Fondamenti di Grafica 3D

The Rasterization Pipeline

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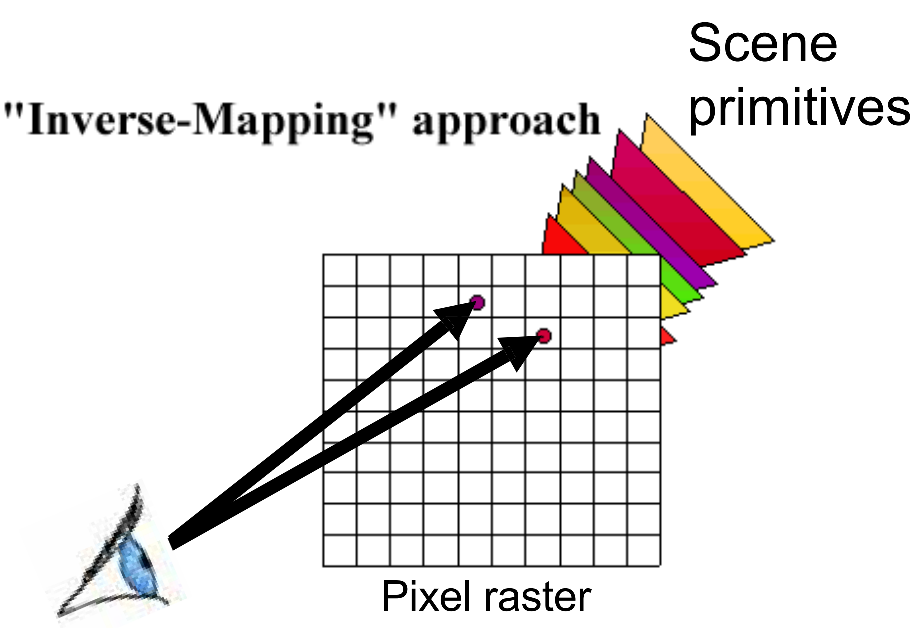
Ray Casting vs. GPUs for Triangles

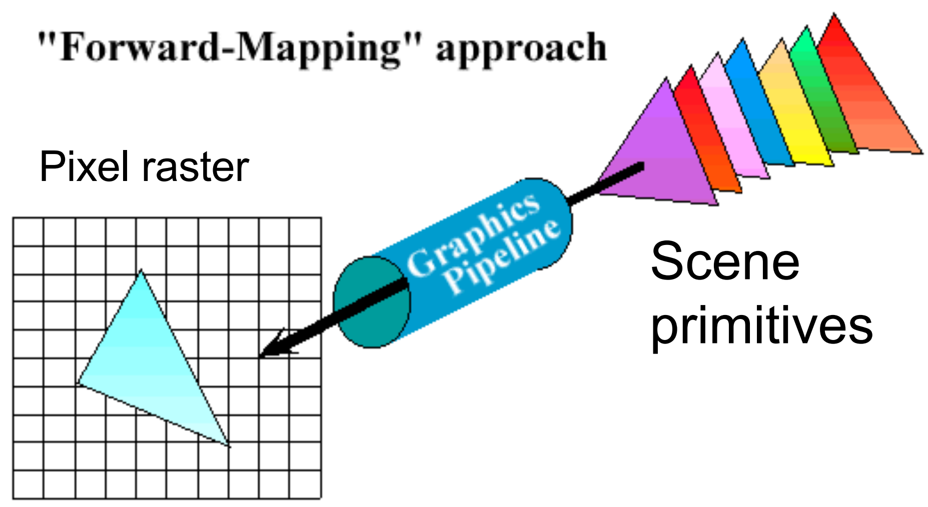
Ray Casting

```
For each pixel (ray)
   For each triangle
      Does ray hit triangle?
```
Keep closest hit

GPU

```
For each triangle
   For each pixel
      Does triangle cover pixel?
```
Keep closest hit





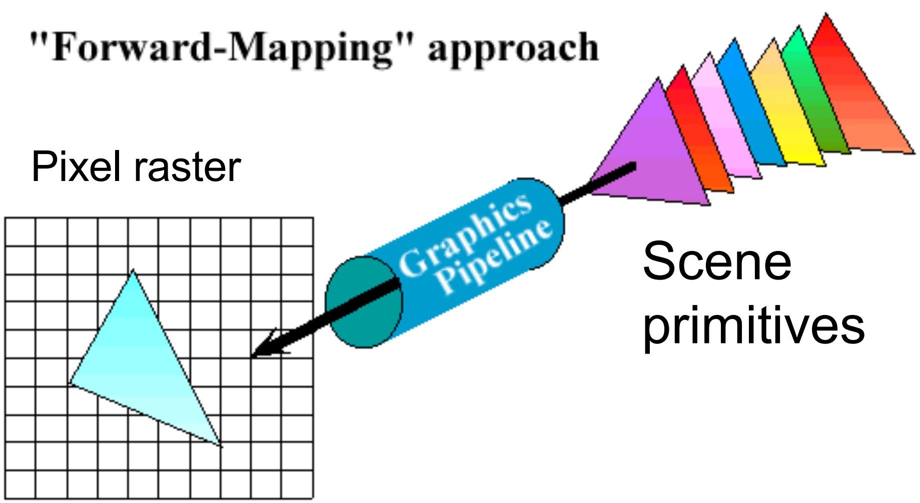
GPUs do Rasterization

- The process of taking a triangle and figuring out which pixels it covers is called **rasterization**

GPU

```
For each triangle
   For each pixel
      Does triangle cover pixel?
```

Keep closest hit

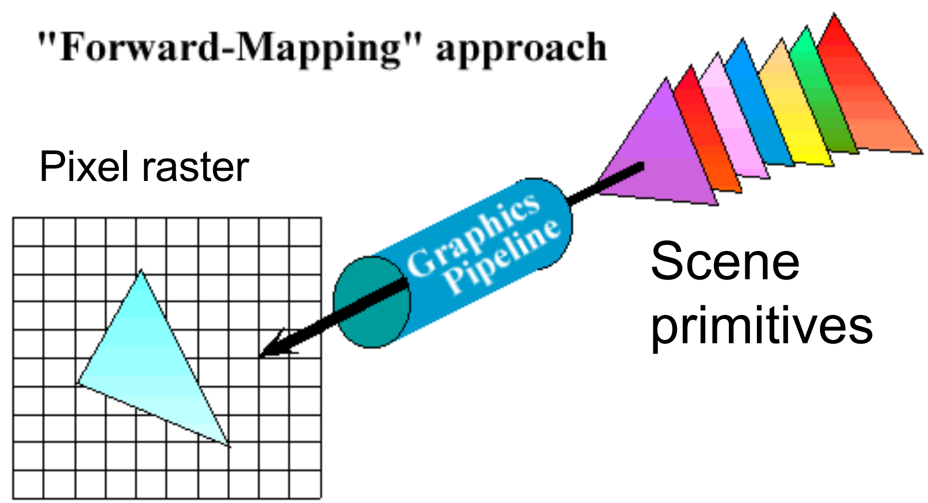


GPUs do Rasterization

- The process of taking a triangle and figuring out which pixels it covers is called **rasterization**
- We will see acceleration structures for ray tracing; rasterization is not stupid either
  - We're not actually going to test *all* pixels for each triangle

GPU

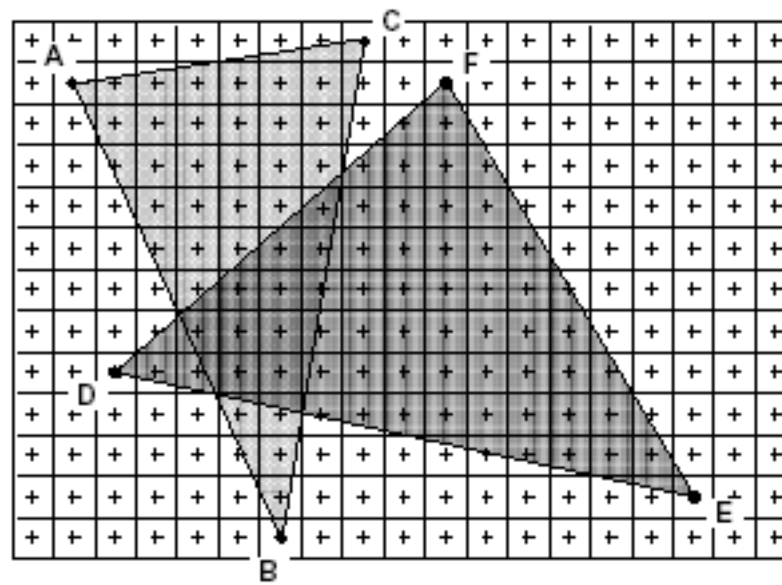
```
For each triangle
   For each pixel
      Does triangle cover pixel?
      Keep closest hit
```



Rasterization ("Scan Conversion")

- Given a triangle's vertices & extra info for shading, figure out which pixels to "turn on" to render the primitive
- Compute illumination values to "fill in" the pixels within the primitive
- At each pixel, keep track of the closest primitive (z-buffer)
  - Only overwrite if triangle being drawn is closer than the previous triangle in that pixel

```
glBegin(GL_TRIANGLES)
glNormal3f(...)
glVertex3f(...)
glVertex3f(...)
glVertex3f(...)
glEnd();
```



What are the Main Differences?

Ray Casting

```
For each pixel (ray)
   For each triangle
      Does ray hit triangle?
```

Keep closest hit

Ray-centric

GPU

```
For each triangle
   For each pixel
      Does triangle cover pixel
```

Keep closest hit

Triangle-centric

- What needs to be stored in memory in each case?

What are the Main Differences?

Ray Casting

```
For each pixel (ray)
   For each triangle
      Does ray hit triangle?
```

Keep closest hit

GPU

```
For each triangle
   For each pixel
      Does triangle cover pixel
```

Keep closest hit

- In this basic form, ray tracing needs the entire scene description in memory at once
  - Then, can sample the image completely freely
- The rasterizer only needs one triangle at a time, *plus* the entire image and associated depth information for all pixels

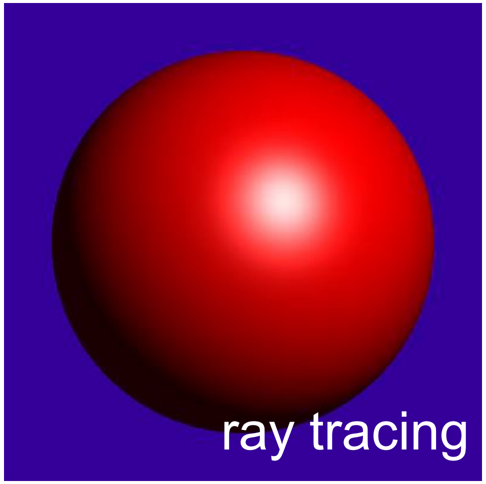
Rasterization Advantages

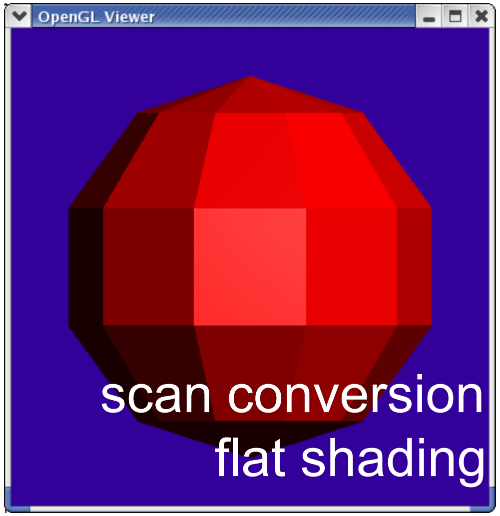
- Modern scenes are more complicated than images
  - A 1920x1080 frame (1080p) at 64-bit color and 32-bit depth per pixel is 24MB (not that much)
    - Of course, if we have more than one sample per pixel (later) this gets larger, but e.g. 4x supersampling is still a relatively comfortable ~100MB
  - Our scenes are routinely larger than this
    - This wasn't always true

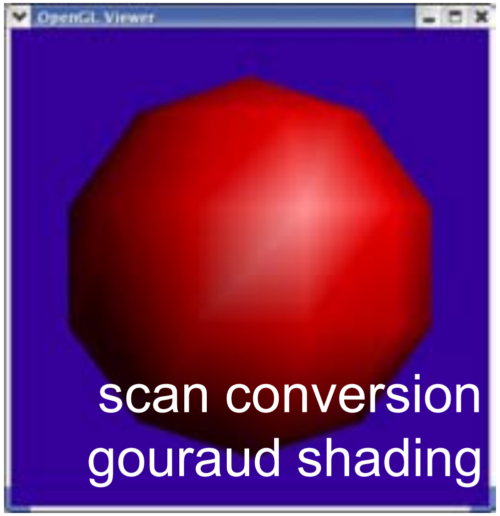
- A rasterization-based renderer can *stream* over the triangles, no need to keep entire dataset around
  - Allows parallelism and optimization of memory systems

Rasterization Limitations

- Restricted to scan-convertible primitives
  - Pretty much: triangles
- Faceting, shading artifacts
  - This is largely going away with programmable per-pixel shading, though
- No unified handling of shadows, reflection, transparency
- Potential problem of overdraw (high depth complexity)
  - Each pixel touched many times



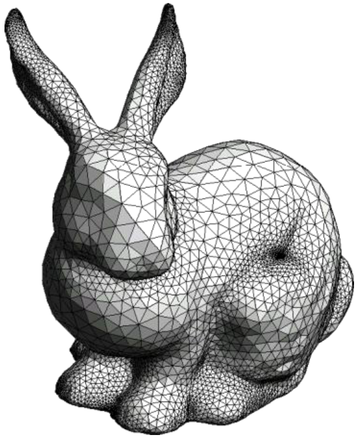




Ray Casting / Tracing

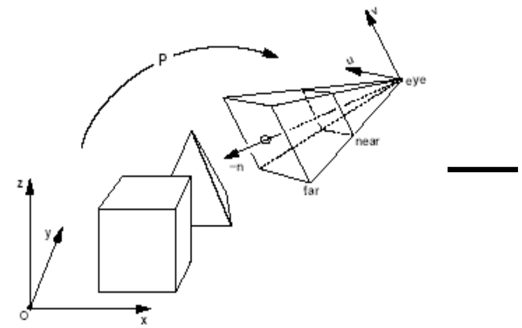
- Advantages
  - Generality: can render anything that can be intersected with a ray
  - Easily allows recursion (shadows, reflections, etc.)
- Disadvantages
  - Hard to implement in hardware (lacks computation coherence, must fit entire scene in memory, bad memory behavior)
    - Not such a big point any more given general purpose GPUs
  - Has traditionally been too slow for interactive applications
  - Both of the above are changing rather rapidly right now!

Modern Graphics Pipeline

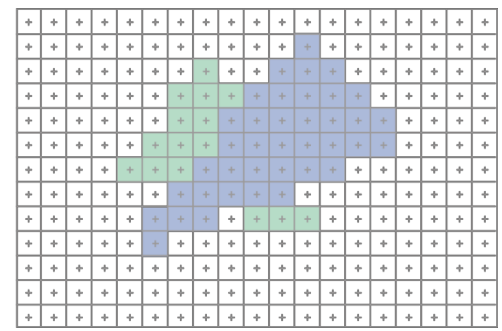


- Input
  - Geometric model
    - Triangle vertices, vertex normals, texture coordinates
  - Lighting/material model (shader)
    - Light source positions, colors, intensities, etc.
    - Texture maps, specular/diffuse coefficients, etc,
  - Viewpoint + projection plane
- Output
  - Color (+depth) per pixel

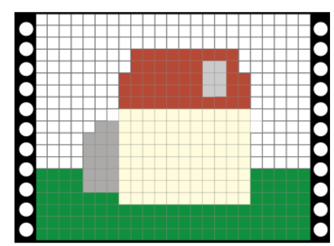
Modern Graphics Pipeline

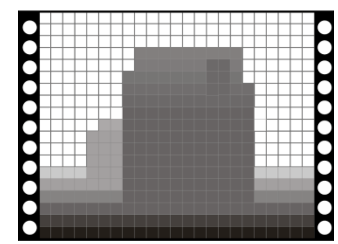


- Project vertices to 2D (image)

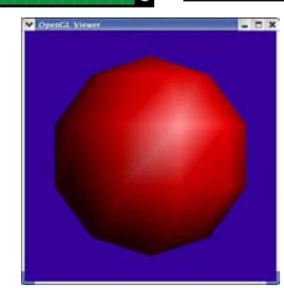


- Rasterize triangle: find which pixels should be lit



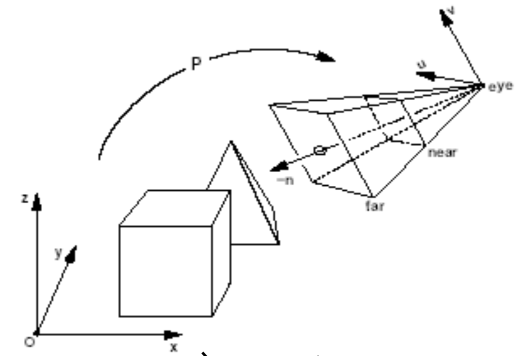


- Test visibility (Z-buffer), update frame buffer color

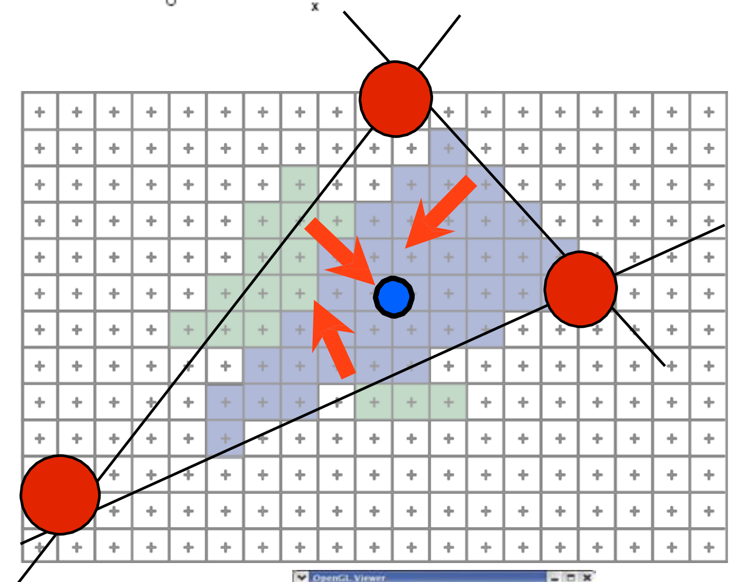


- Compute per-pixel color

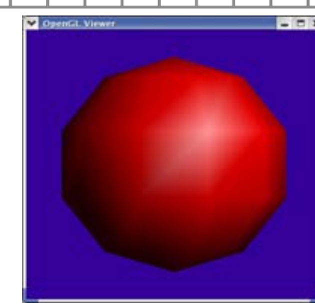
Modern Graphics Pipeline



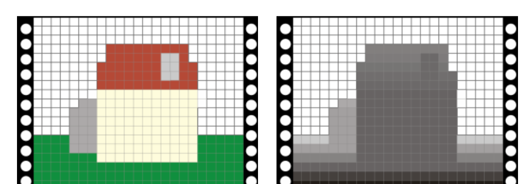
- Project vertices to 2D (image)



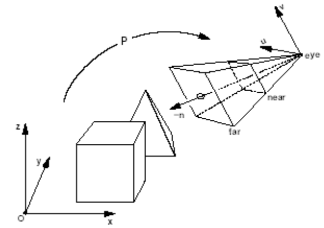
- Rasterize triangle: find which pixels should be lit
  - For each pixel, test 3 edge equations
    - if all pass, draw pixel



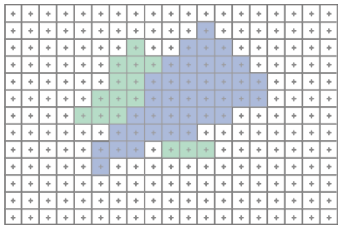
- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer color



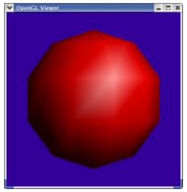
Modern Graphics Pipeline



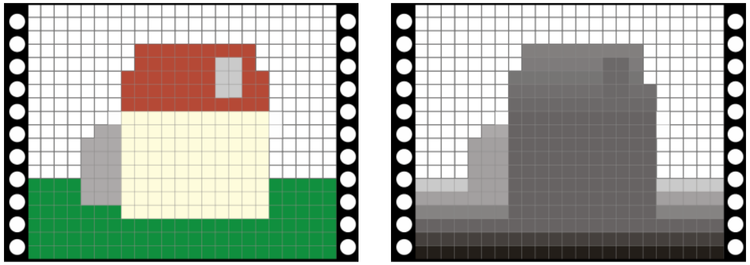
- Perform projection of vertices
- Rasterize triangle: find which pixels should be lit



- Compute per-pixel color



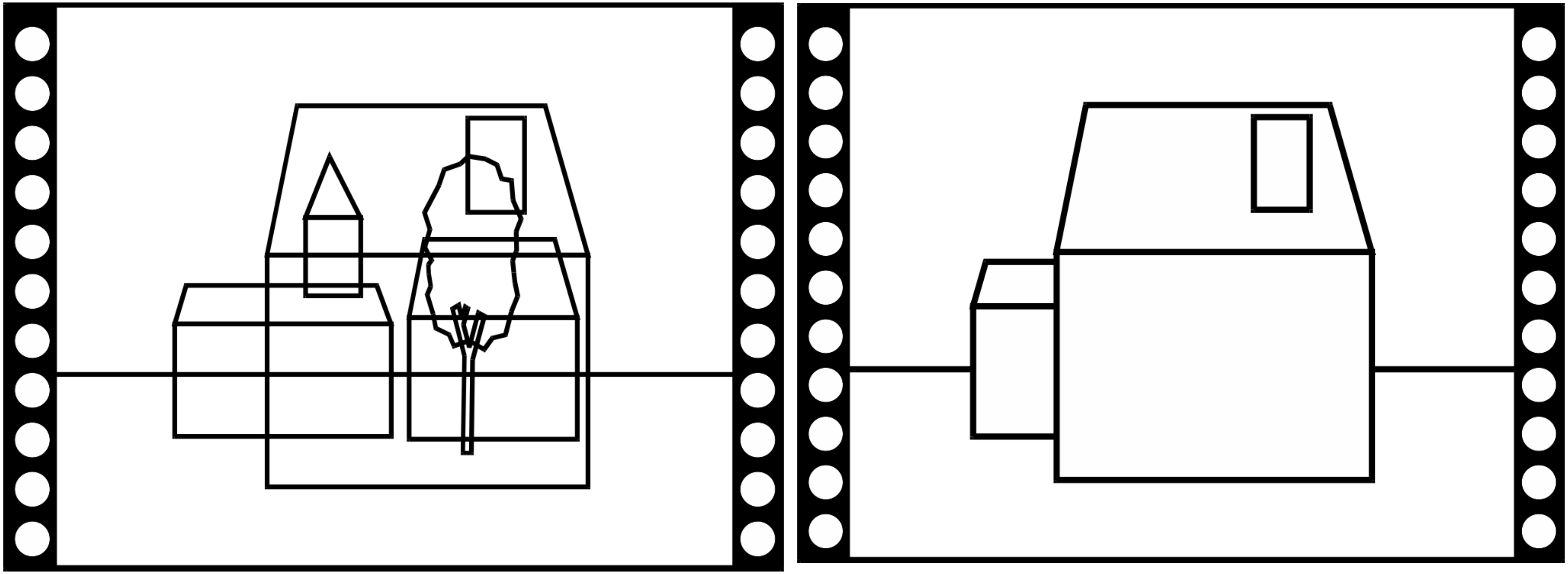
- Test visibility, update frame buffer color
  - Store minimum distance to camera for each pixel in "Z-buffer"
    - ~same as t_{min} in ray casting!
  - **if** *newz* < *zbuffer*[*x,y*]
    zbuffer[x,y]=new_z
    framebuffer[x,y]=new_color



frame buffer Z buffer

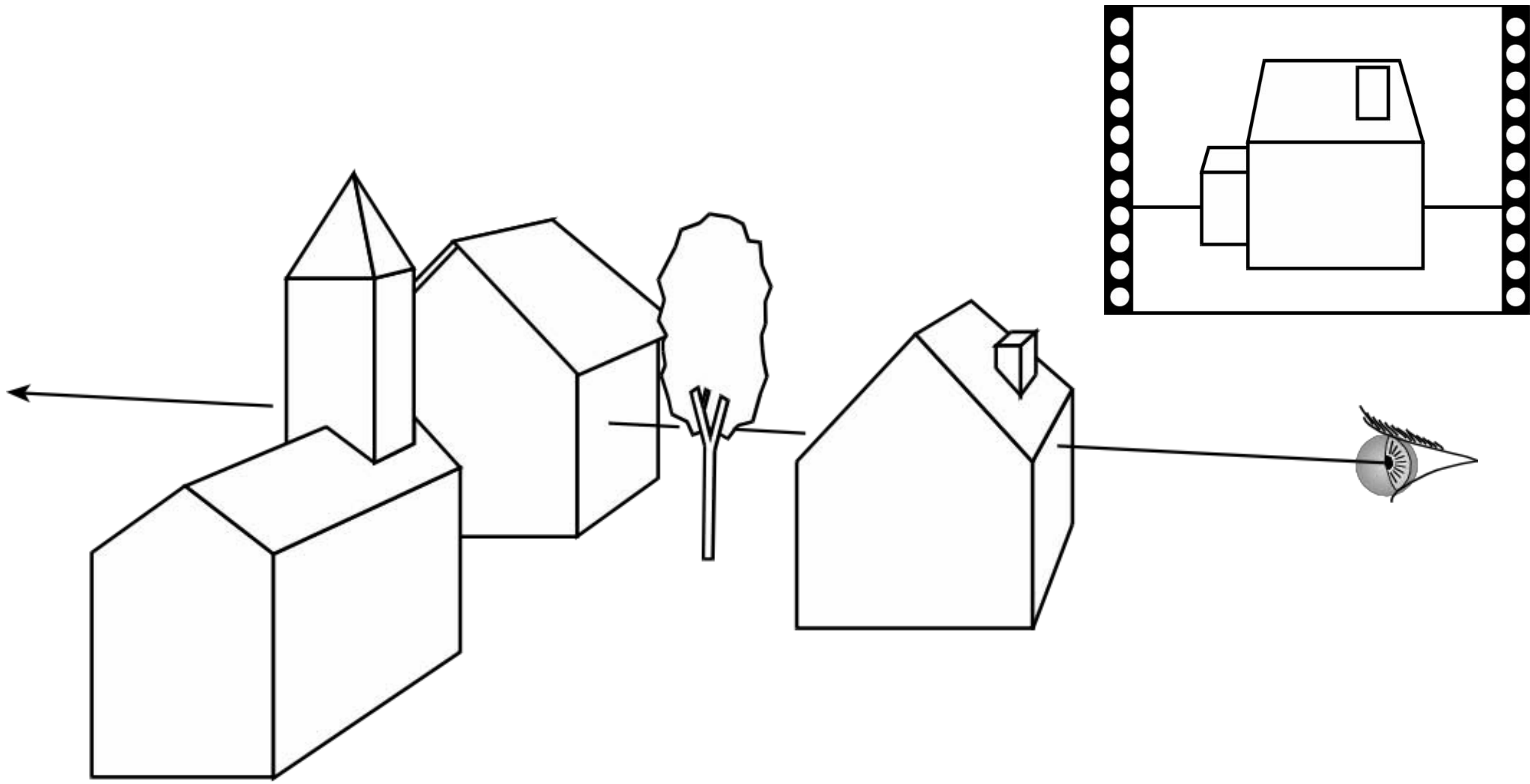
Note sul problema della visibilità

- How do we know which parts are visible/in front?



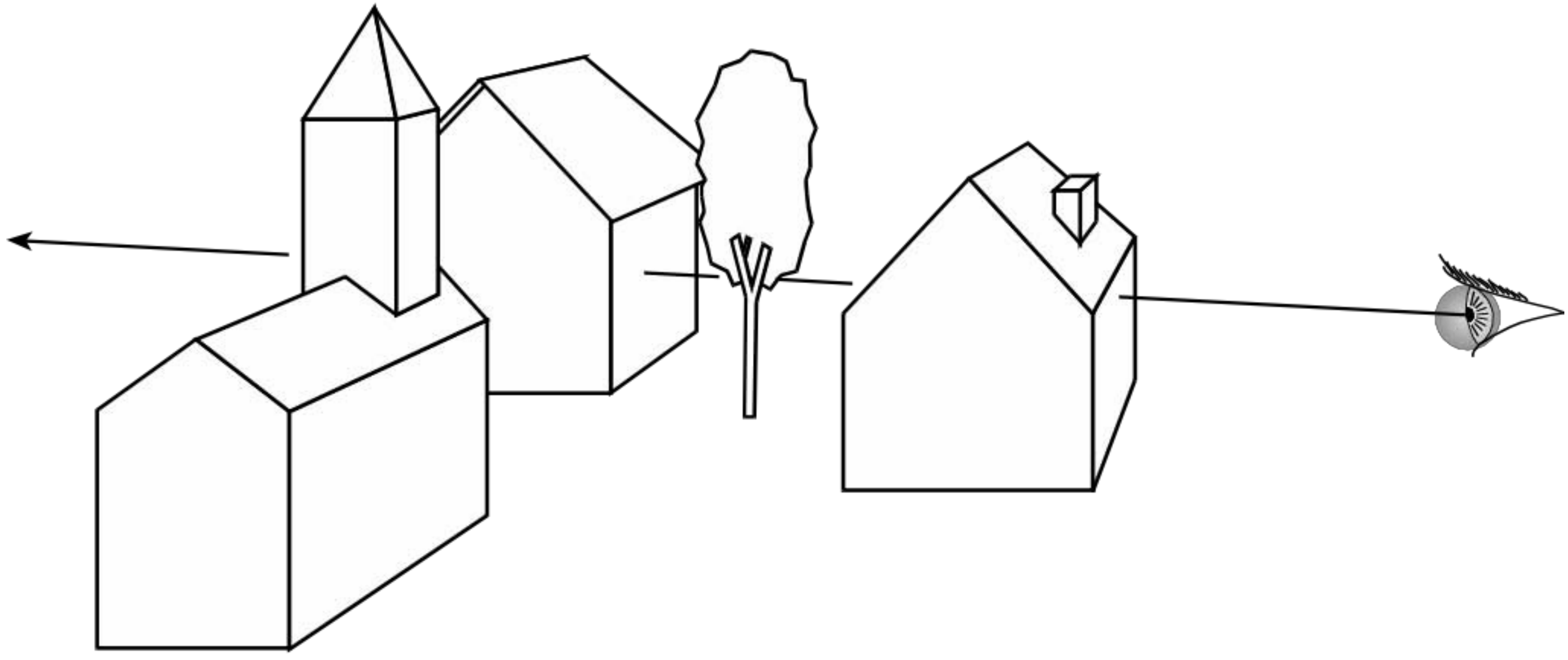
Ray Casting

- Maintain intersection with closest object



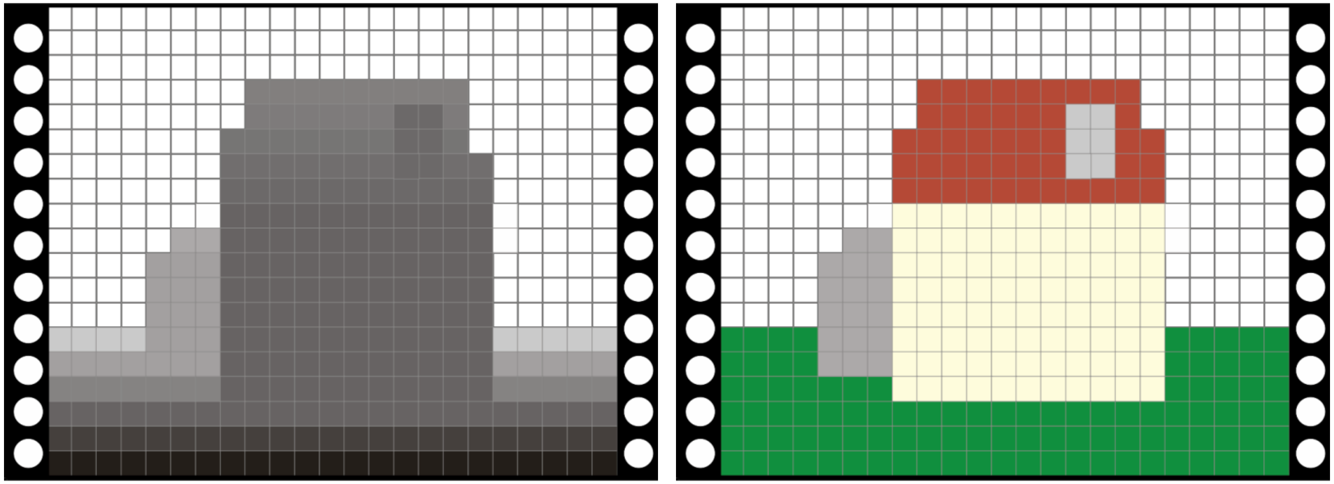
Visibility

- In ray casting, use intersection with closest *t*
- Now we have swapped the loops (pixel, object)
- What do we do?



Z buffer

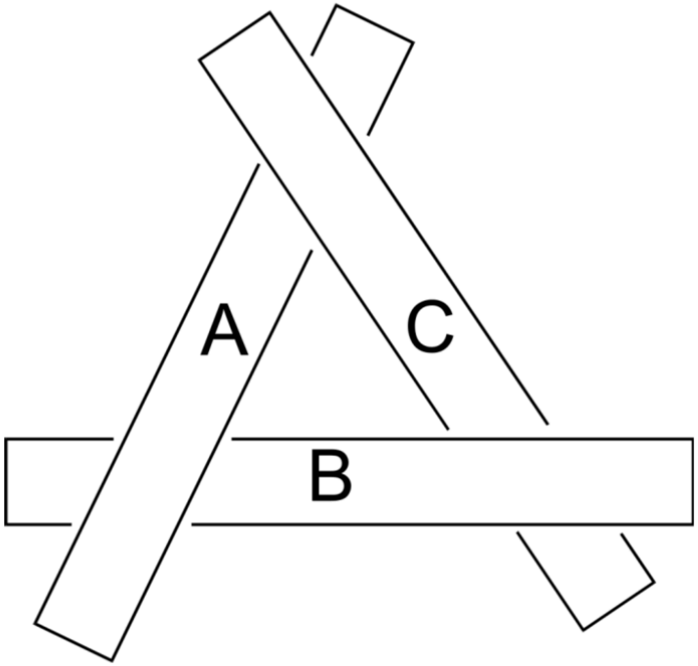
- In addition to frame buffer (R, G, B)
- Store distance to camera (*z*-buffer)
- Pixel is updated only if *newz* is closer than *z*-buffer value

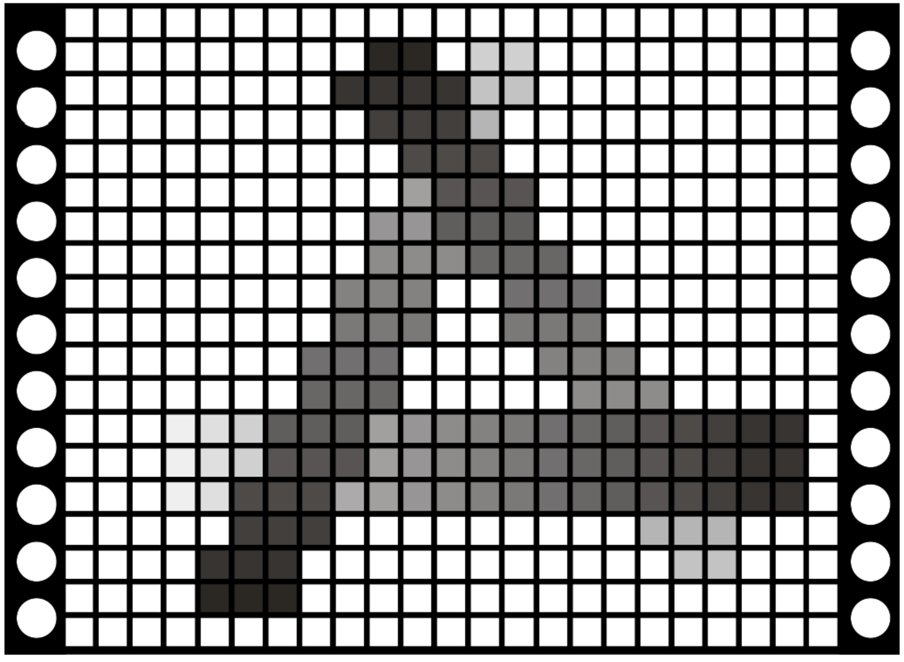


Z-buffer pseudo code

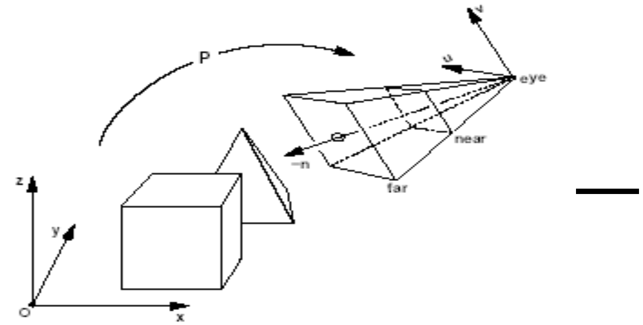
```
For every triangle
   Compute Projection, color at vertices
   Setup line equations
   Compute bbox, clip bbox to screen limits
   For all pixels in bbox
      Increment line equations
      Compute curentZ
      Compute currentColor
      If all line equations>0 //pixel [x,y] in triangle
         If currentZ<zBuffer[x,y] //pixel is visible
            Framebuffer[x,y]=currentColor
            zBuffer[x,y]=currentZ
```

Works for hard cases!

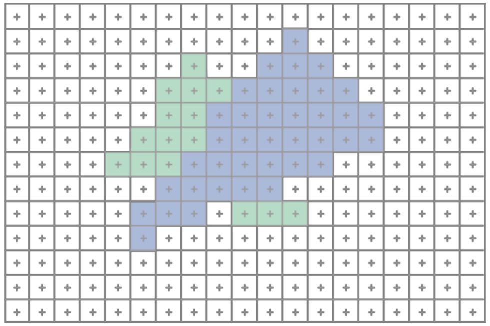




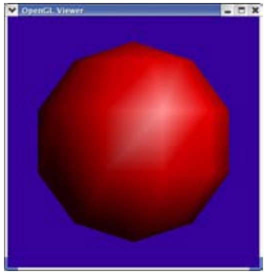
Projection



- Project vertices to 2D (image)

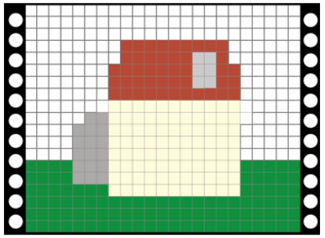


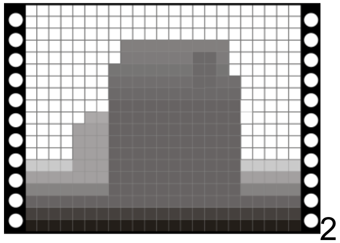
- Rasterize triangle: find which pixels should be lit



- Compute per-pixel color

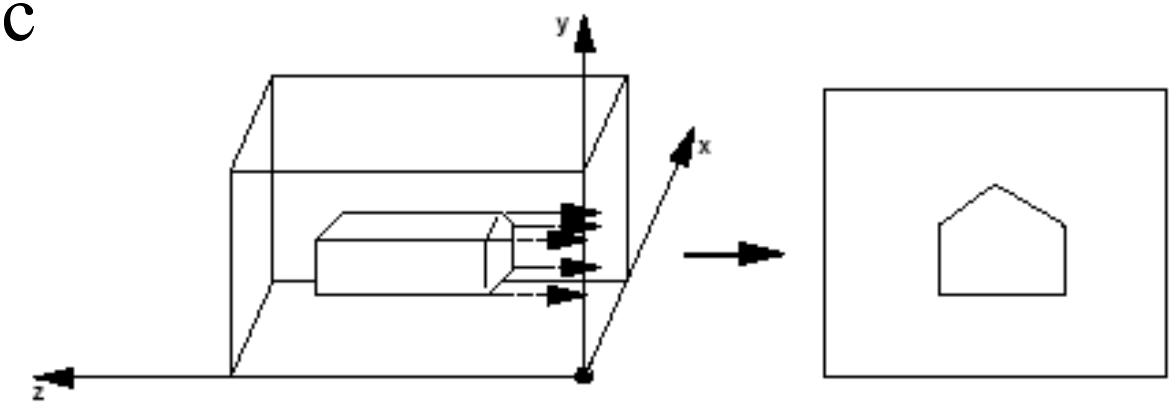
- Test visibility (Z-buffer), update frame buffer



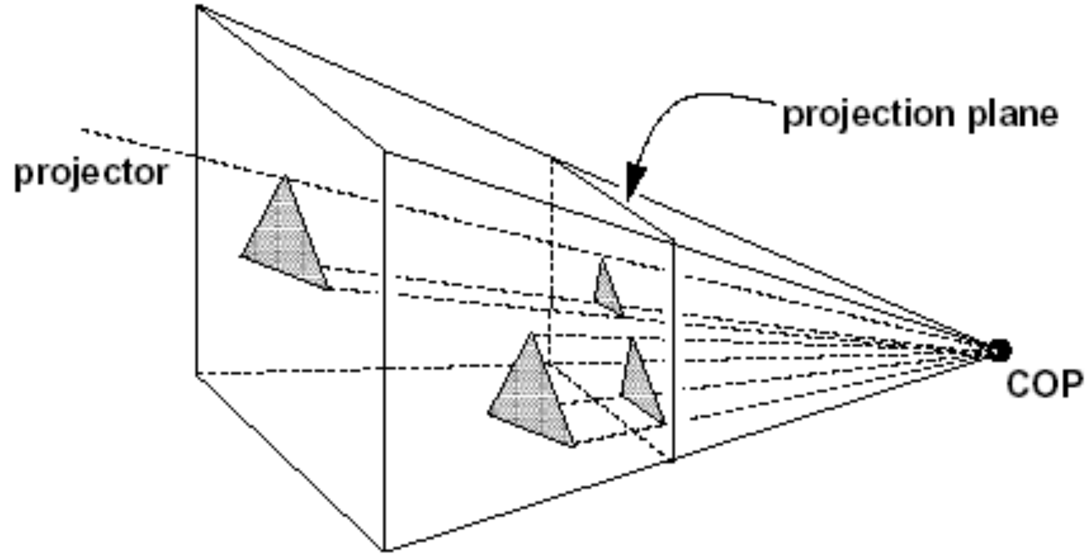


Orthographic vs. Perspective

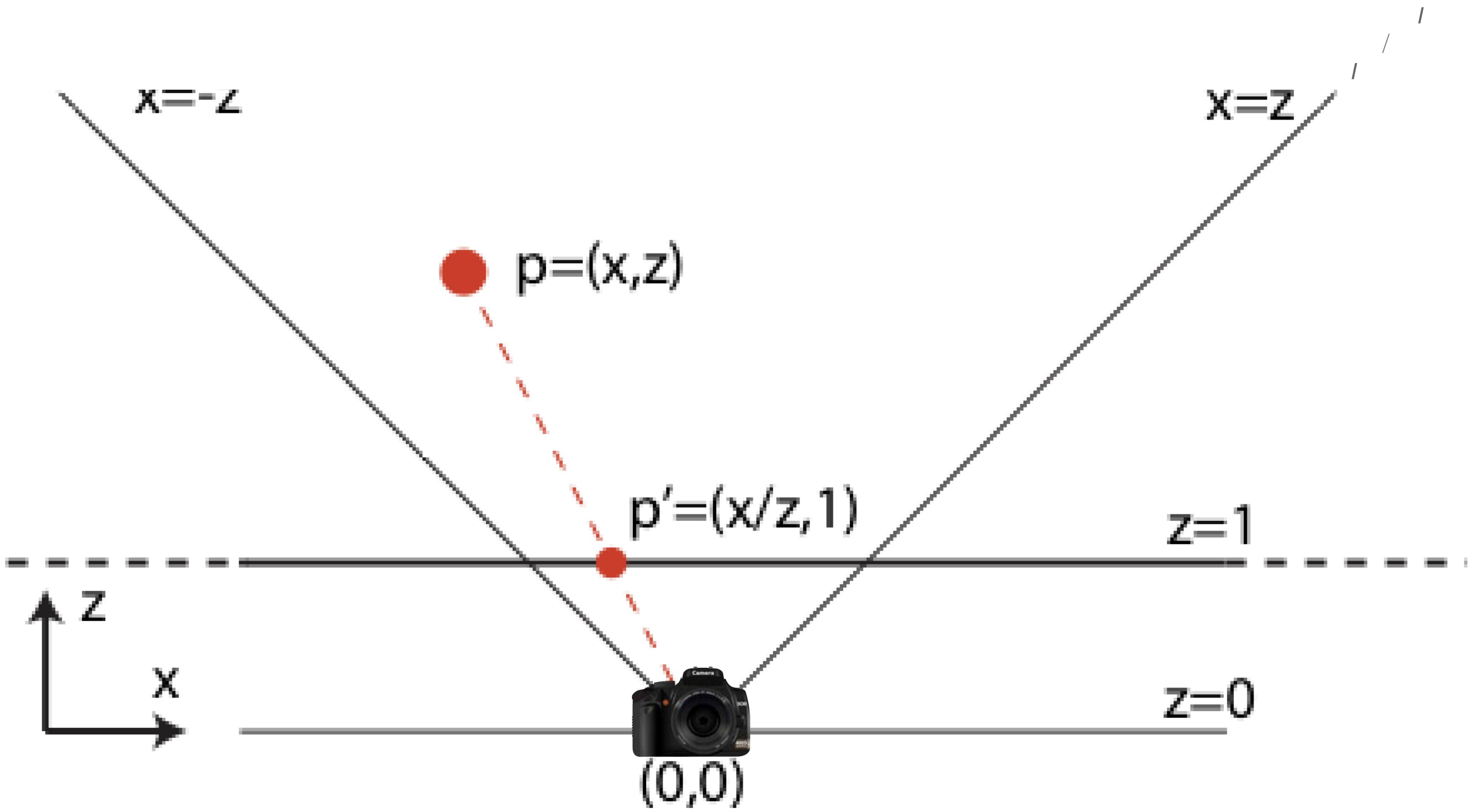
- Orthographic



- Perspective



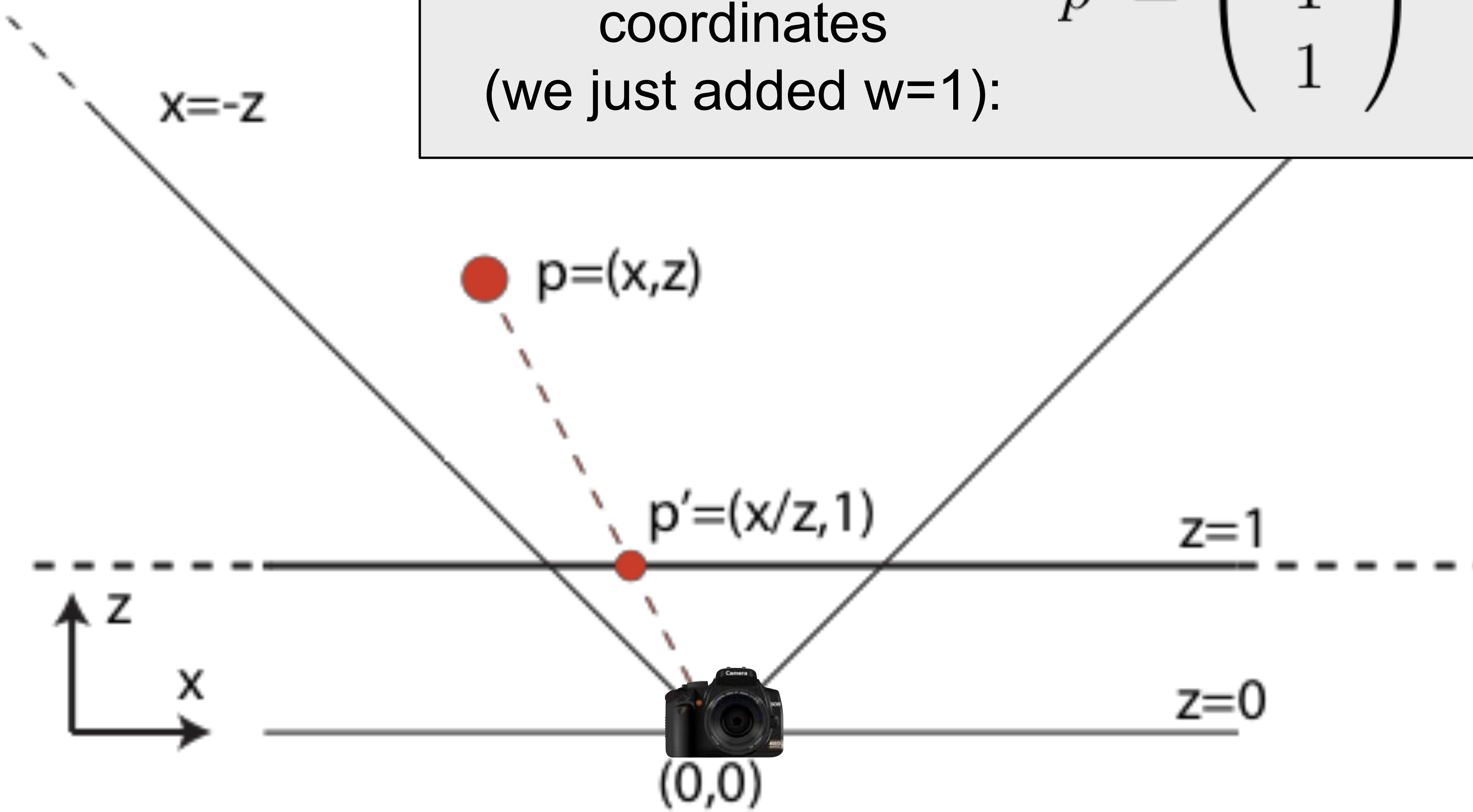
Perspective in 2D



Perspective in 2D

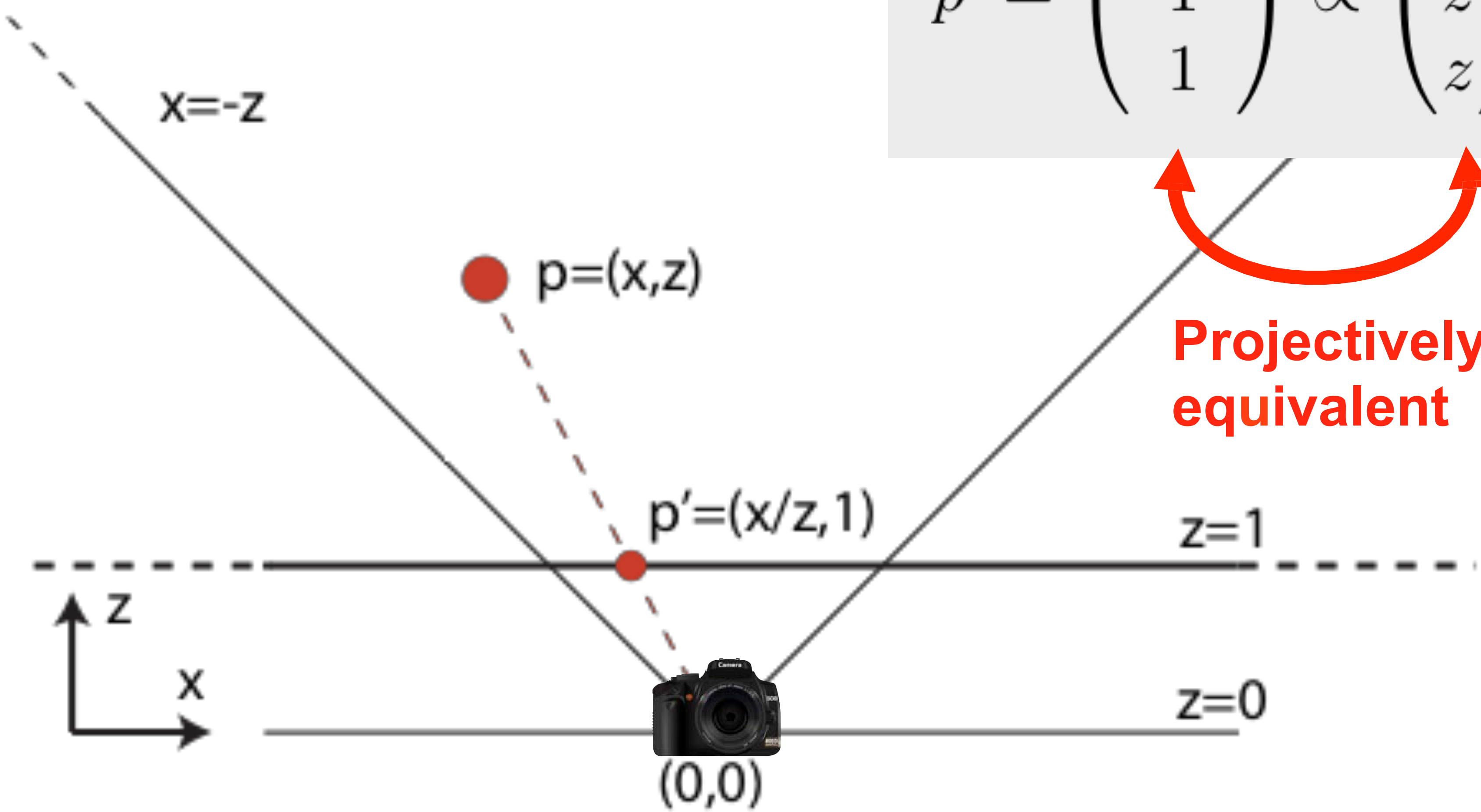
The projected point in homogeneous coordinates (we just added w=1):

$$p' = \begin{pmatrix} x/z \\ 1 \\ 1 \end{pmatrix}$$



Perspective in 2D

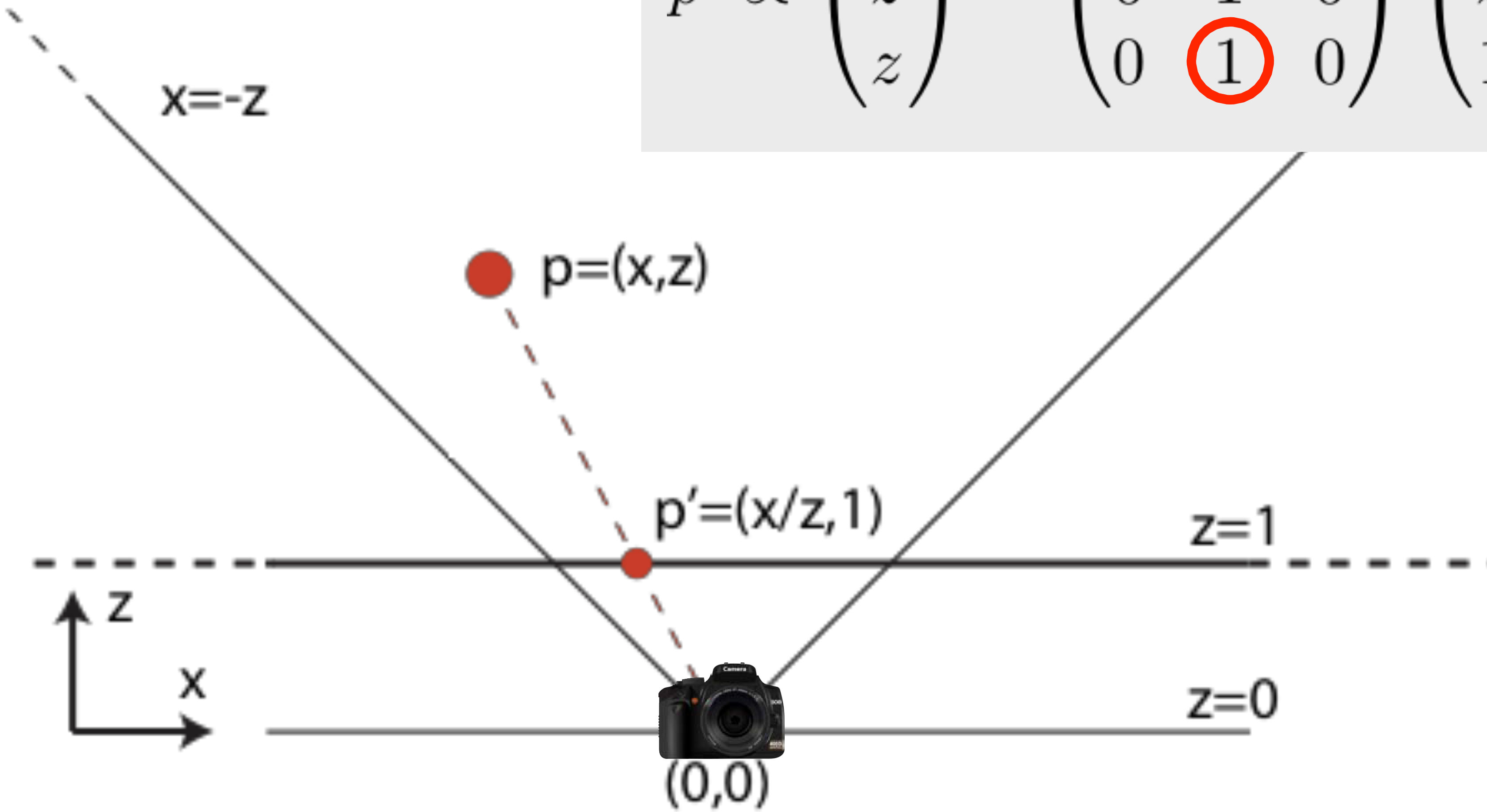
$$p' = \begin{pmatrix} x/z \\ 1 \\ 1 \end{pmatrix} \propto \begin{pmatrix} x \\ z \\ z \end{pmatrix}$$



Perspective in 2D

We'll just copy z to w, and get the projected point after homogenization!

$$p' \propto \begin{pmatrix} x \\ z \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ z \\ 1 \end{pmatrix}$$



Extension to 3D

- Trivial: add another dimension *y* and treat it like *x*
- Different fields of view and non-square image aspect ratios can be accomplished by simple scaling of the *x* and *y* axes.

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Caveat

- These projections matrices work perfectly in the sense that you get the proper 2D projections of 3D points.
- However, since we are flattening the scene onto the z=1 plane, we've lost all information about the distance to camera.
  - We need the distance for Z buffering, i.e., figuring out what is in front of what!

Basic Idea: store 1/*z*

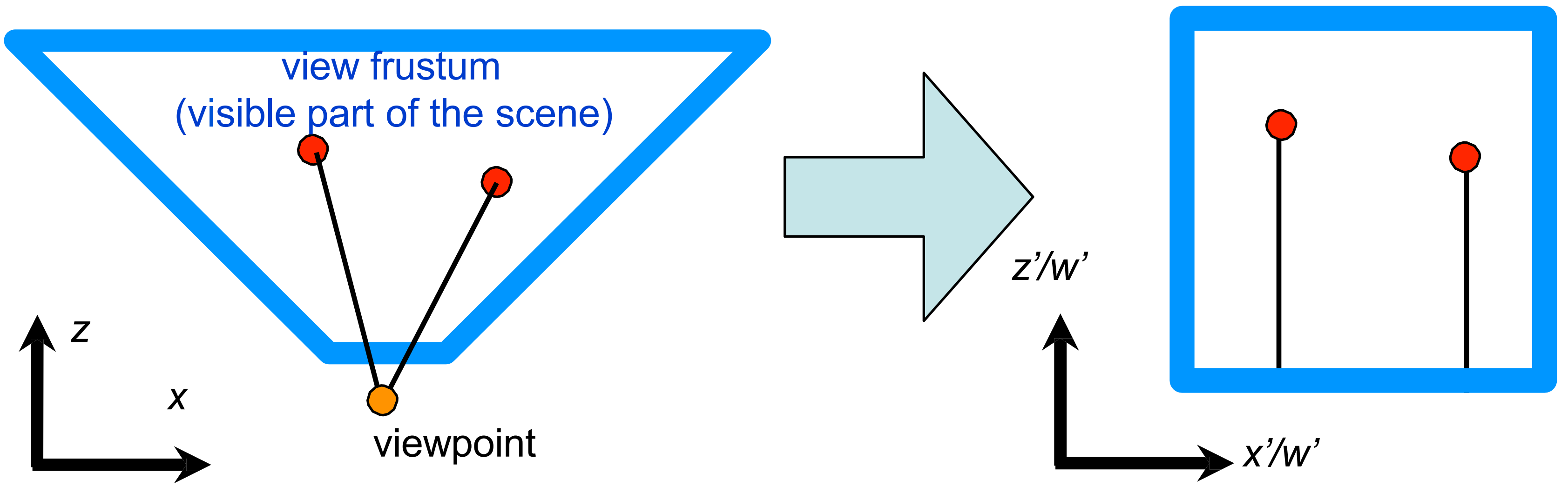
$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \\ z \end{pmatrix}$$

- $z' = 1$ before homogenization
- $z' = 1/z$ after homogenization

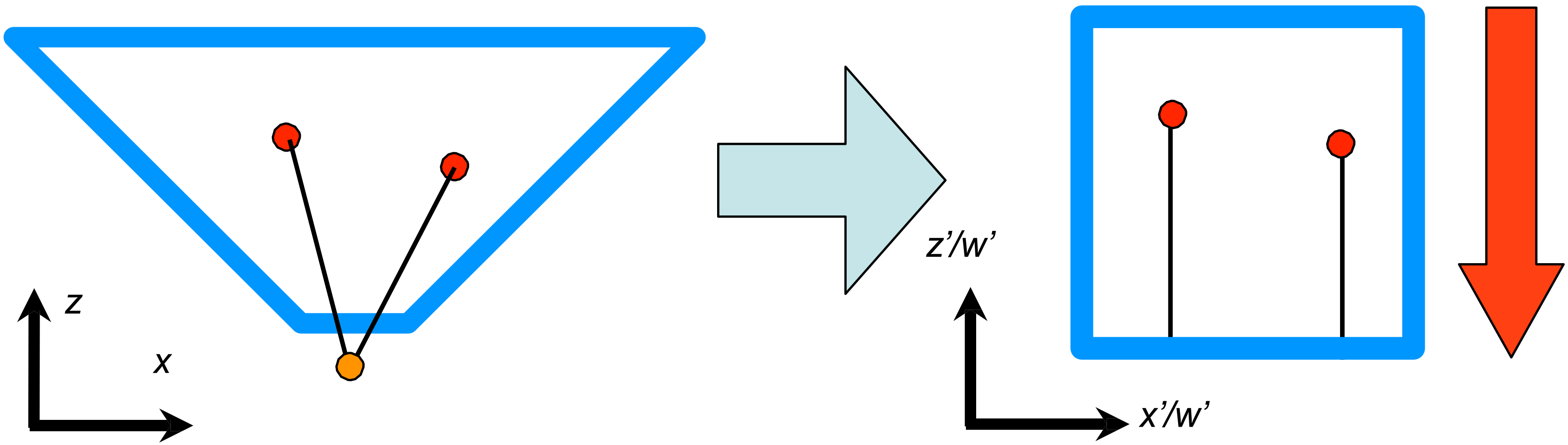
Full Idea: Remap the View Frustum

- We can transform the frustum by a modified projection in a way that makes it a square (cube in 3D) after division by *w*'.



The View Frustum in 2D

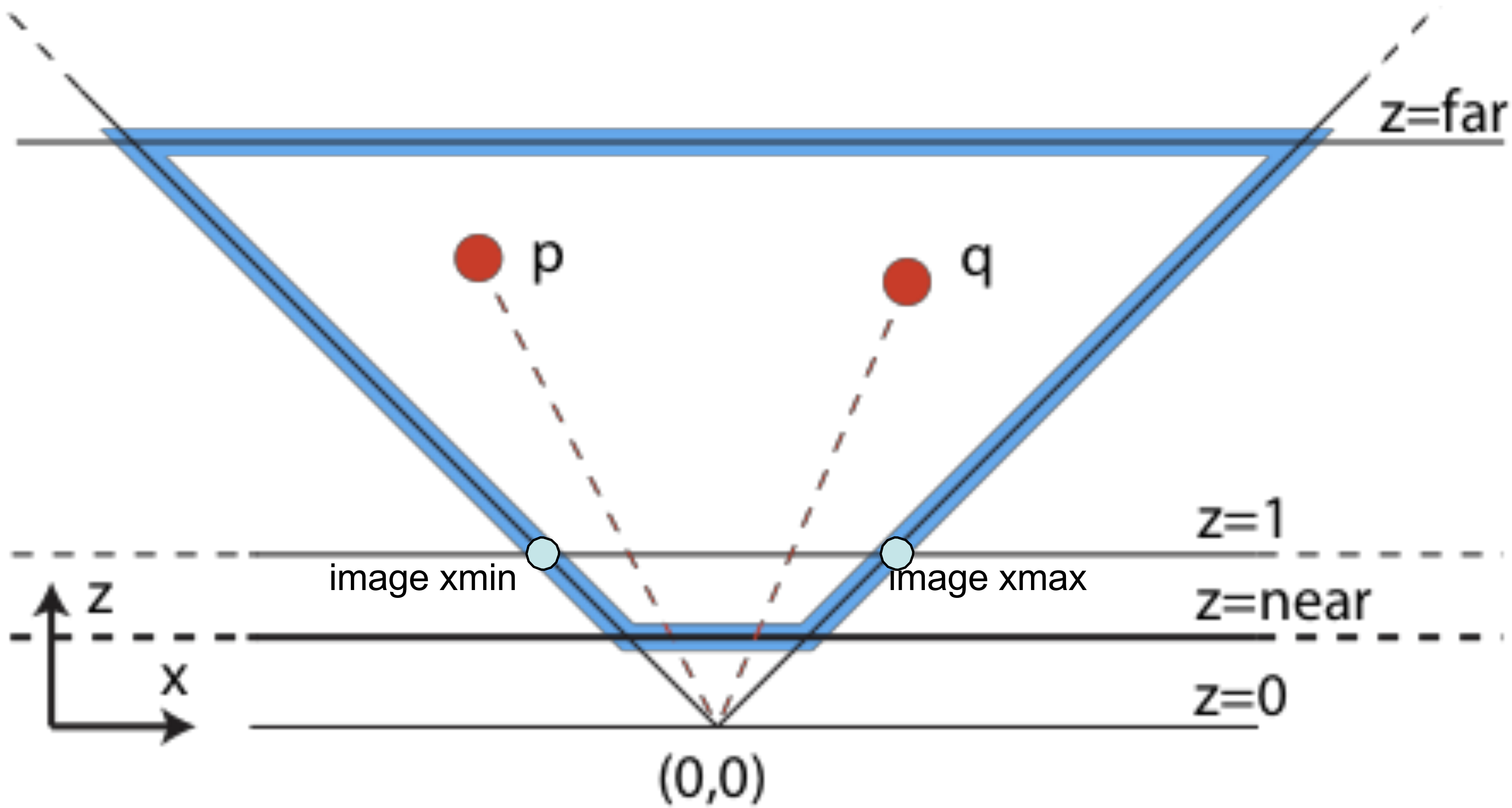
- We can transform the frustum by a modified projection in a way that makes it a square (cube in 3D) after division by *w*'.



The final image is obtained by merely dropping the z coordinate after projection (orthogonal projection)

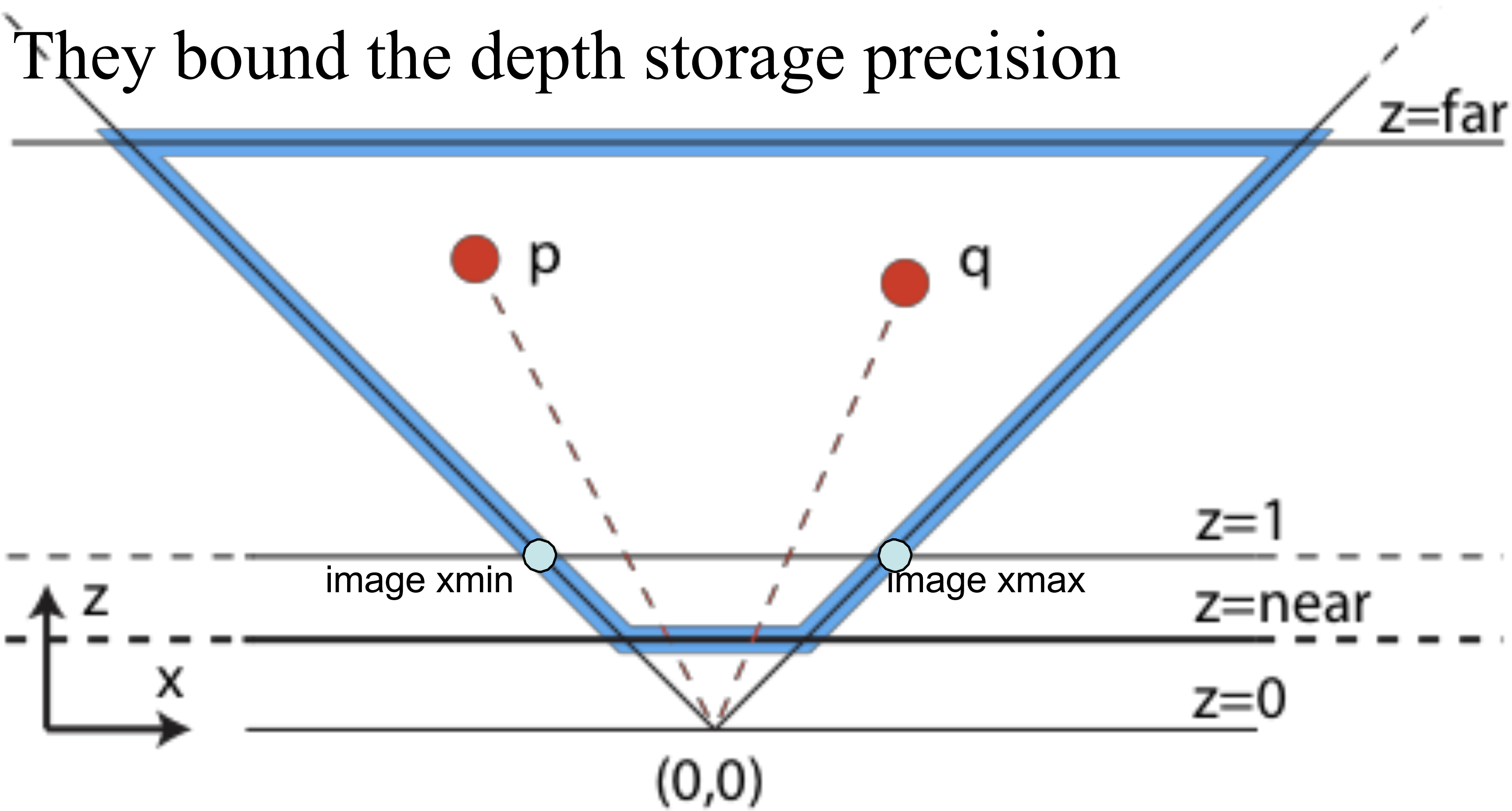
The View Frustum in 2D

- (In 3D this is a truncated pyramid.)

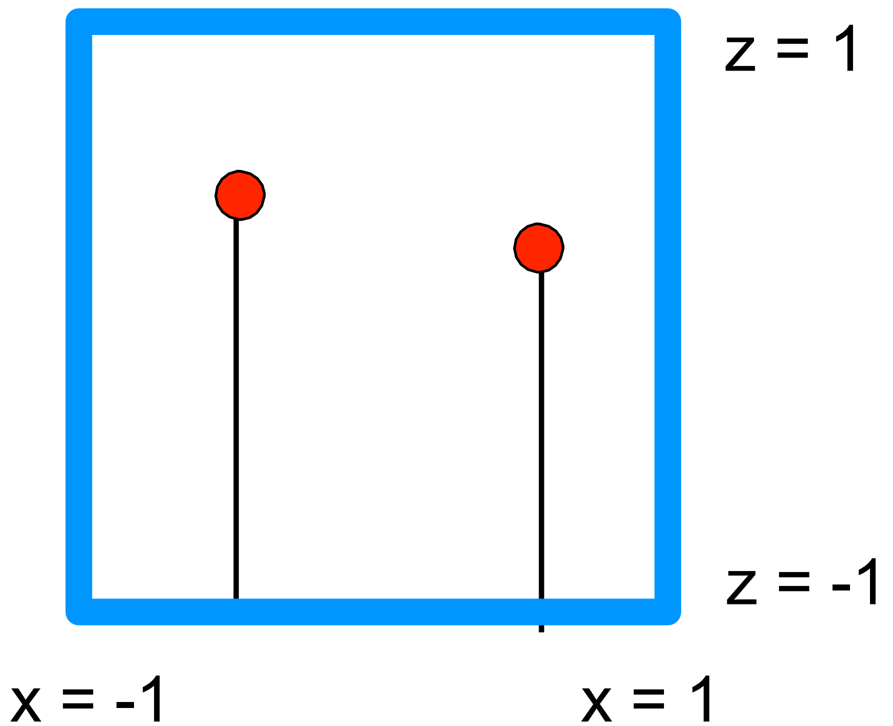


The View Frustum in 2D

- Far and near are kind of arbitrary
- They bound the depth storage precision



The Canonical View Volume



- Point of the exercise: This gives screen coordinates and depth values for Z-buffering with unified math
  - Caveat: OpenGL and DirectX define Z differently [0,1] vs.[-1,1]

OpenGL Form of the Projection

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\mathrm{far}+\mathrm{near}}{\mathrm{far}-\mathrm{near}} & -\frac{2*\mathrm{far}*\mathrm{near}}{\mathrm{far}-\mathrm{near}} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Homogeneous coordinates within canonical view volume

Input point in view coordinates

OpenGL Form of the Projection

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2*\text{far}*\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

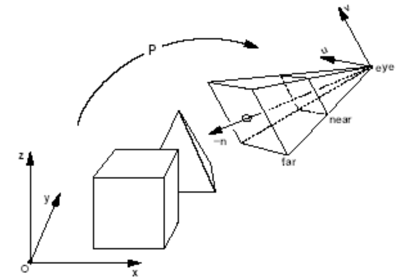
- z'=(az+b)/z =a+b/z
  - where a & b depend on near & far
- Similar enough to our basic idea:
  - z'=1/z

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

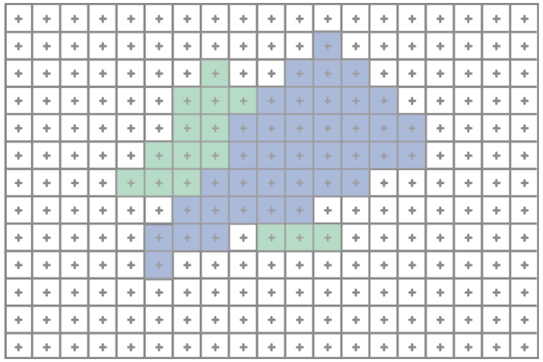
Recap: Projection

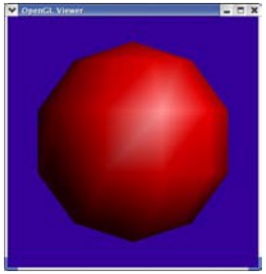
- Perform rotation/translation/other transforms to put viewpoint at origin and view direction along z axis
  - This is the OpenGL “modelview” matrix
- Combine with projection matrix (perspective or orthographic)
  - Homogenization achieves foreshortening
  - This is the OpenGL “projection” matrix
- **Corollary**: The entire transform from object space to canonical view volume $[-1,1]^3$ is a single matrix

Modern Graphics Pipeline

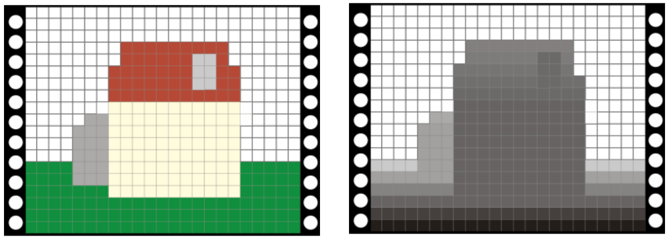


- Project vertices to 2D (image)
  - We now have screen coordinates
- Rasterize triangle: find which pixels should be lit



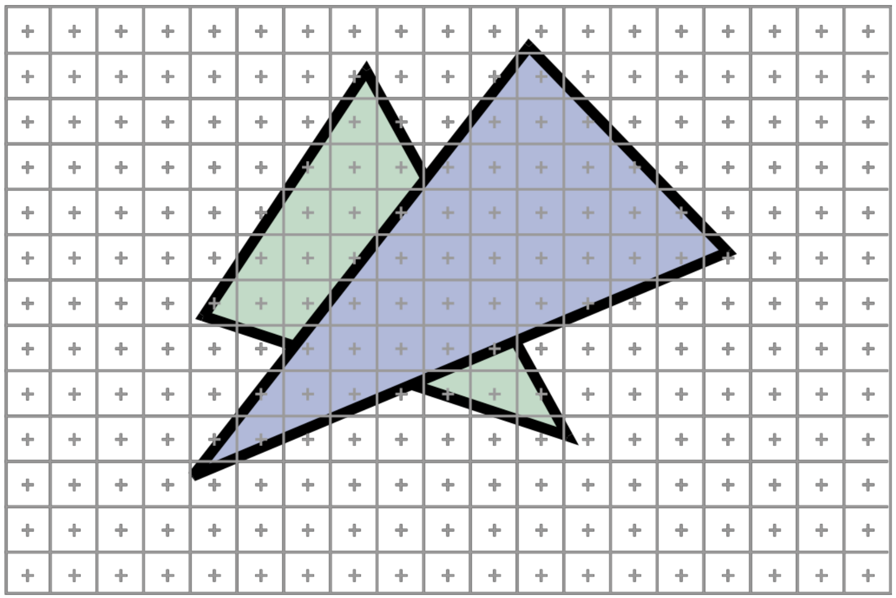


- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer



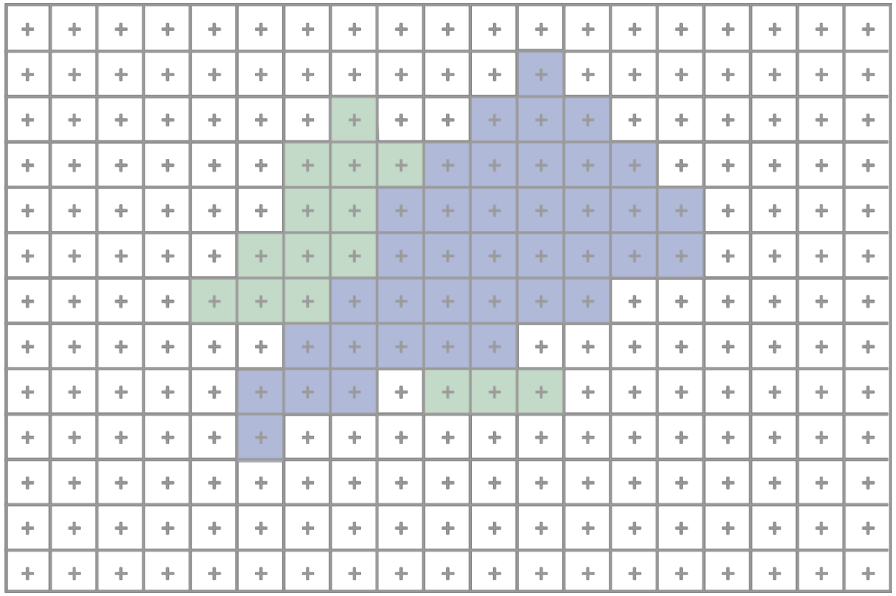
2D Scan Conversion

- Primitives are "continuous" geometric objects; screen is discrete (pixels)



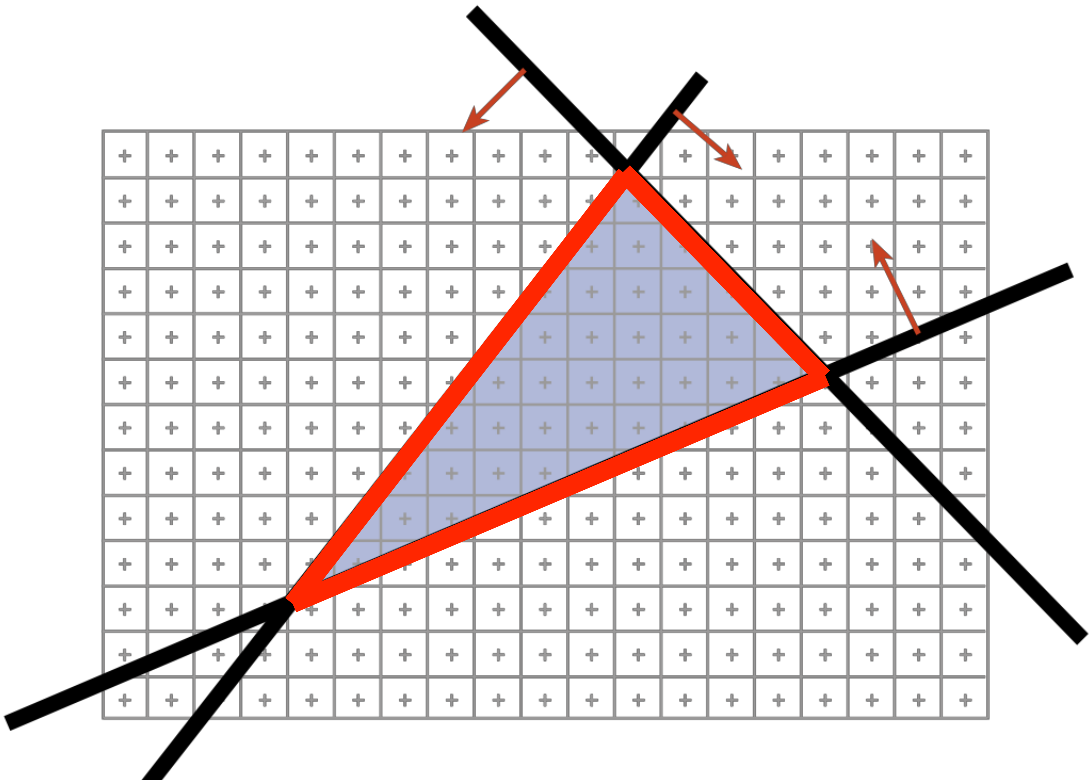
2D Scan Conversion

- Primitives are "continuous" geometric objects; screen is discrete (pixels)
- Rasterization computes a discrete approximation in terms of pixels **(how?)**



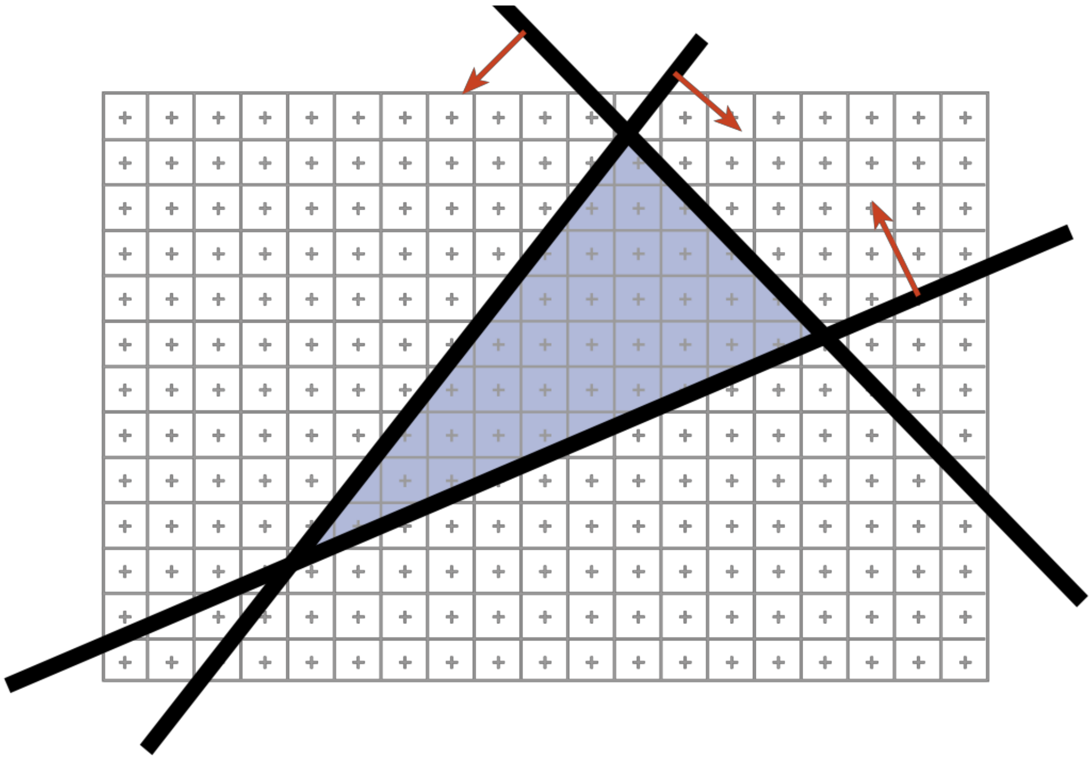
Edge Functions

- The triangle's 3D edges project to line segments in the image (thanks to planar perspective)
  - Lines map to lines, not curves



Edge Functions

- The triangle's 3D edges project to line segments in the image (thanks to planar perspective)
- The interior of the triangle is the set of points that is inside all three halfspaces defined by these lines



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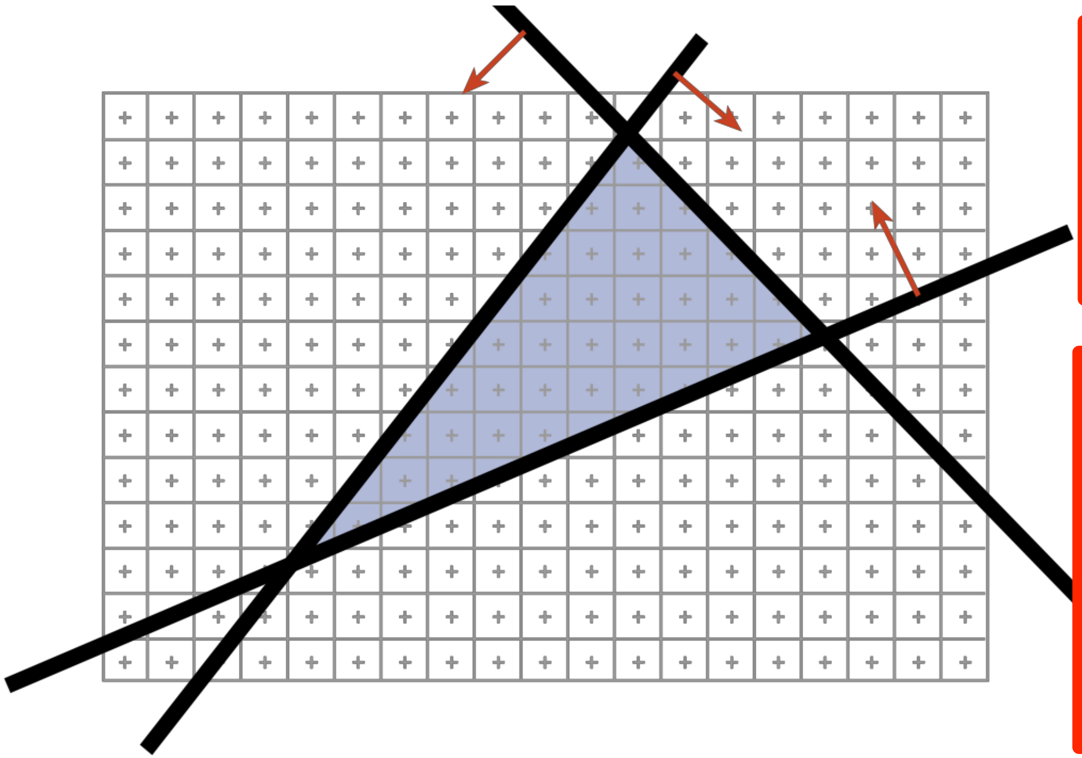
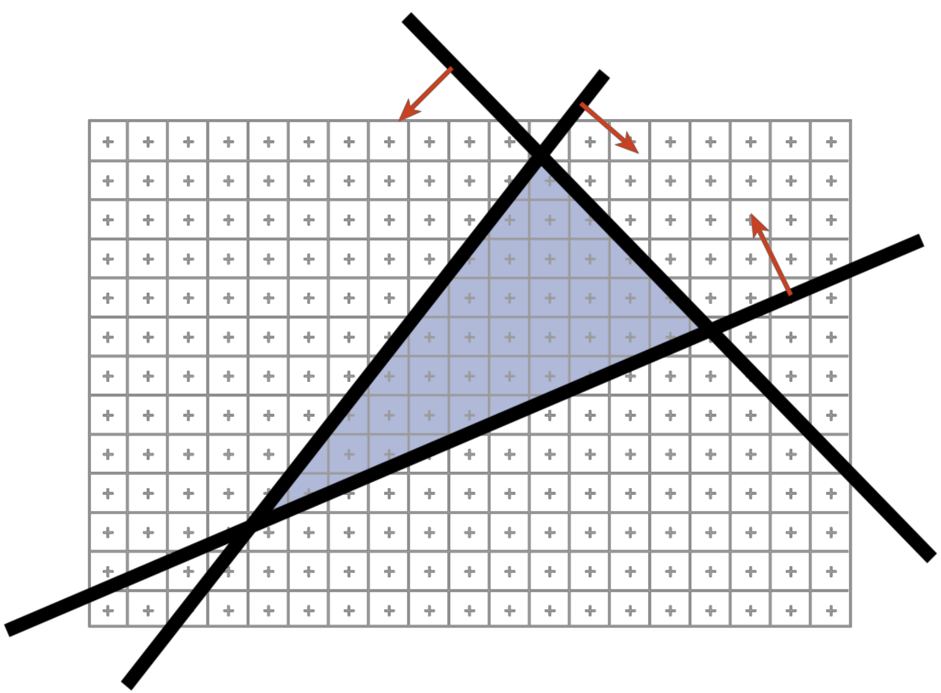


$$E_i(x, y) = a_i x + b_i y + c_i$$

$$(x, y) \text{ within triangle} \Leftrightarrow E_i(x, y) \geq 0, \quad \forall i = 1, 2, 3$$

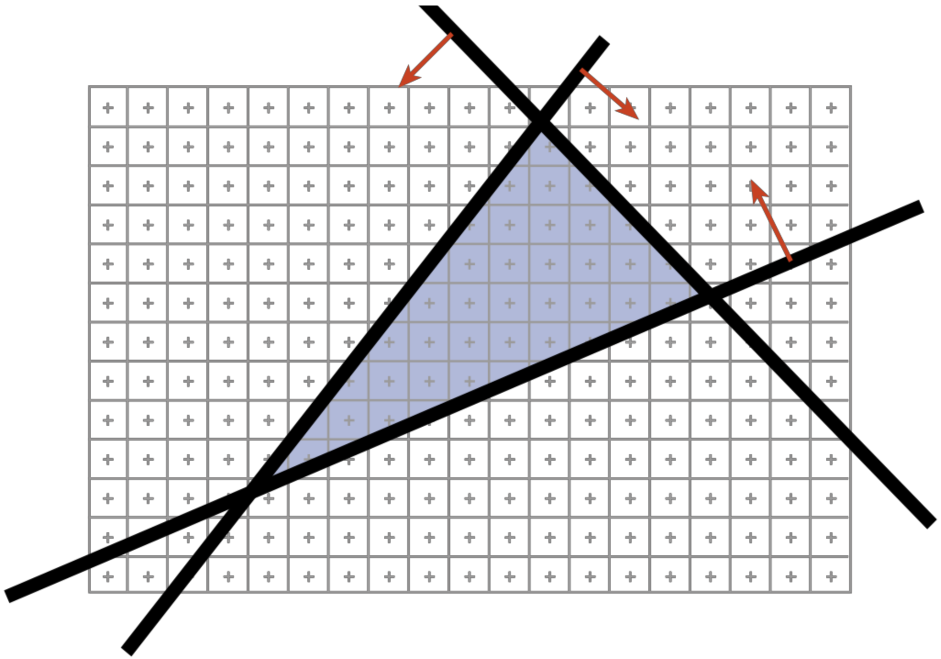
Brute Force Rasterizer

- Compute E_1, E_2 , E_3 coefficients from projected vertices
  – Called "triangle setup", yields a_i, b_i, c_i for i=1,2,3



Brute Force Rasterizer

- Compute E_1, E_2 , E_3 coefficients from projected vertices
- For each pixel (x, y)
  - Evaluate edge functions at pixel center
  - If all non-negative, pixel is in!



Problem?

Brute Force Rasterizer

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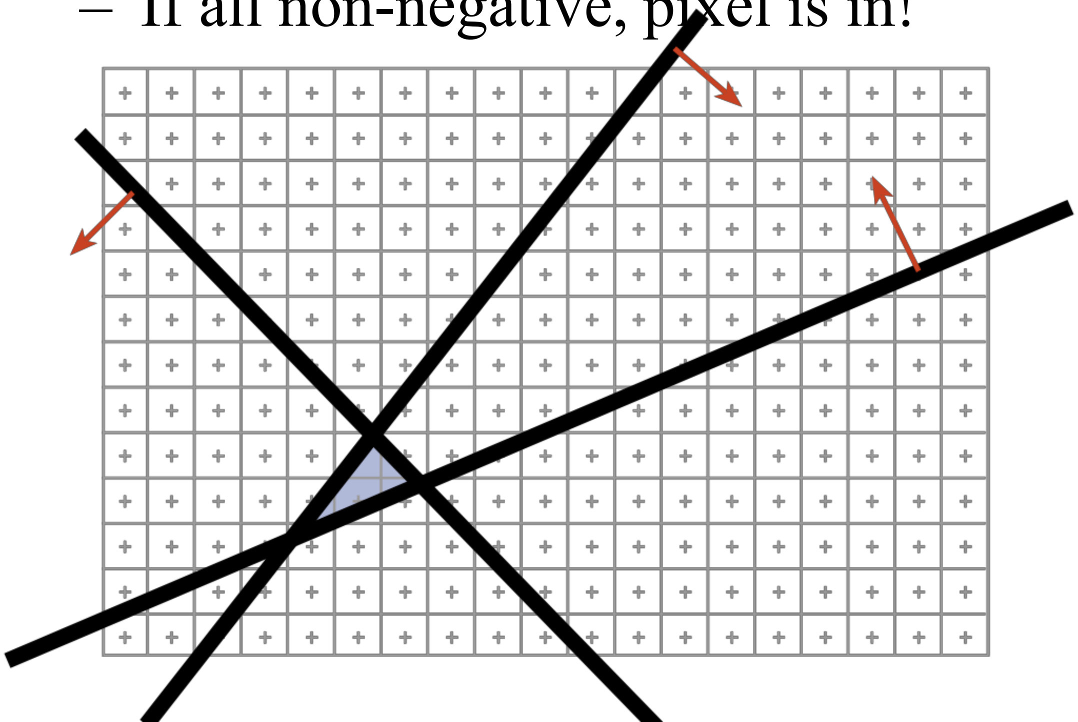
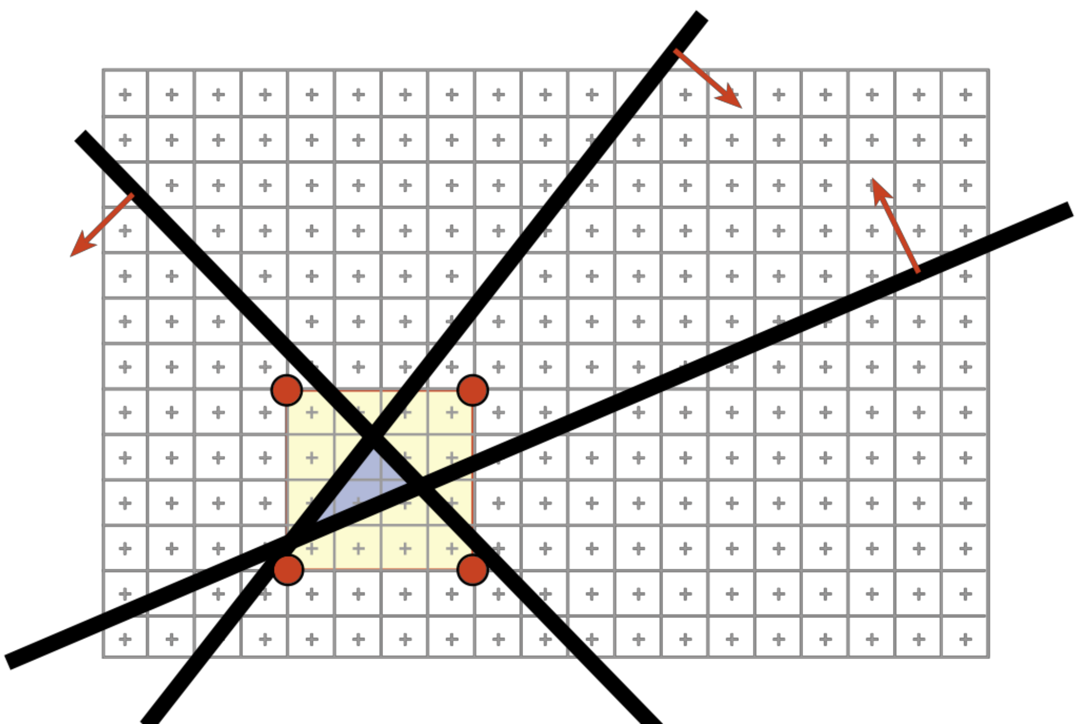


If the triangle is small, lots of useless computation if we really test all pixels

Easy Optimization

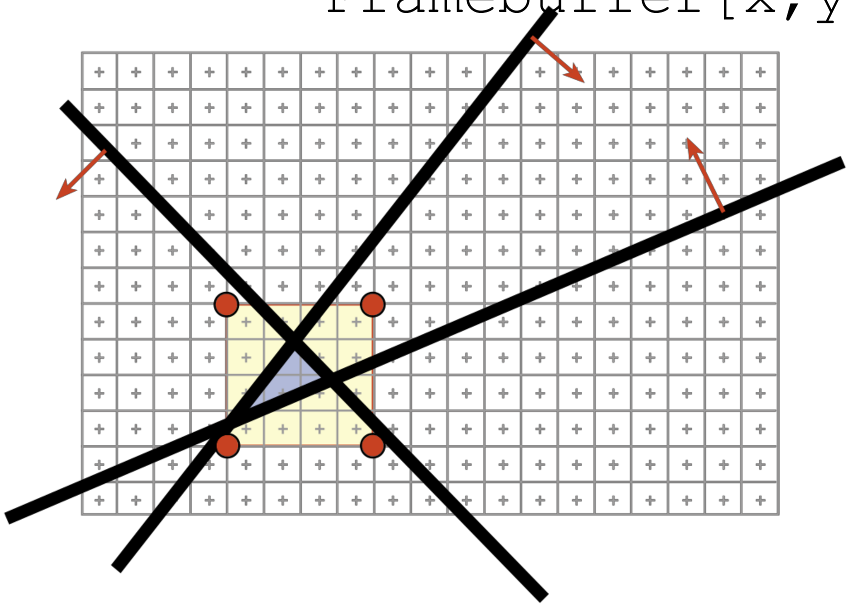
- Improvement: Scan over only the pixels that overlap the *screen bounding box* of the triangle
- How do we get such a bounding box?
  - X_{min}, X_{max}, Y_{min}, Y_{max} of the projected triangle vertices



Rasterization Pseudocode

Note: No visibility

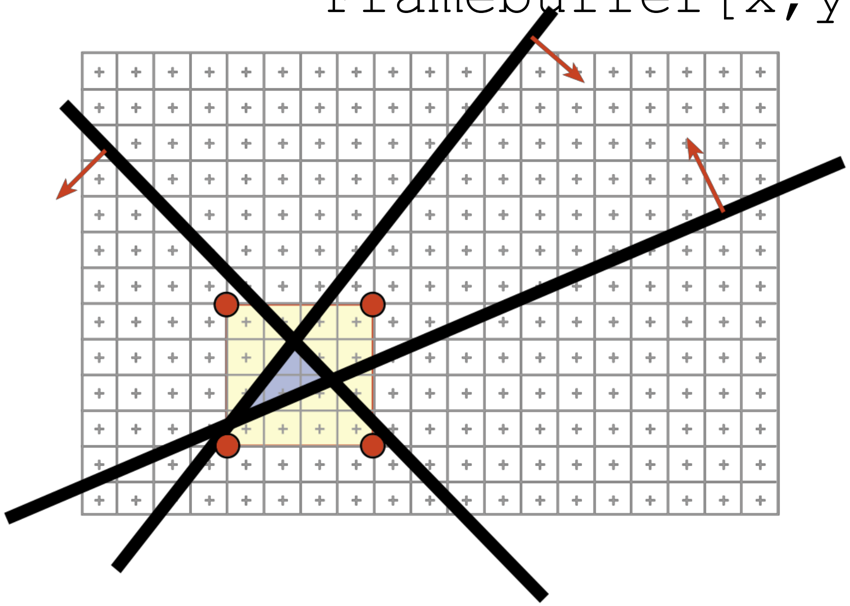
```
For every triangle
    Compute projection for vertices, compute the Ei
    Compute bbox, clip bbox to screen limits
    For all pixels in bbox
        Evaluate edge functions Ei
        If all > 0
            Framebuffer[x,y ] = triangleColor
```



Bounding box clipping is easy, just clamp the coordinates to the screen rectangle

Can We Do Better?

```
For every triangle
    Compute projection for vertices, compute the Ei
    Compute bbox, clip bbox to screen limits
    For all pixels in bbox
        Evaluate edge functions aix + biy + ci
        If all > 0
            Framebuffer[x,y ] = triangleColor
```



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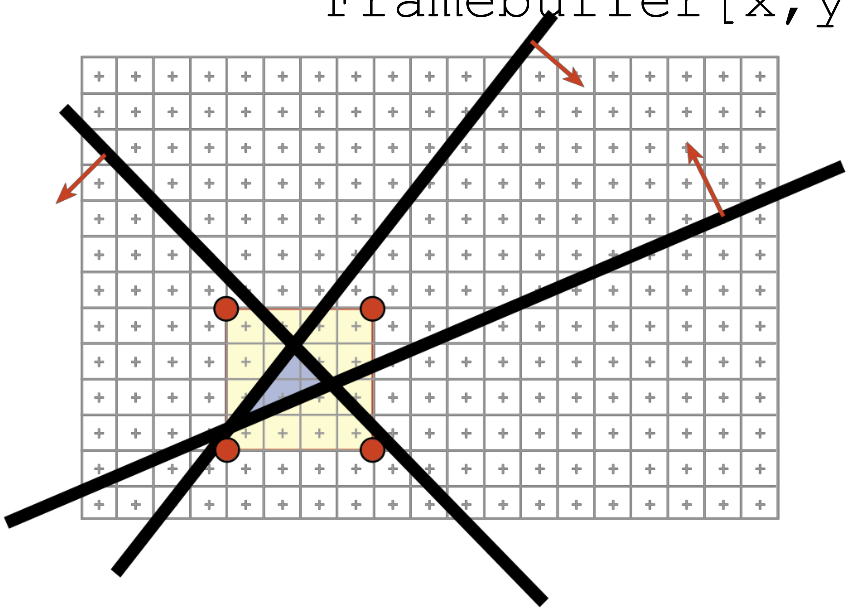

These are linear functions of the pixel coordinates (x,y), i.e., they only change by a constant amount when we step from x to x+1 (resp. y to y+1)

Incremental Edge Functions

```
For every triangle
   ComputeProjection
   Compute bbox, clip bbox to screen limits
   For all scanlines y in bbox
      Evaluate all Ei's at (x0,y): Ei = aix0 + biy + ci
      For all pixels x in bbox
          If all Ei>0
              Framebuffer[x,y ] = triangleColor
          Increment line equations: Ei += ai
```

- We save ~two multiplications and two additions per pixel when the triangle is large

Incremental Edge Functions

```
For every triangle
    ComputeProjection
    Compute bbox, clip bbox to screen limits
    For all scanlines y in bbox
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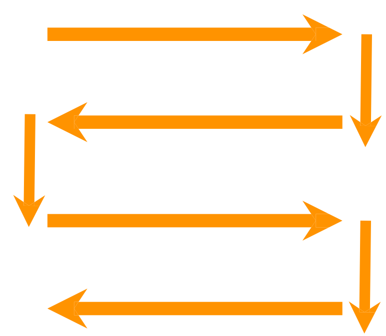
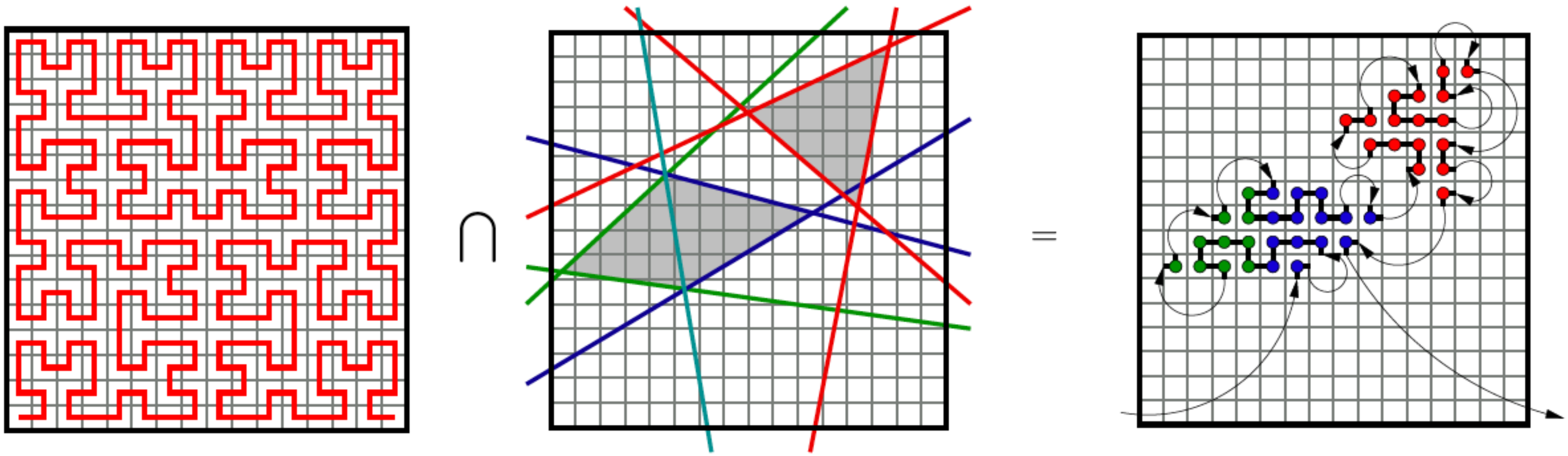


- We save ~two multiplications and two additions per pixel when the triangle is large

Can also zig-zag to avoid reinitialization per scanline, just initialize once at x0, y0

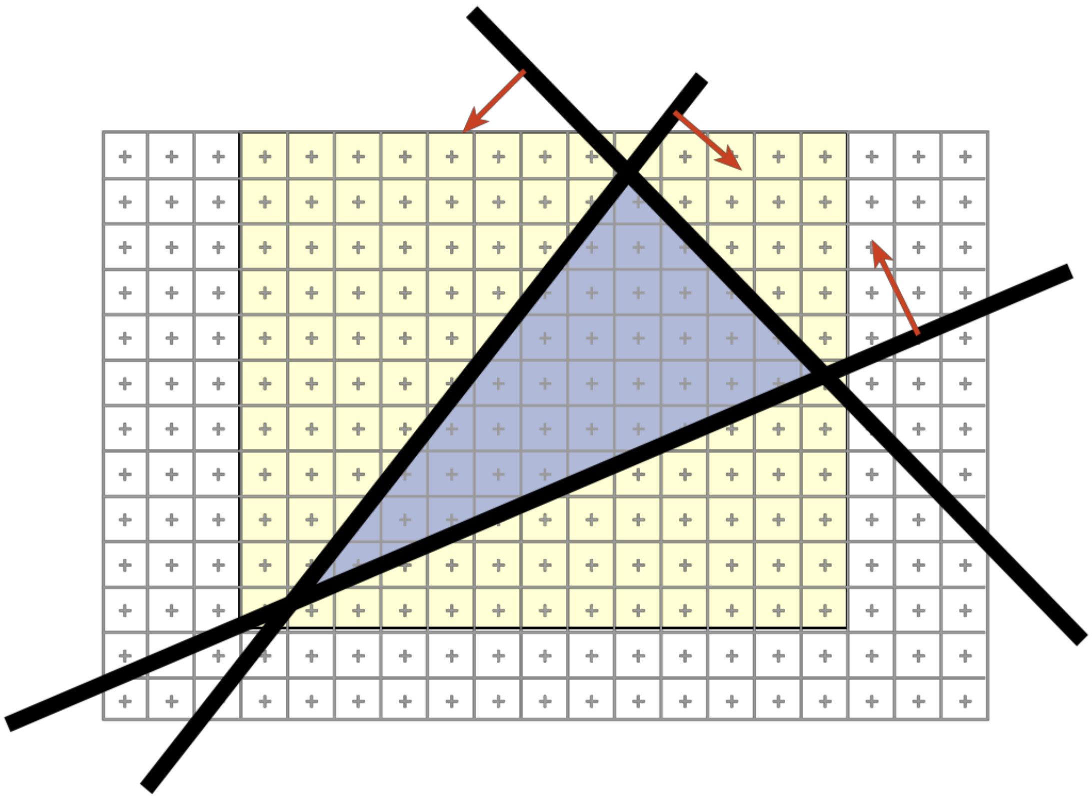
Questions?

- For a really HC piece of rasterizer engineering, see the hierarchical Hilbert curve rasterizer by McCool, Wales and Moule.
  - (Hierarchical? We'll look at that next..)

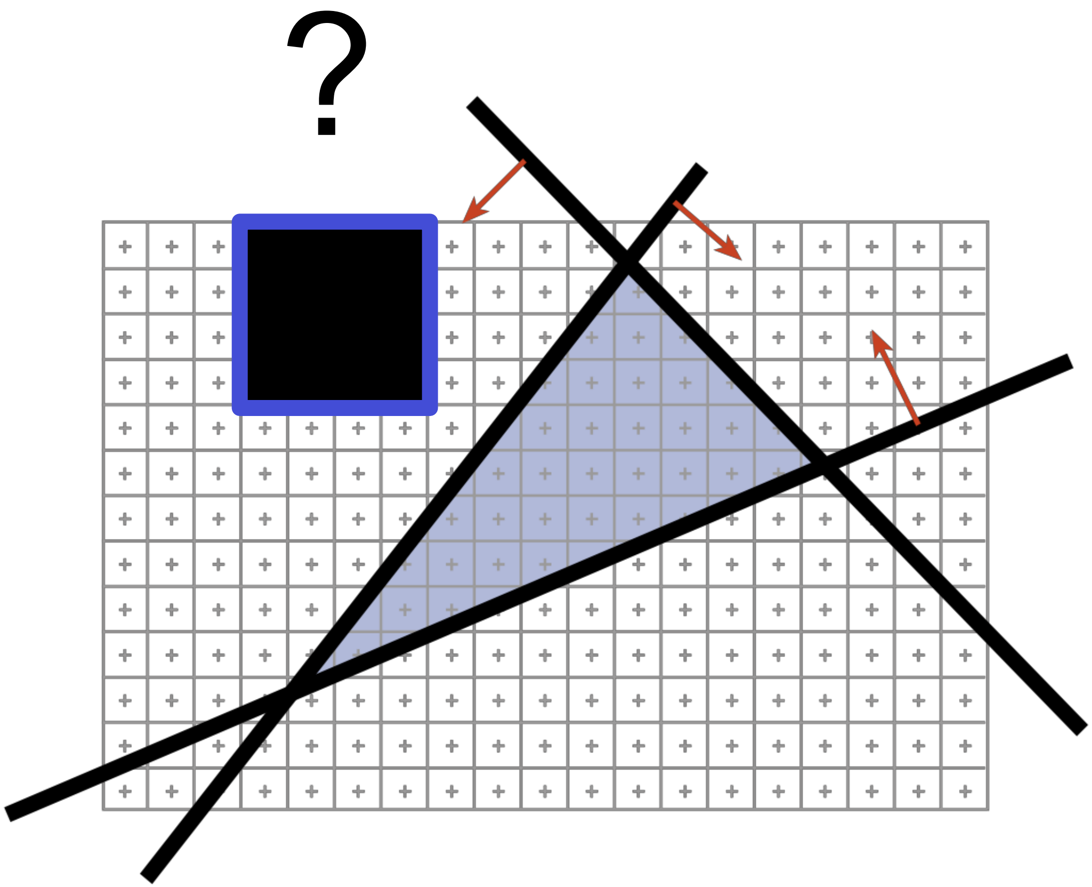


Can We Do Even Better?

- We compute the line equation for many useless pixels
- What could we do?

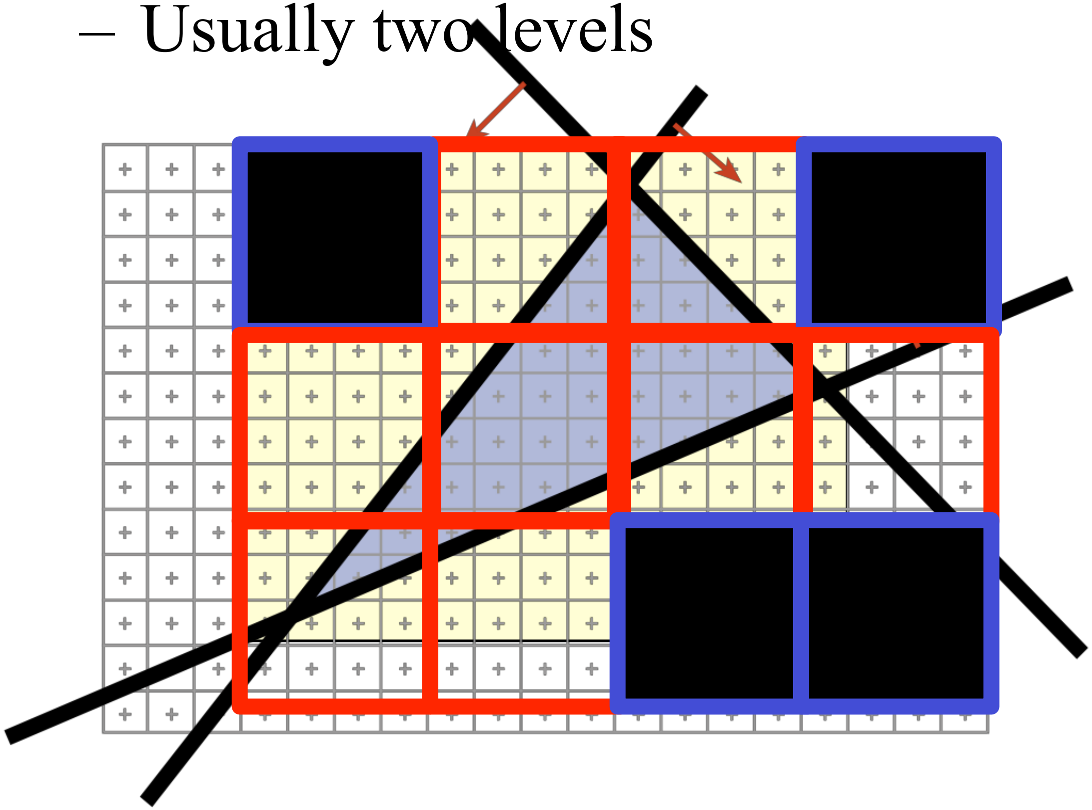


Indeed, We Can Be Smarter



Indeed, We Can Be Smarter

- Hierarchical rasterization!
  - Conservatively test **blocks of pixels** before going to per-pixel level (can skip large blocks at once)
  - Usually two levels



Conservative tests of axis-aligned blocks vs. edge functions are not very hard, thanks to linearity. See [Akenine-Möller and Aila, Journal of Graphics Tools 10(3), 2005.]()

Indeed, We Can Be Smarter

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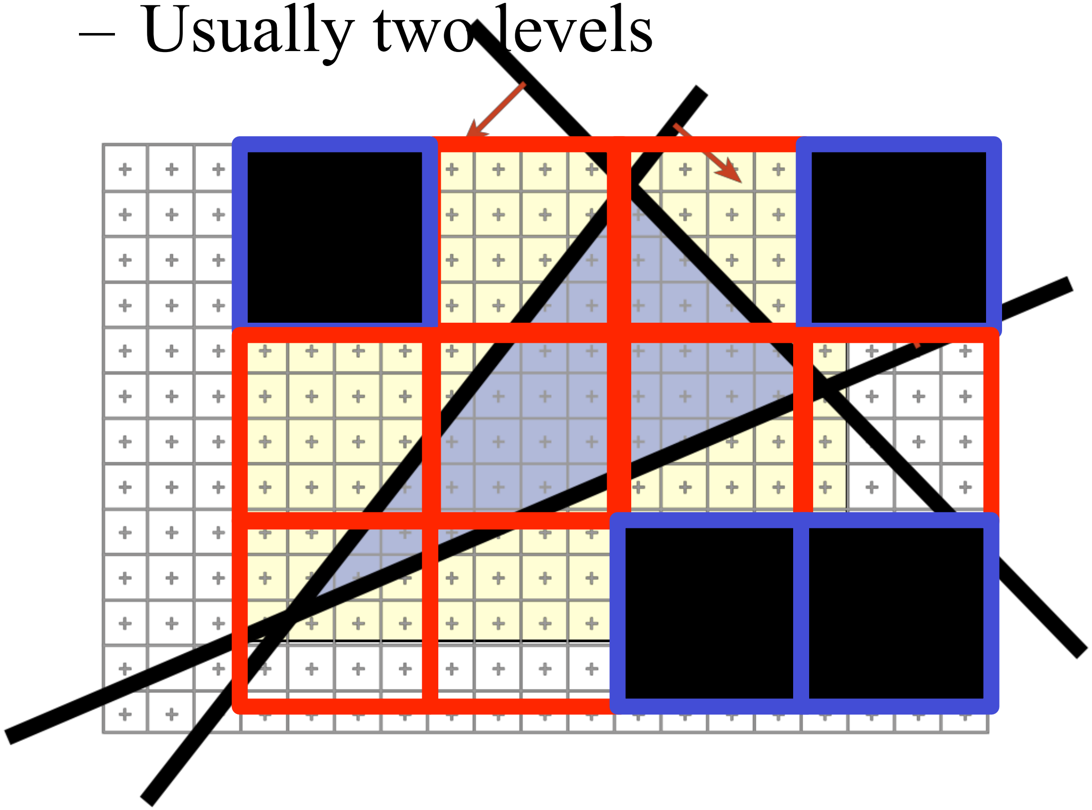
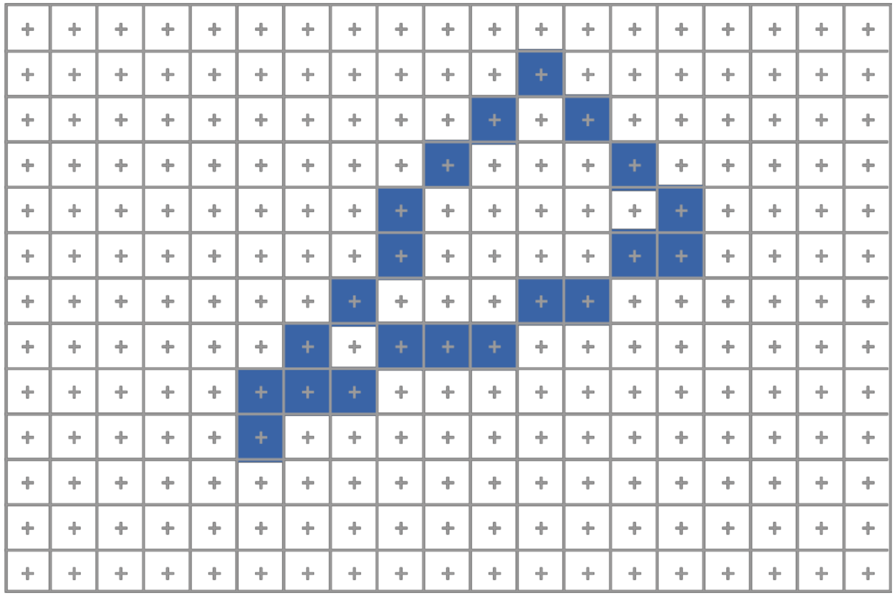


Can also test if an entire block is **inside** the triangle; then, can skip edge functions tests for all pixels for even further speedups.(Must still test Z, because they might still be occluded.)

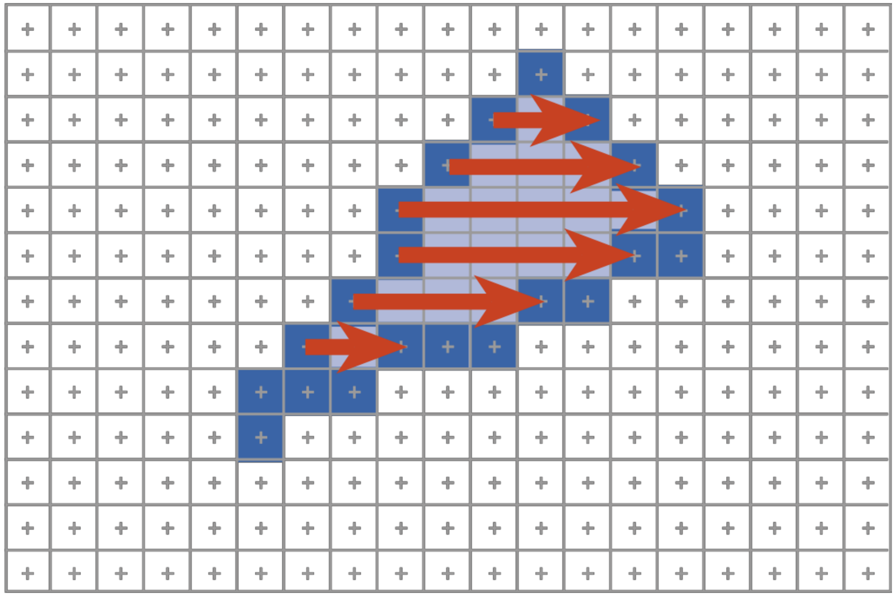
Oldschool Rasterization

- Compute the boundary pixels using line rasterization



Oldschool Rasterization

- Compute the boundary pixels using line rasterization
- Fill the spans



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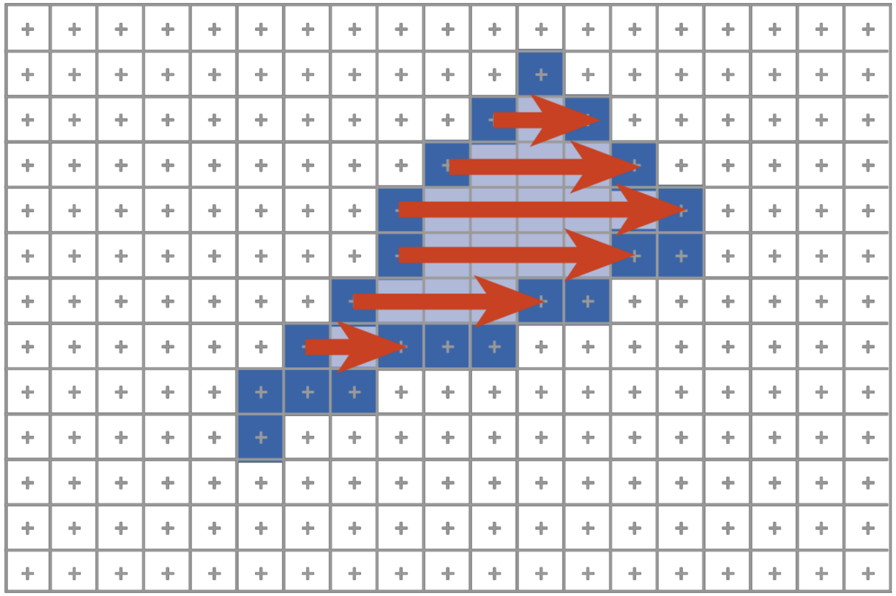

More annoying to implement than edge functions

Not faster unless triangles are huge

Oldschool Rasterization Questions?

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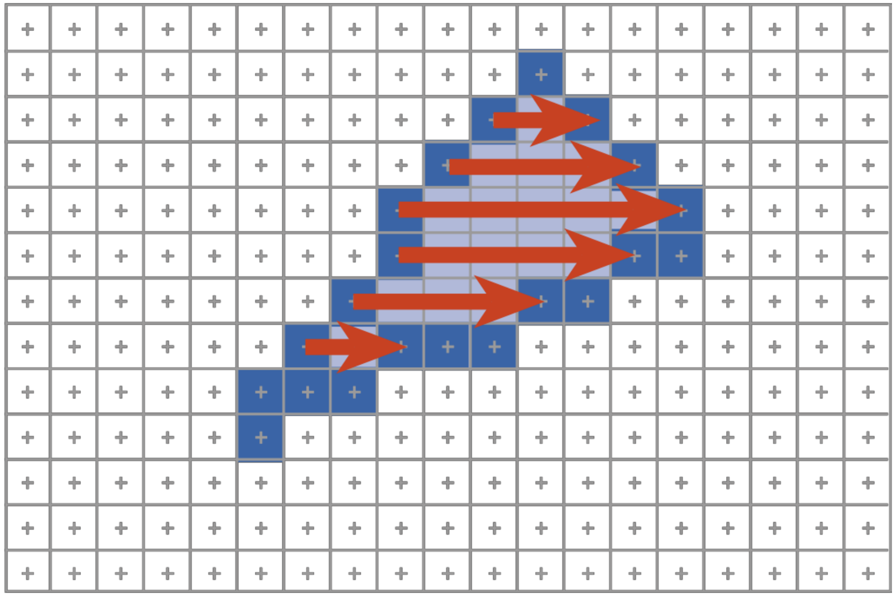

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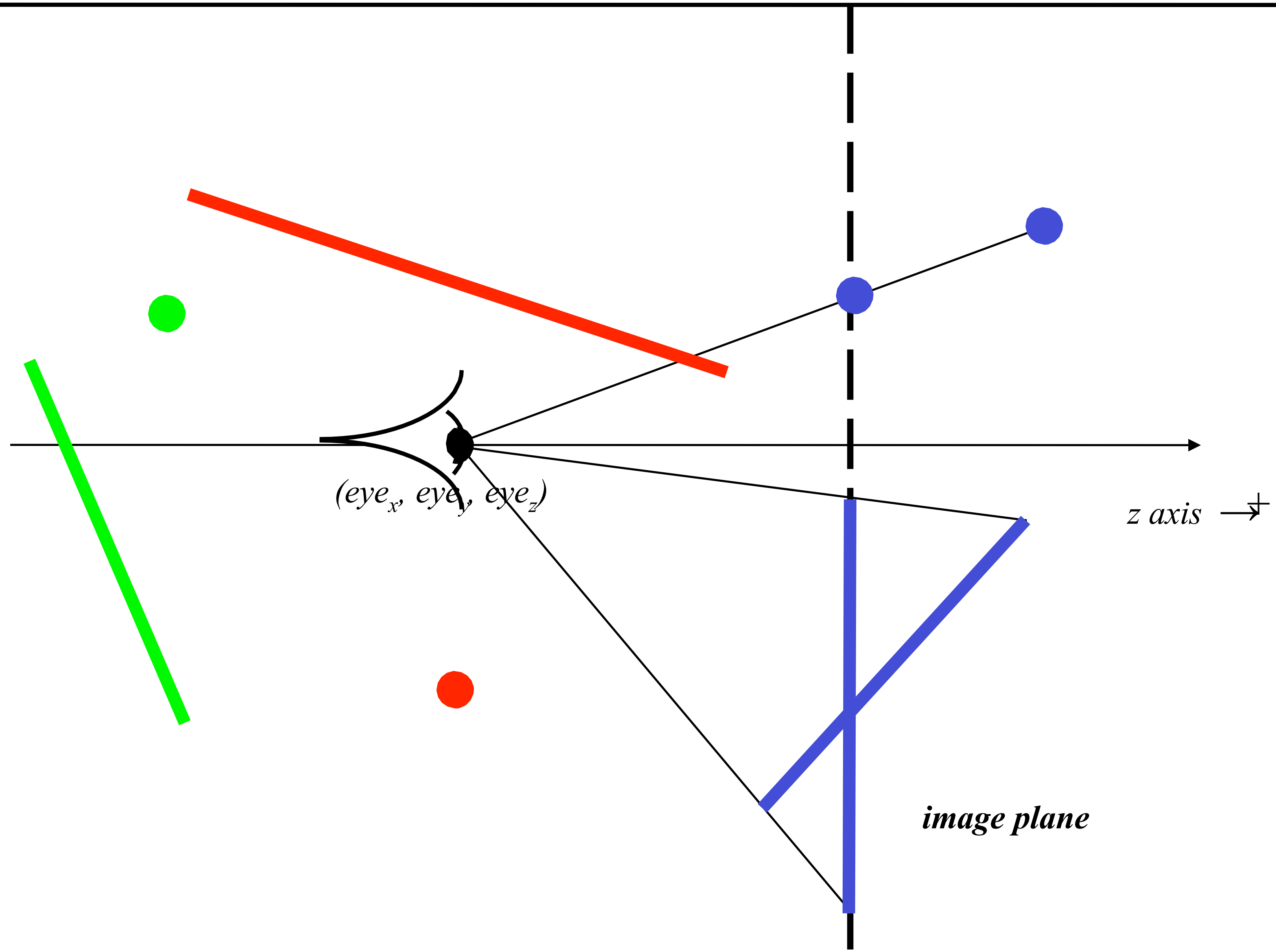
What if the p_z is > eye_z?



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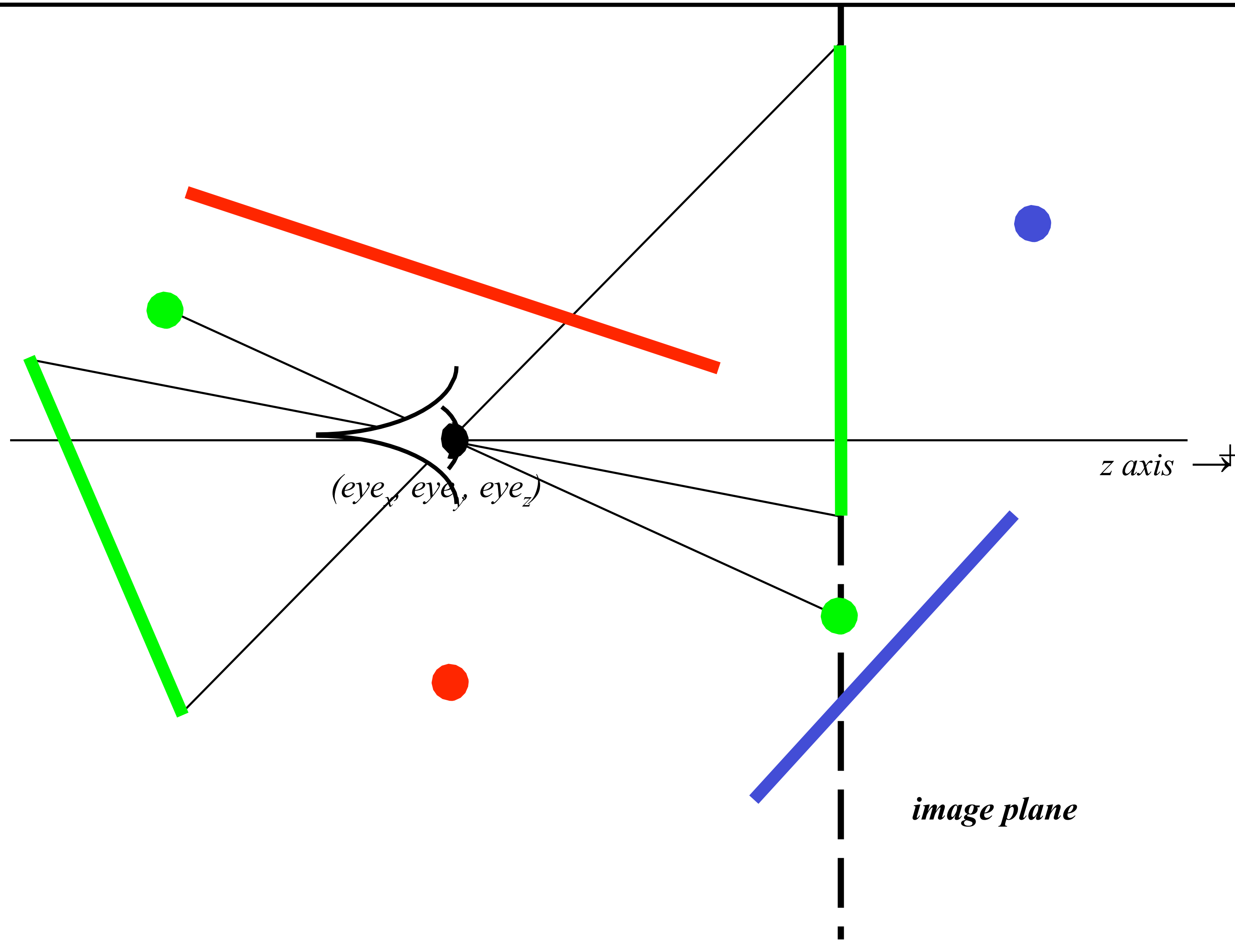
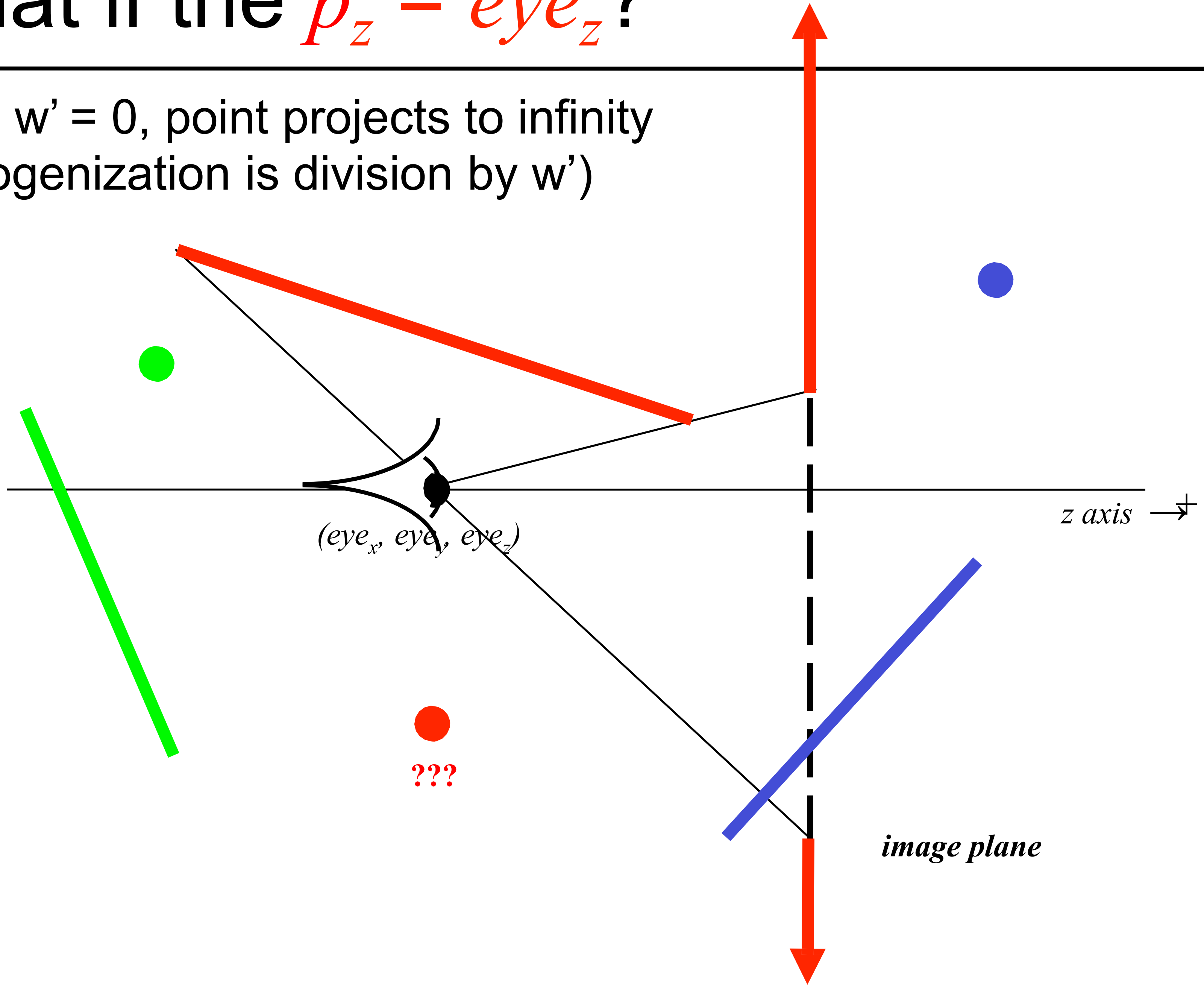
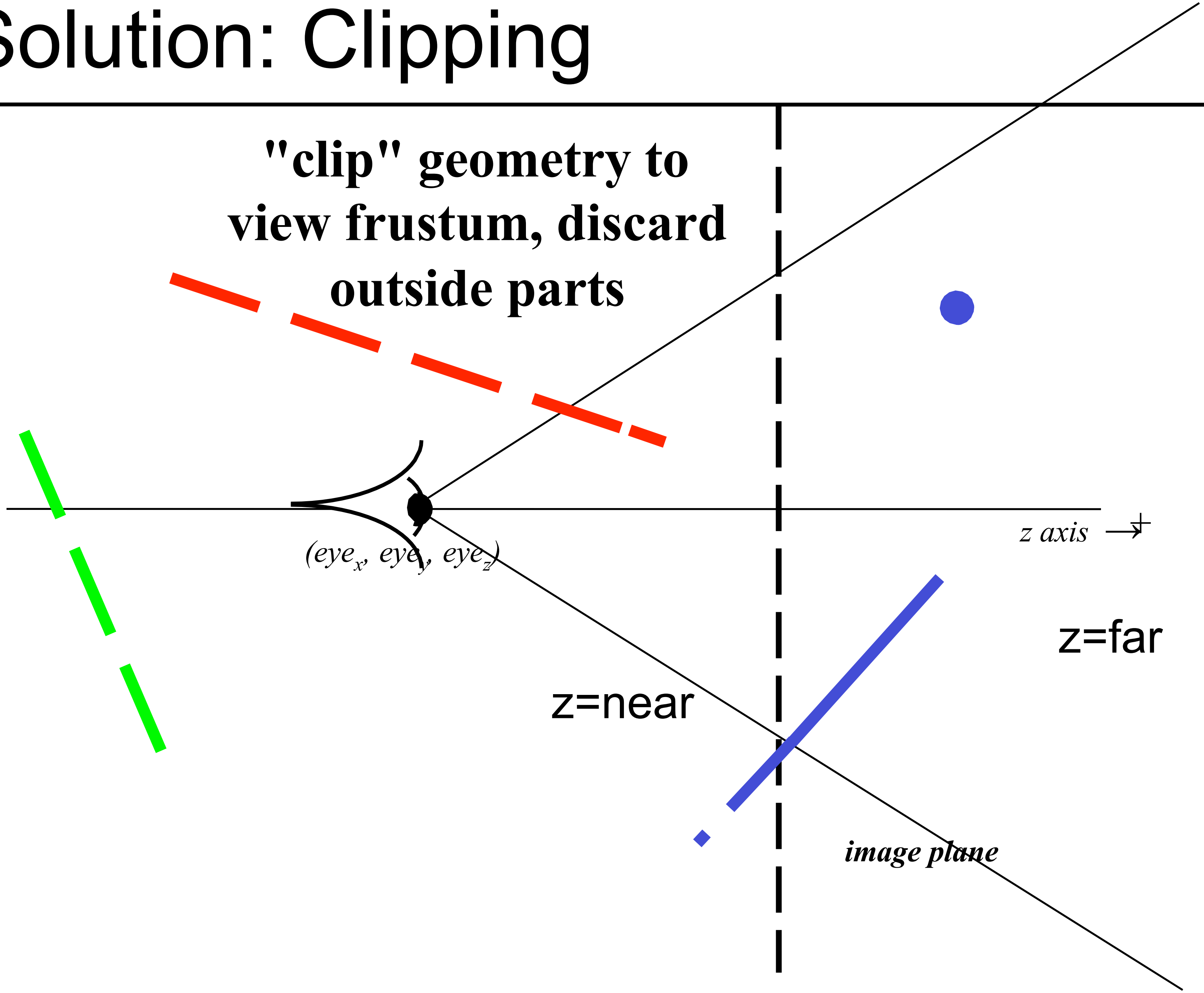


What if the $p_z = eye_z$?

When w' = 0, point projects to infinity
(homogenization is division by w')

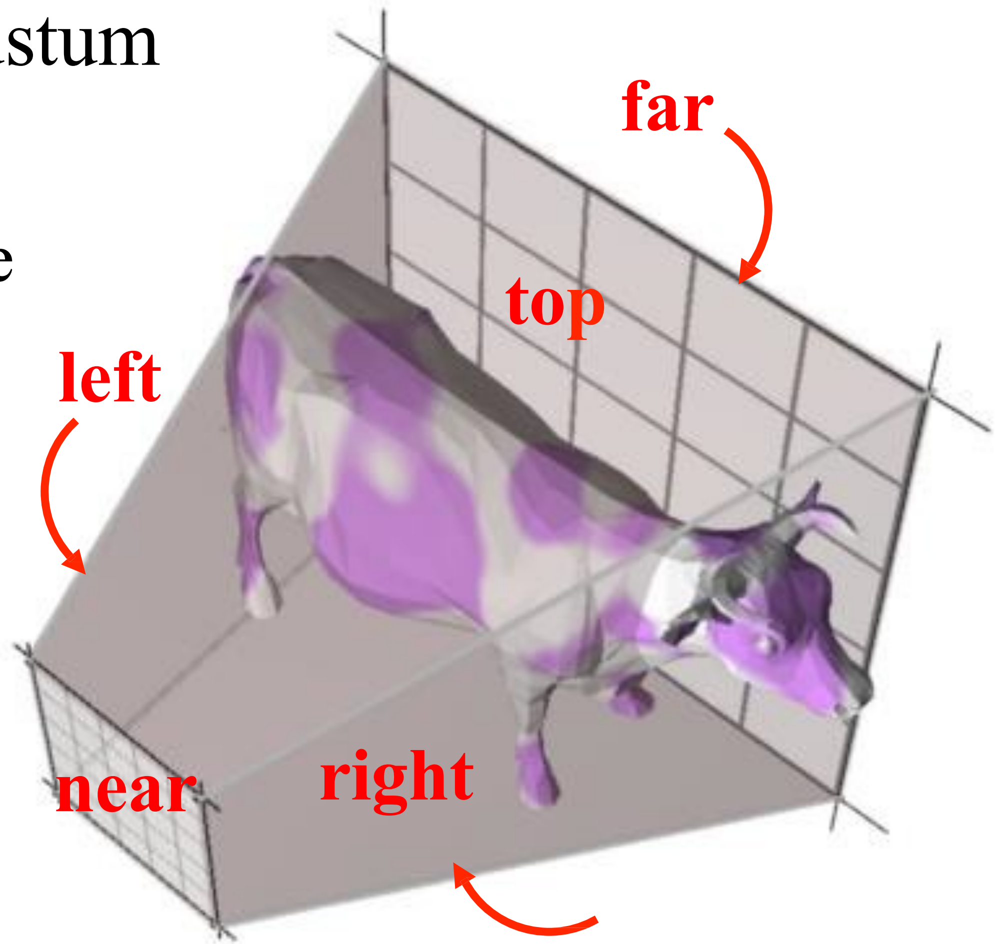


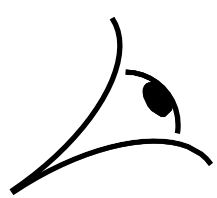
A Solution: Clipping



Clipping

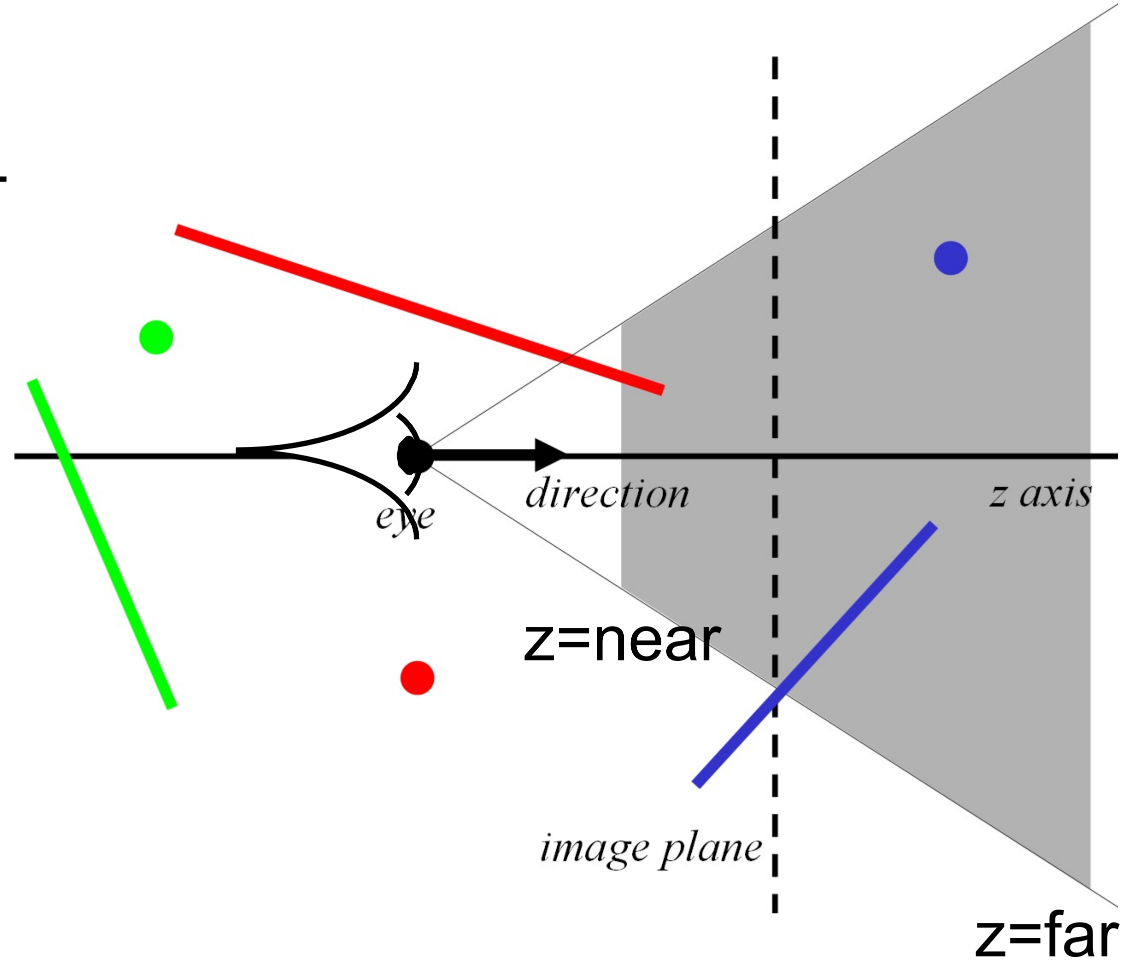
- Eliminate portions of objects outside the viewing frustum
- View Frustum
  - boundaries of the image plane projected in 3D
  - a near & far clipping plane
- User may define additional clipping planes





Why Clip?

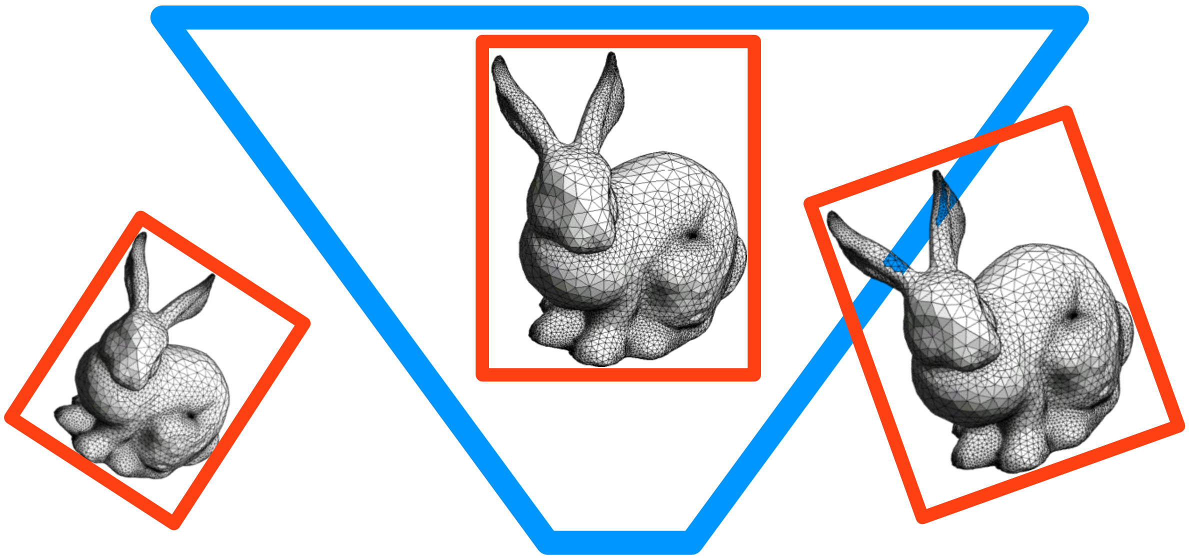
- Avoid degeneracies
  - Don't draw stuff behind the eye
  - Avoid division by 0 and overflow



Related Idea

- "View Frustum Culling"
  - Use bounding volumes/hierarchies to test whether any part of an object is within the view frustum
    - Need "frustum vs. bounding volume" intersection test
    - Crucial to do hierarchically when scene has *lots* of objects!
    - Early rejection (different from clipping)

See e.g. [Optimized view frustum culling algorithms for bounding boxes](), Ulf Assarsson and Tomas Möller, journal of graphics tools, 2000.



Related Idea

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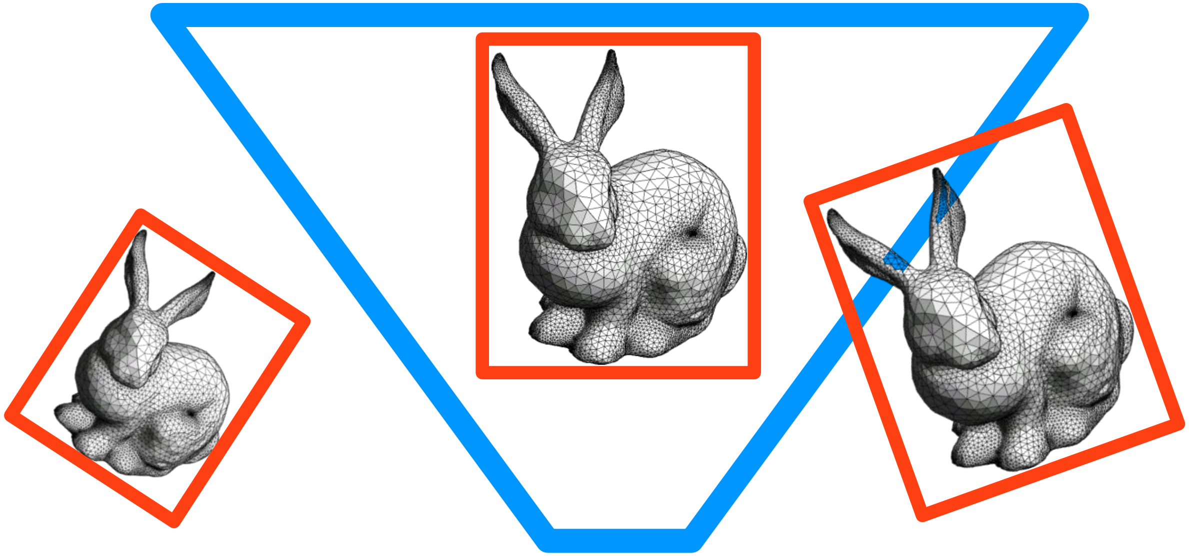


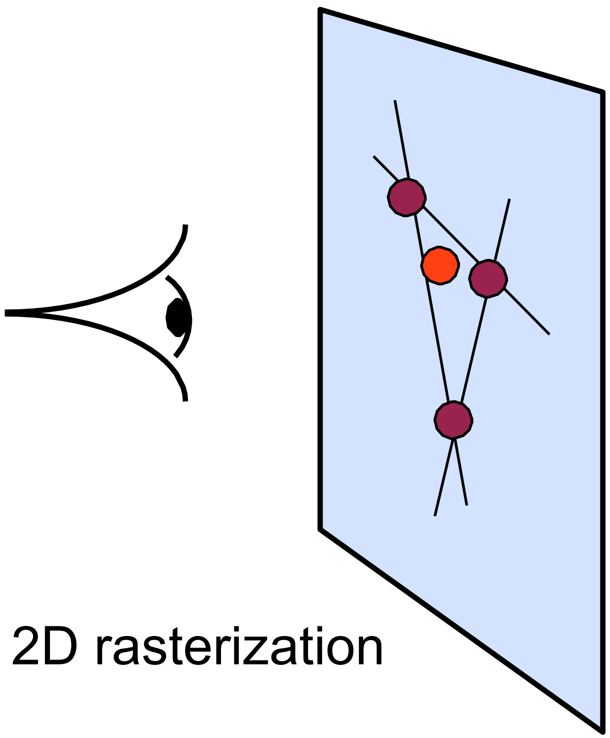
Homogeneous Rasterization

- Idea: avoid projection (and division by zero) by performing rasterization in 3D
  - Or equivalently, use 2D homogenous coordinates (w'=z after the projection matrix, remember)

- **Motivation: clipping is annoying**

- Marc Olano, Trey Greer: Triangle scan conversion using 2D homogeneous coordinates, Proc. ACM SIGGRAPH/Eurographics Workshop on Graphics Hardware 1997

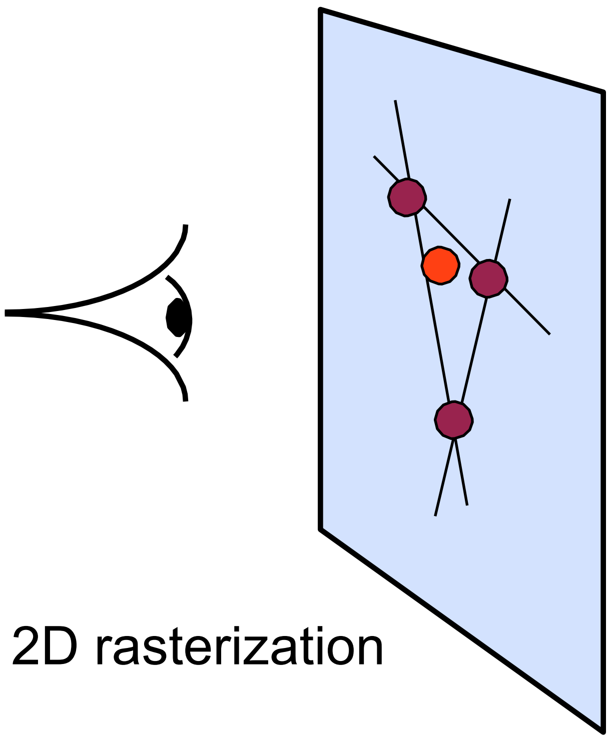
Homogeneous Rasterization



2D rasterization

Homogeneous Rasterization

- Replace 2D edge equation by 3D plane equation
  - Plane going through 3D edge and viewpoint
  - Still a halfspace, just 3D



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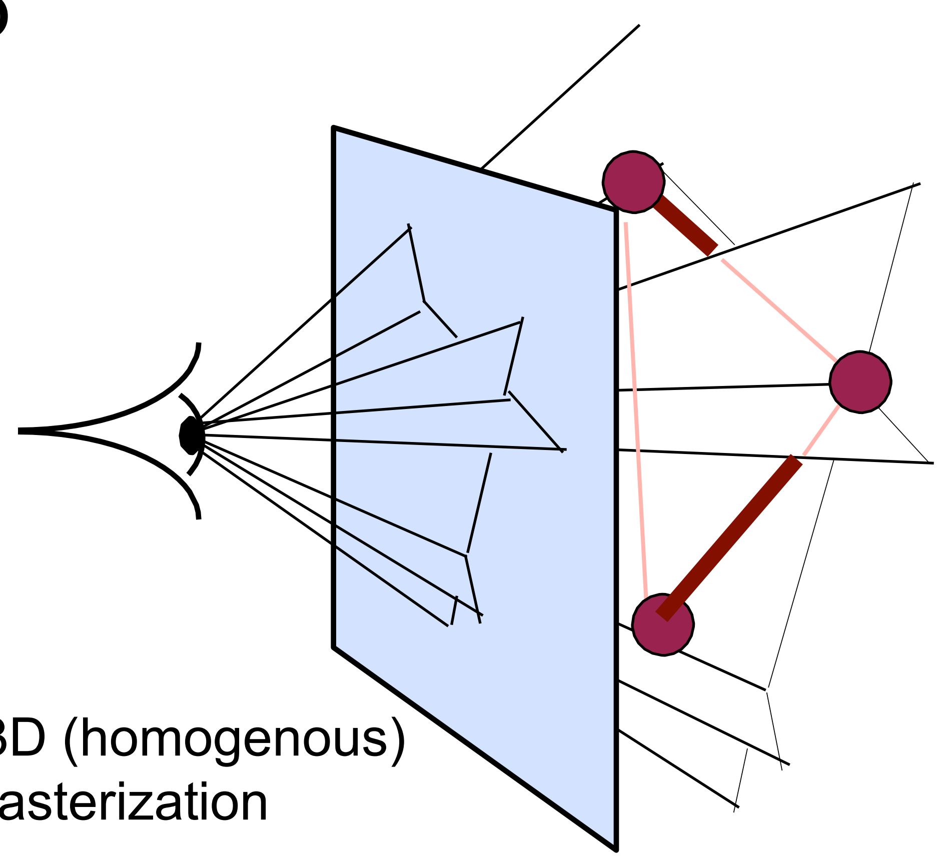
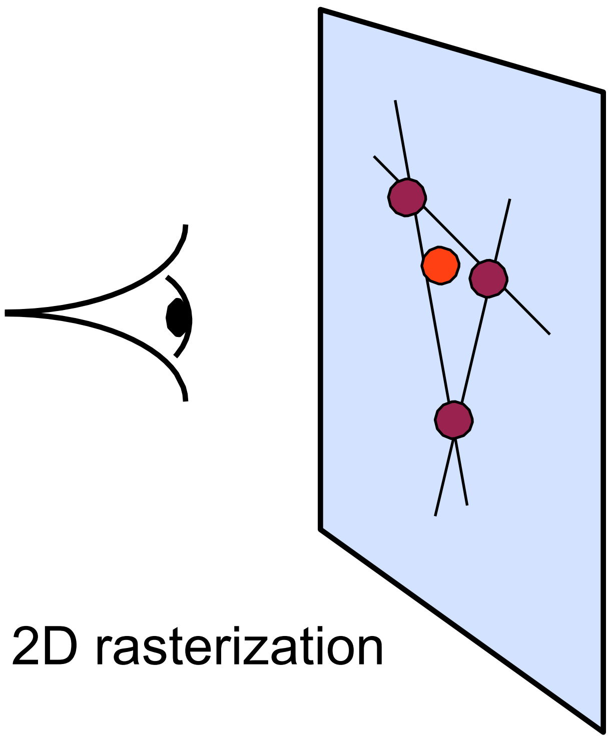


3D (homogenous) rasterization

Homogeneous Rasterization

- Replace 2D edge equation by 3D plane equation
  - Treat pixels as 3D points (x, y, 1) on image plane, test for containment in 3 halfspaces just like edge functions



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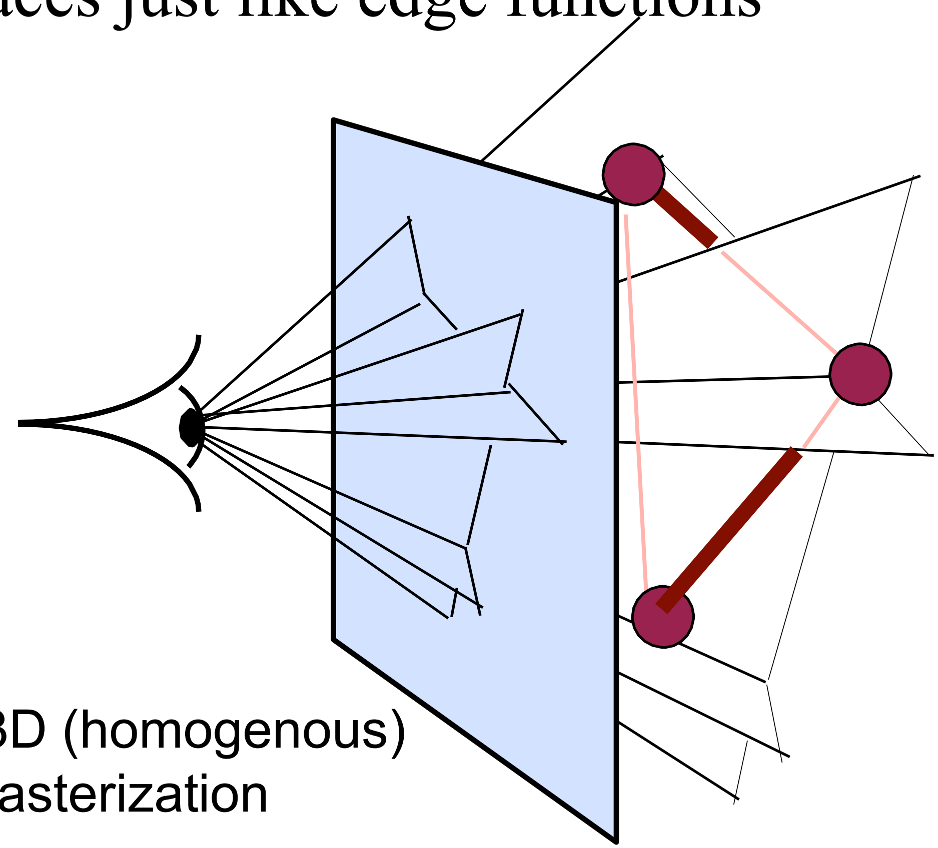


3D (homogenous) rasterization

Homogeneous Rasterization

Given 3D triangle

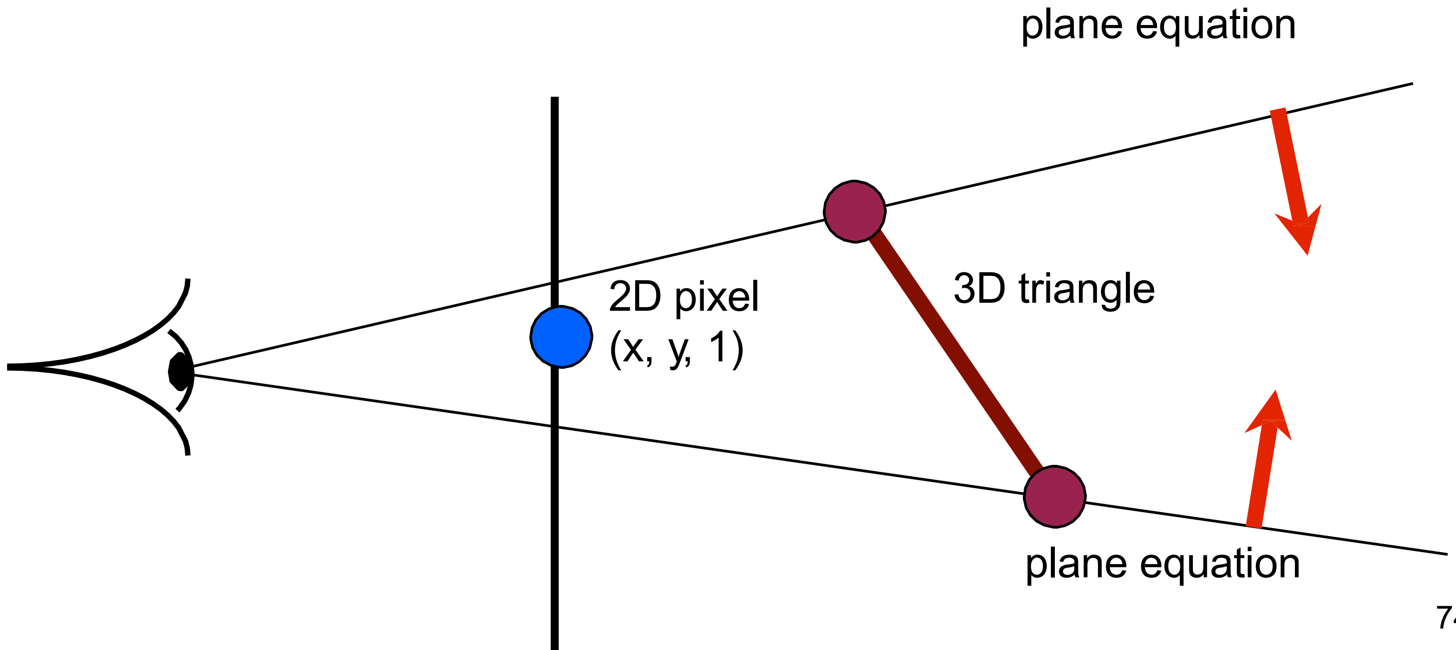
setup plane equations

(plane through viewpoint & triangle edge)

For each pixel x,y

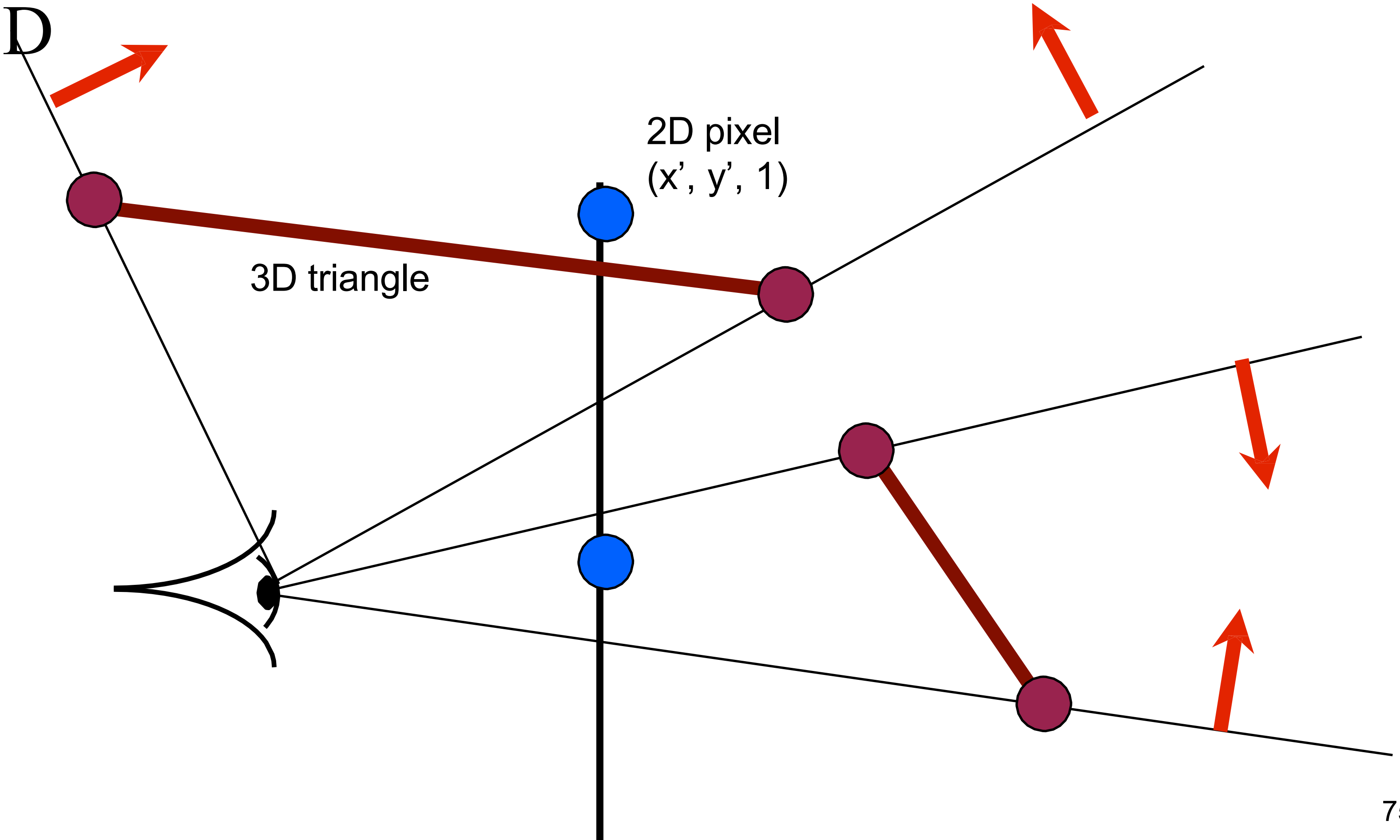
compute plane equations for (x,y,1)

if all pass, draw pixel



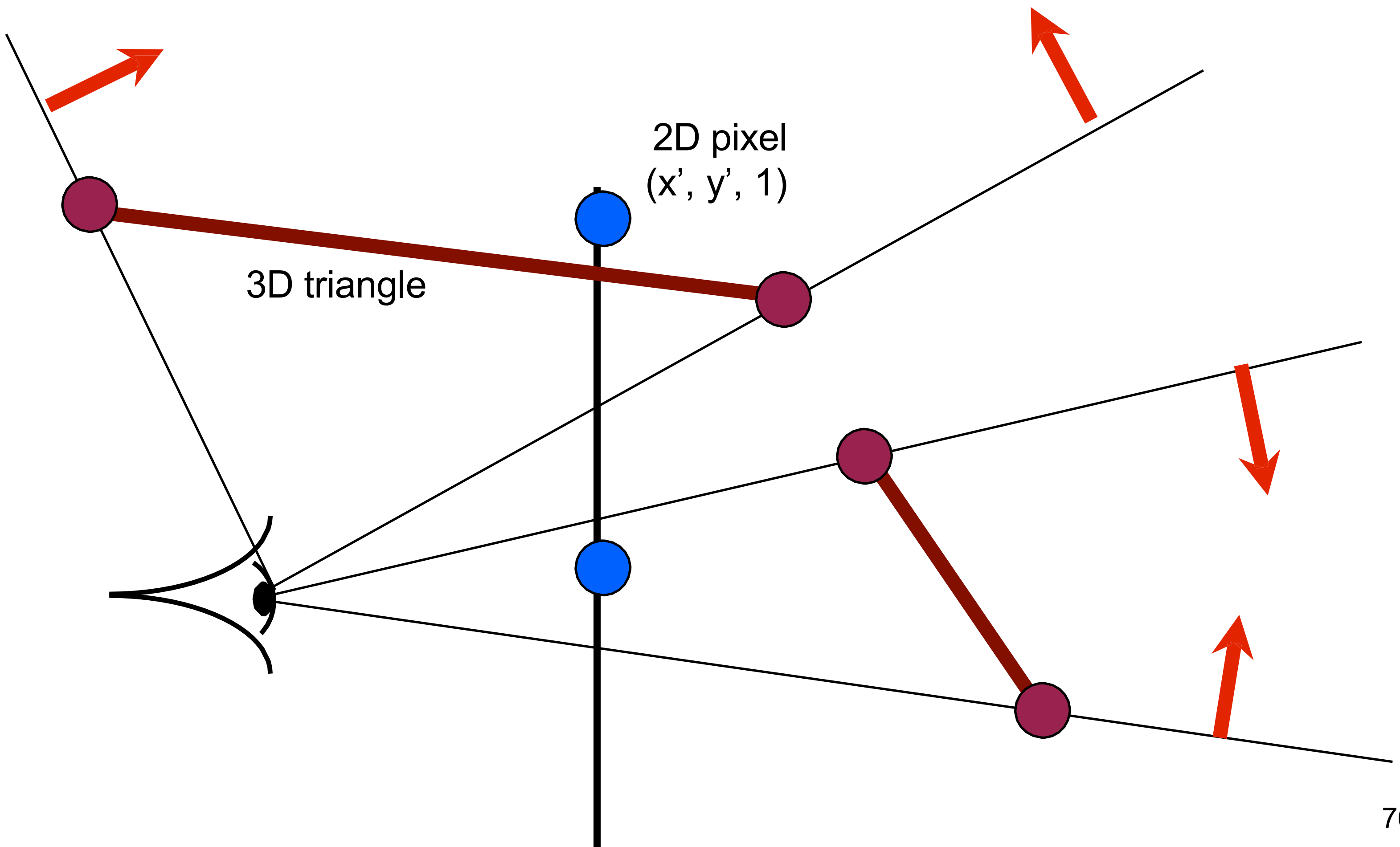
Homogeneous Rasterization

- Works for triangles behind eye
- Still linear, can evaluate incrementally/hierarchically like 2D



Homogeneous Rasterization Recap

- Rasterizes with plane tests instead of edge tests
- **Removes the need for clipping!**



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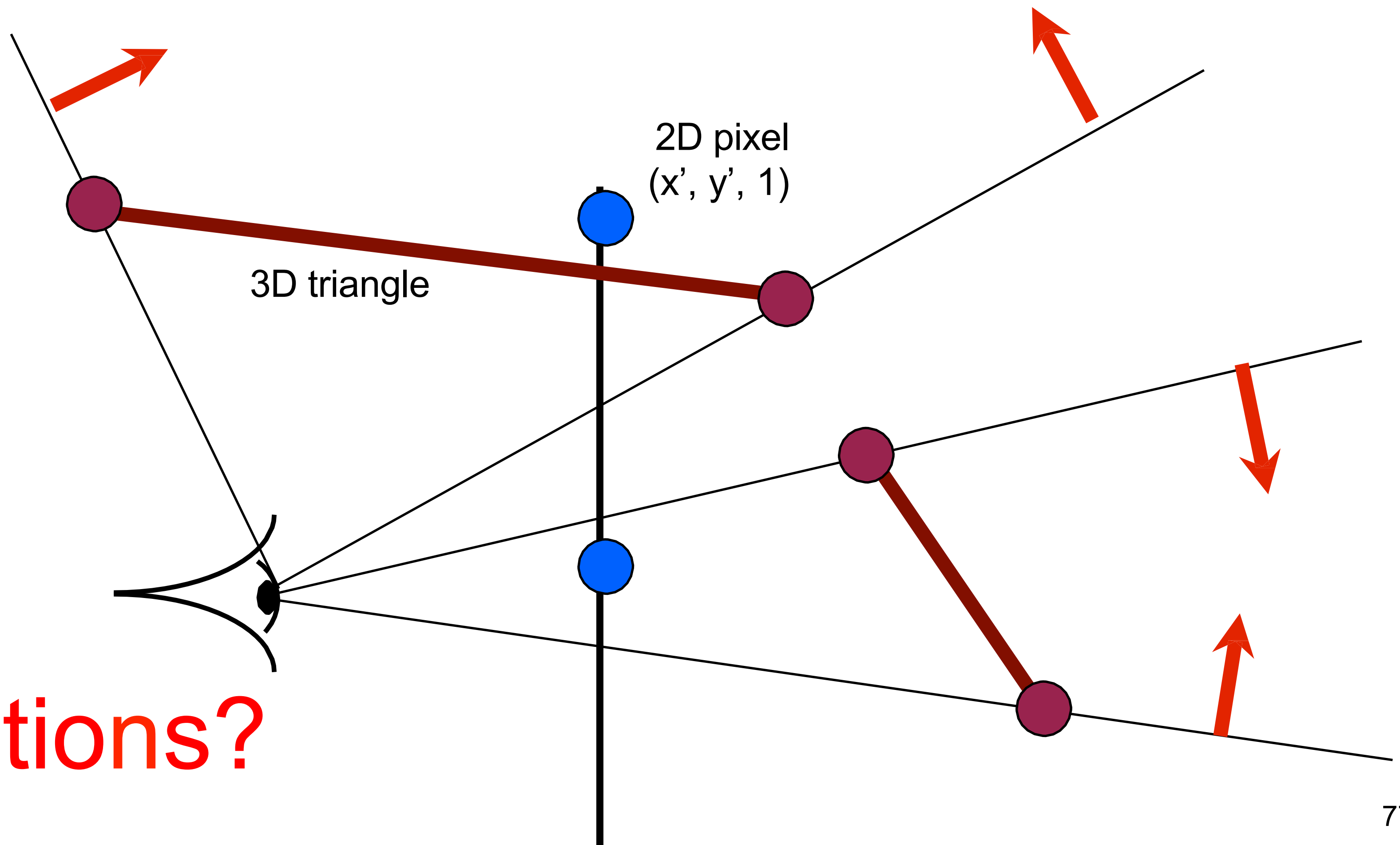
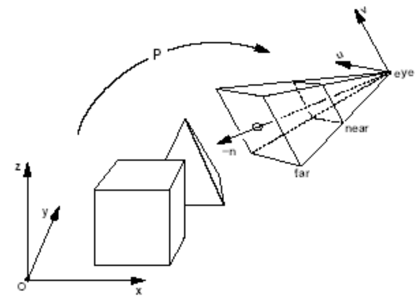


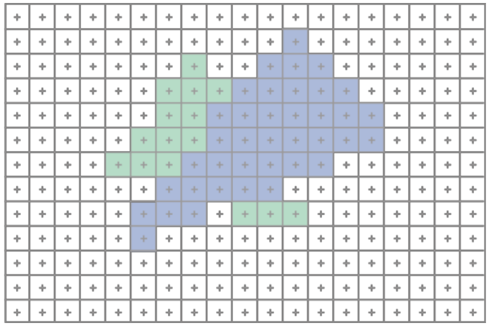
Questions?

Modern Graphics Pipeline

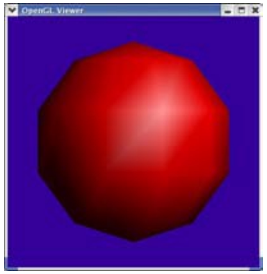
- Perform projection of vertices



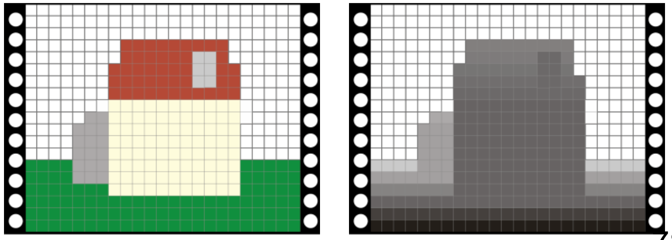
- Rasterize triangle: find which pixels should be lit



- Compute per-pixel color

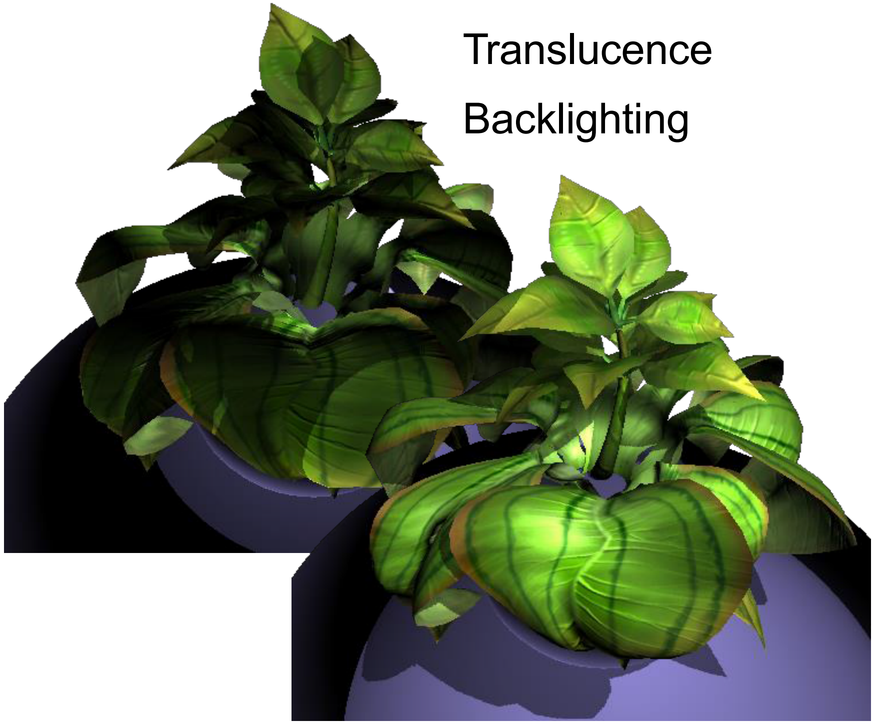


- Test visibility, update frame buffer



Pixel Shaders

- Modern graphics hardware enables the execution of rather complex programs to compute the color of every single pixel
  - More later



Translucence

Backlighting

Procedural texture,
Anisotropic brdf

iridescence