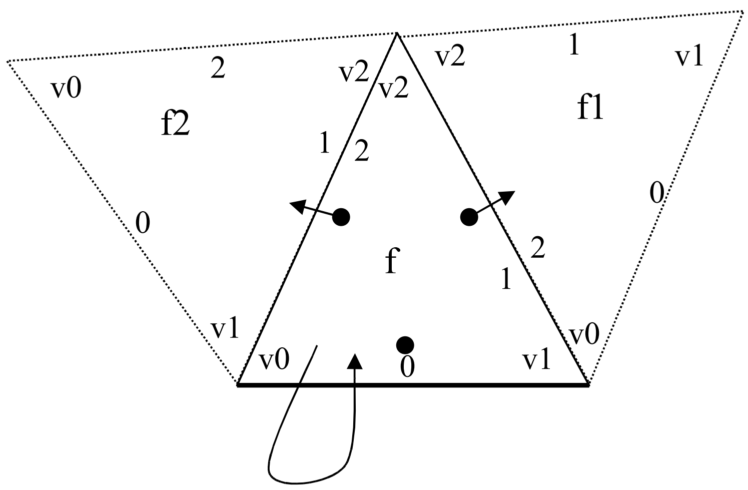
`<vcg/simplex/face/pos.h>`

```
template <typename FaceType>
class Pos {
    ...
        void NextE()
        {
            assert( f->V(z)==v || f->V((z+1)%3)==v );
            FlipE();
            FlipF();
            assert( f->V(z)==v || f->V((z+1)%3)==v );
        }
    ...
};
```



FF Implementation

- Three pointers to face and three integers in each face:




```
f.FFp(1)==&f1
f.FFi(1) == 2

f.FFp(2) == &f2
f.FFi(2) == 1

f.FFp(0) == &f
f.FFi(0) == -1

f.FFp(i)->f.FFp(f.FFi(i)) == &f
```

FF implementation

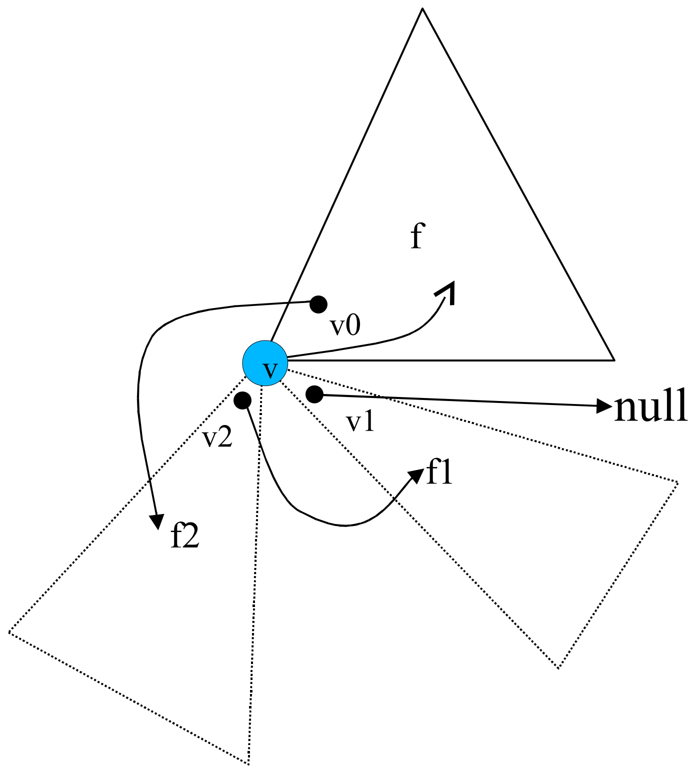
- Works also for non manifold situations.
  - The pointers in the FF forms a circular list ordered, monodirectional list.

  - It does not hold anymore that

    `f.FFp(i)->f.FFp(f.FFi(i)) == &f`
  - The pos flip property do not hold any more...

VF Implementation

- The list is distributed over the involved faces: No dynamic allocation




```
v.VFp()=&f
v.VFi()=0

f.VFp(0)==&f2
f.VFi(0) == 2

f2.VFp(2) == &f2
f2.VFi(2) == 1

f1.VFp(1) == null
f1.VFi(1) == -1
```

References

- http://vcg.sf.net
  - A wiki with tutorials
  - Feel free to spot out missing parts.
- vcglib/apps/samples
  - A set of more or less simple examples.