

FGT: 2015/2016

# Parametrization & Remeshing

Nico Pietroni ([nico.pietroni@isti.cnr.it](mailto:nico.pietroni@isti.cnr.it))

ISTI – CNR

# What is a parametrization?



# What is a parametrization?



Mollweide-Projektion



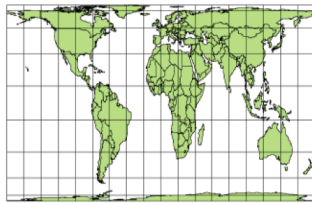
Mercator-Projektion



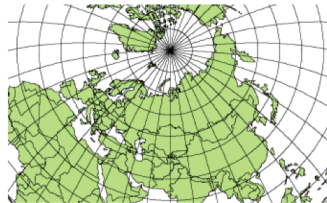
Zylinderprojektion nach Miller



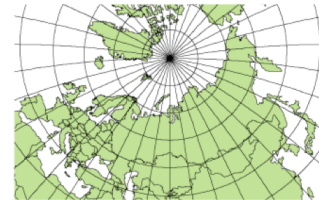
Hammer-Aitoff-Projektion



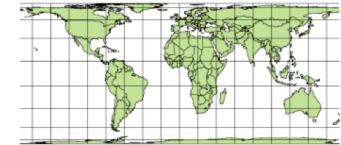
Peters-Projektion



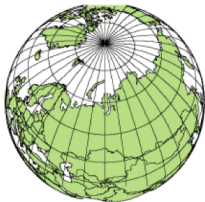
Längentreue Azimuthalprojektion



Stereographische Projektion



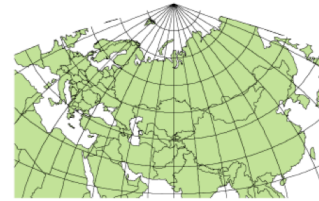
Behrmann-Projektion



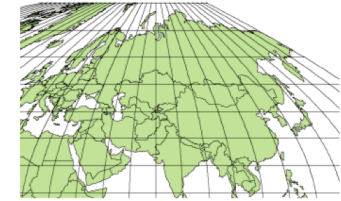
Senkrechte Umgebungsperspektive



Robinson-Projektion



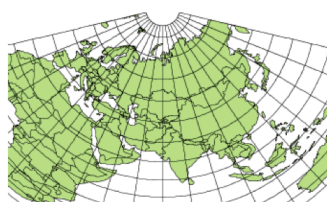
Hotine Oblique Mercator-Projektion



Sinusoidale Projektion



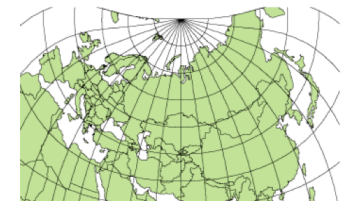
Gnomonische Projektion



Flächentreue Kegelprojektion



Transverse Mercator-Projektion



Cassini-Soldner-Projektion

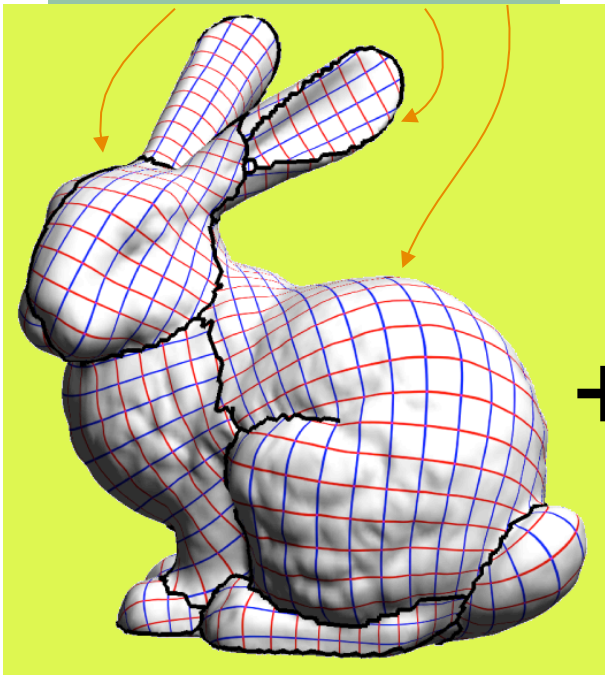
...

..

# Why Parametrization?

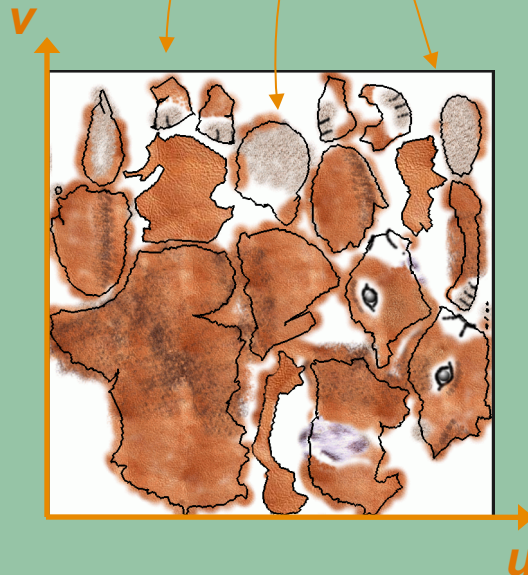
## ▣ Texture Mapping

disk-like patches



3D mesh

texture charts



2D texture image

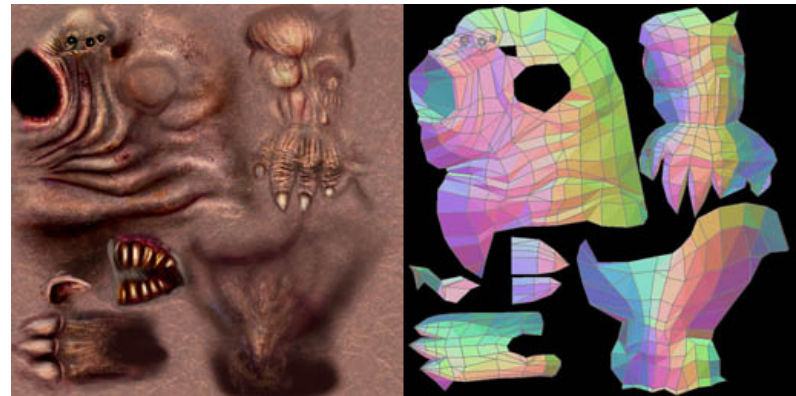
=



textured bunny

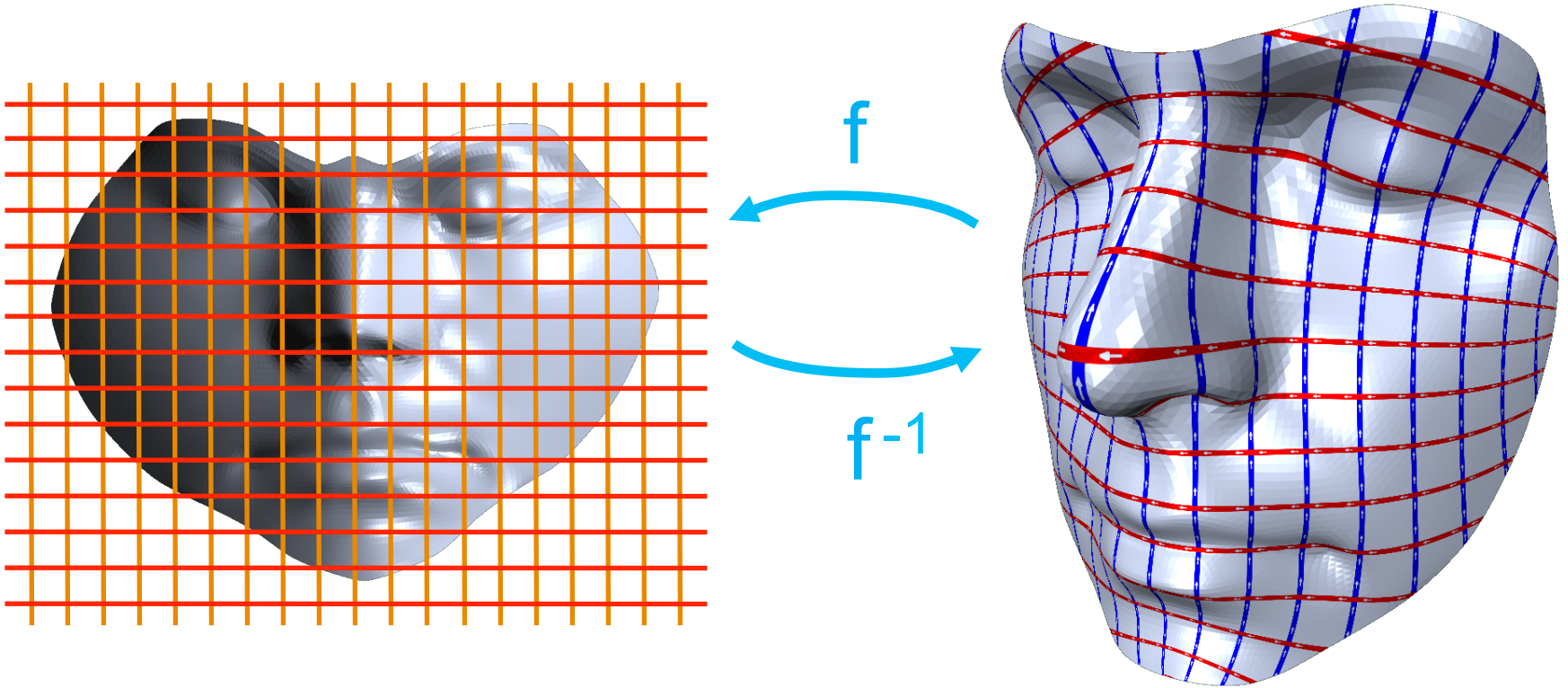
# Why Parametrization?

- Manual UV mapping
- An advanced artistic skill



# Why Parametrization?

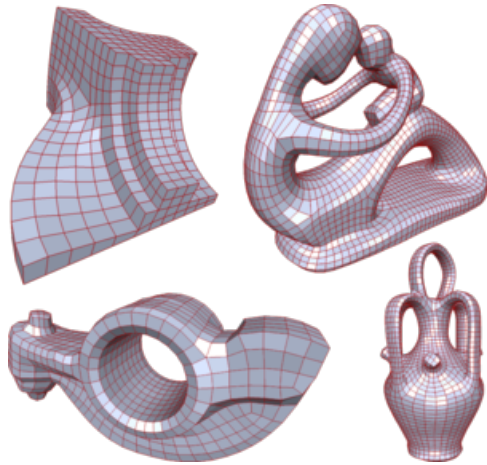
## ▣ Remeshing



# Why Parametrization?

## ▣ Remeshing

### QUADRILATERAL



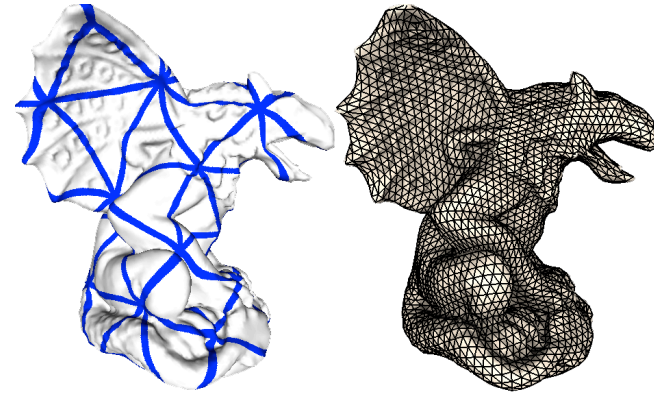
Bommes, et AL.: *Mixed Integer Quadrangulation*

### HEXAGONAL



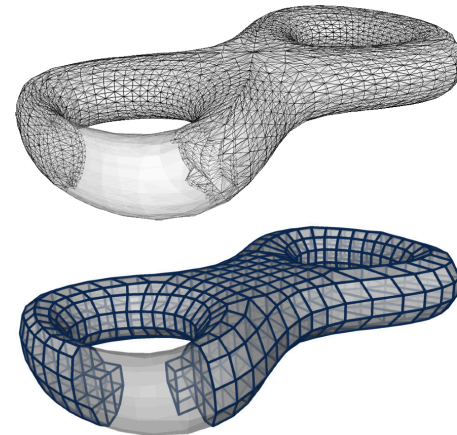
Nieser et al.: *Hexagonal Global Parameterization of Arbitrary Surfaces*

### TRIANGULAR



Pietroni, et AL.: *Almost isometric mesh parameterization through abstract domains*

### HEXAHERAL



Nieser, et AL. : *CUBECOVER – Parameterization of 3D Volumes*

# Why Parametrization?

- Analysys.... 2D is easier than 3D

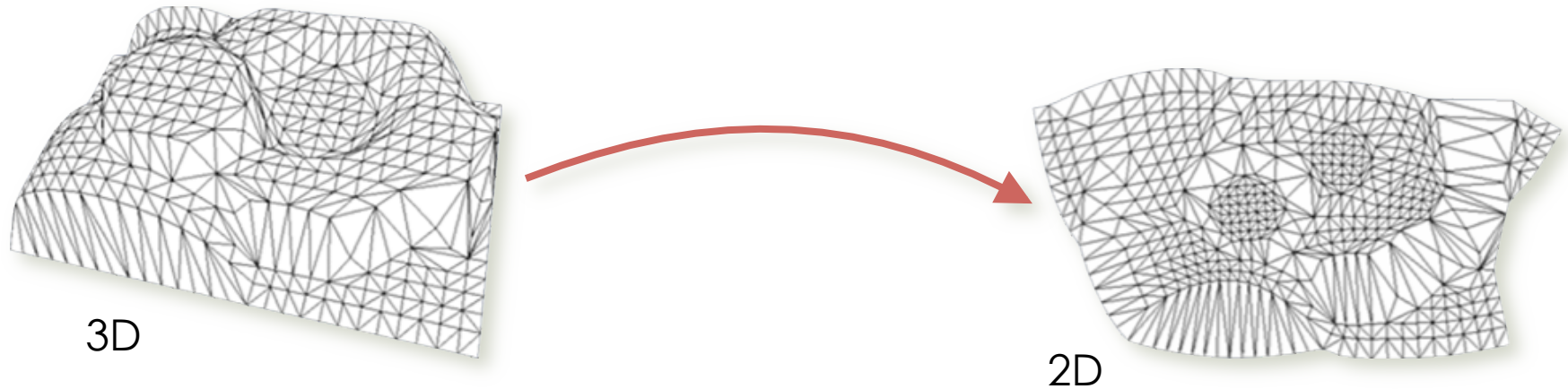


Pietroni, et AL.: *An Interactive Local Flattening Operator to Support Digital Investigations on Artwork Surfaces*

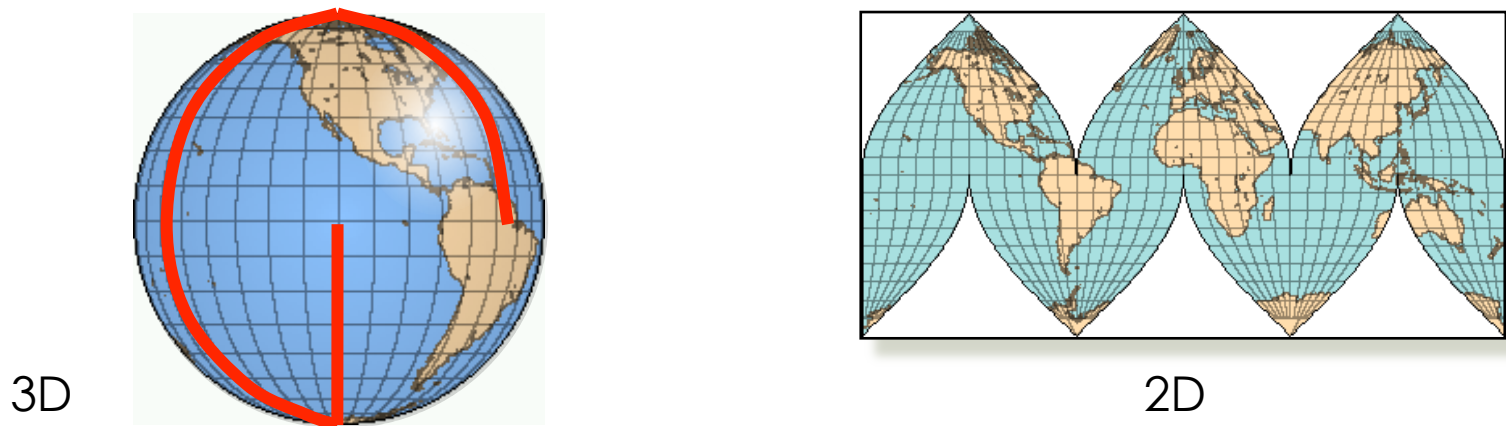


# Parametrization: what we need?

- A strategy to flatten a 3D surface on 2D domain
  - Introducing as few distortion as possible

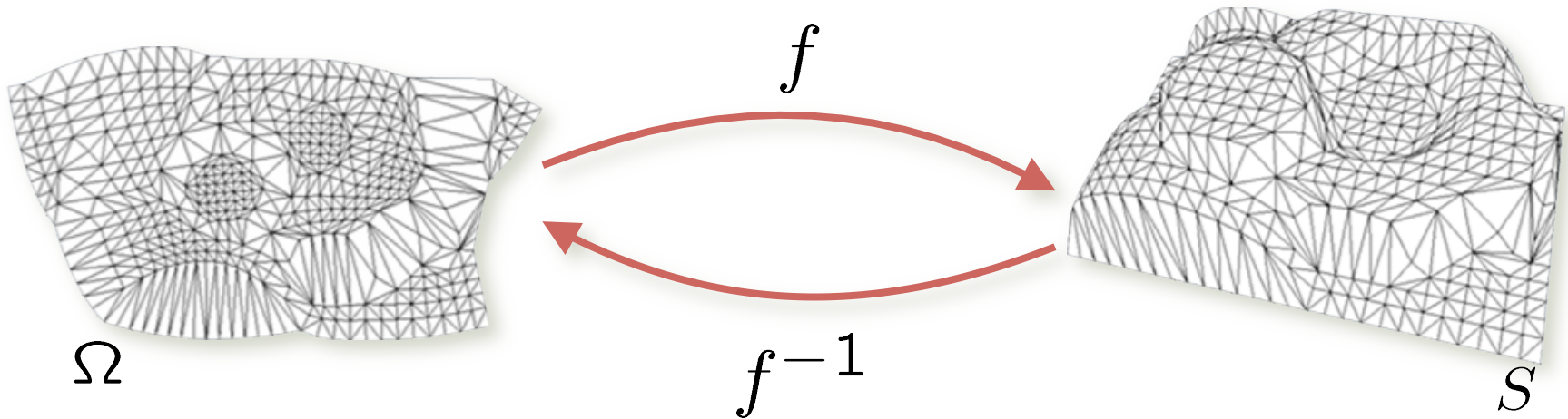


- A strategy to introduce cuts

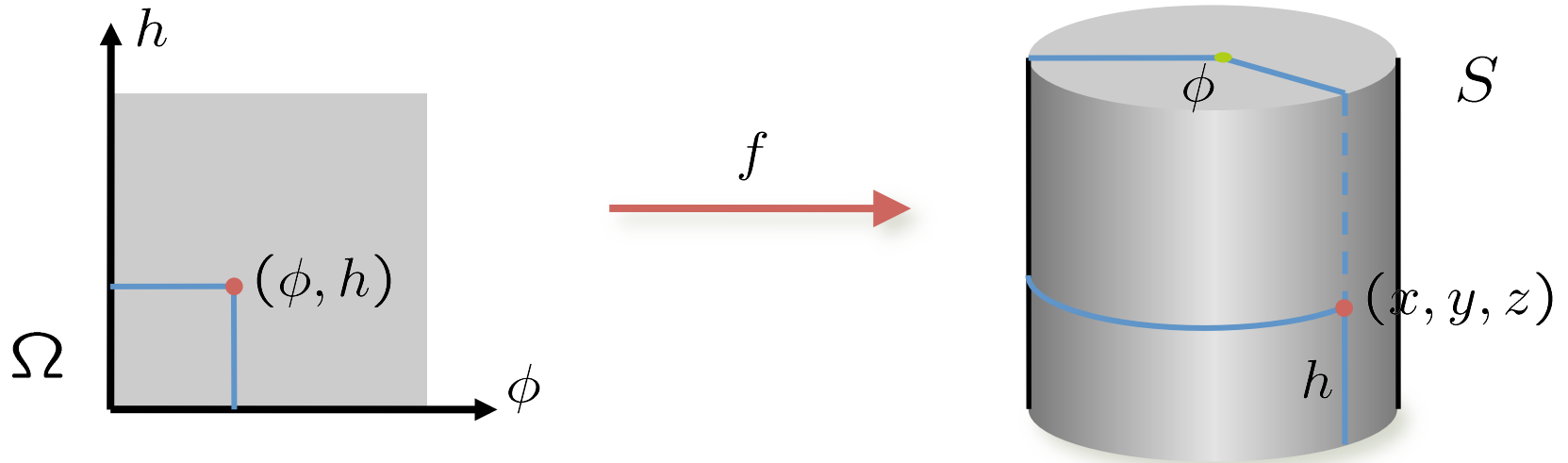


# Flattening a surface

- surface  $S \subset \mathbb{R}^3$
- parameter domain  $\Omega \subset \mathbb{R}^2$
- mapping  $f : \Omega \rightarrow S$  and  $f^{-1} : S \rightarrow \Omega$



# Parametrization: Cylindrical coords

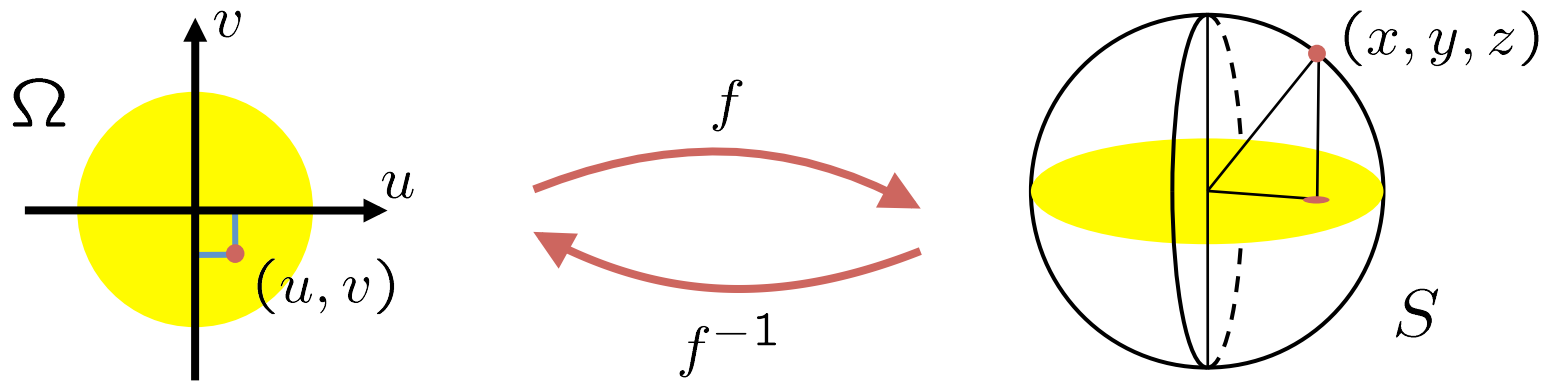


$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1, z \in [0, 1]\}$$

$$\Omega = \{(\phi, h) \in \mathbb{R}^2 : \phi \in [0, 2\pi), h \in [0, 1]\}$$

$$f(\phi, h) = (\sin \phi, \cos \phi, h)$$

# Parametrization: Ortho Projection



$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, z \geq 0\}$$

$$\Omega = \{(u, v) \in \mathbb{R}^2 : u^2 + v^2 \leq 1\}$$

$$f^{-1}(x, y, z) = (x, y)$$

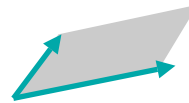
$$f(u, v) = (u, v, \sqrt{1 - u^2 - v^2})$$

# Minimize Distortion

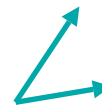
■ Angle preservation: conformal



■ Area preservation: equiareal

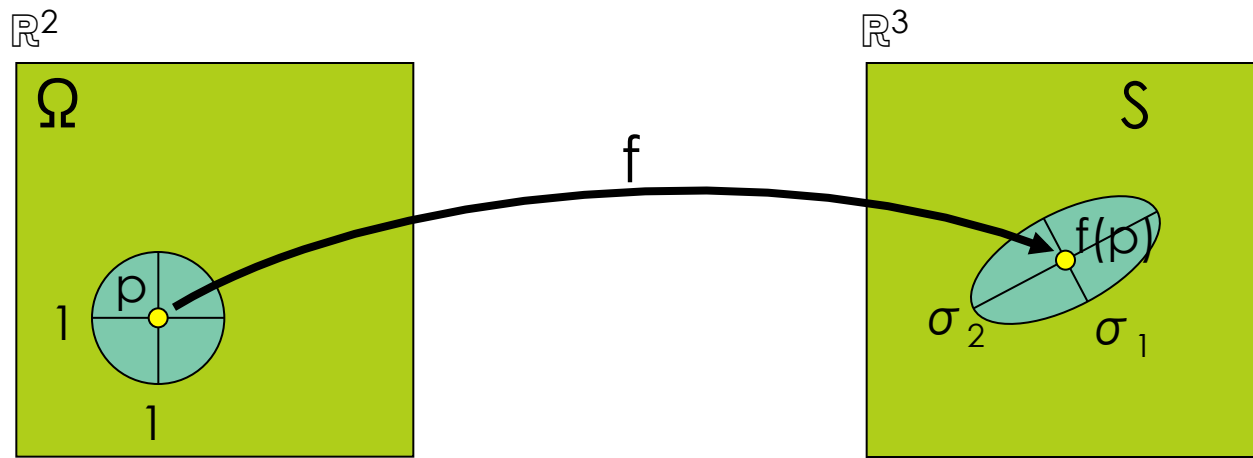


■ Area and Angle: Isometric



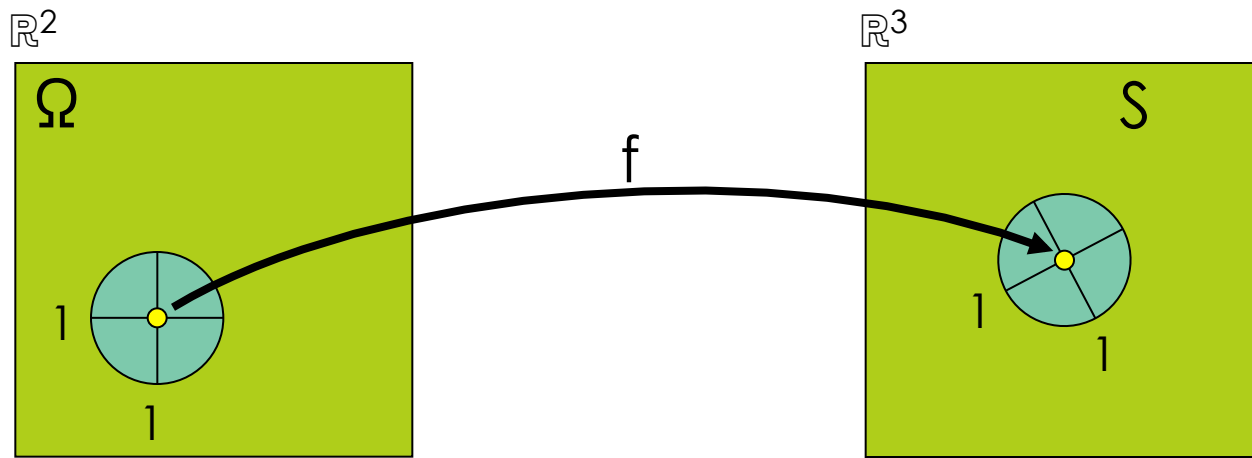
# Distortion

- $\sigma_1$  and  $\sigma_2$  describe local deformations



# Isometric Mapping

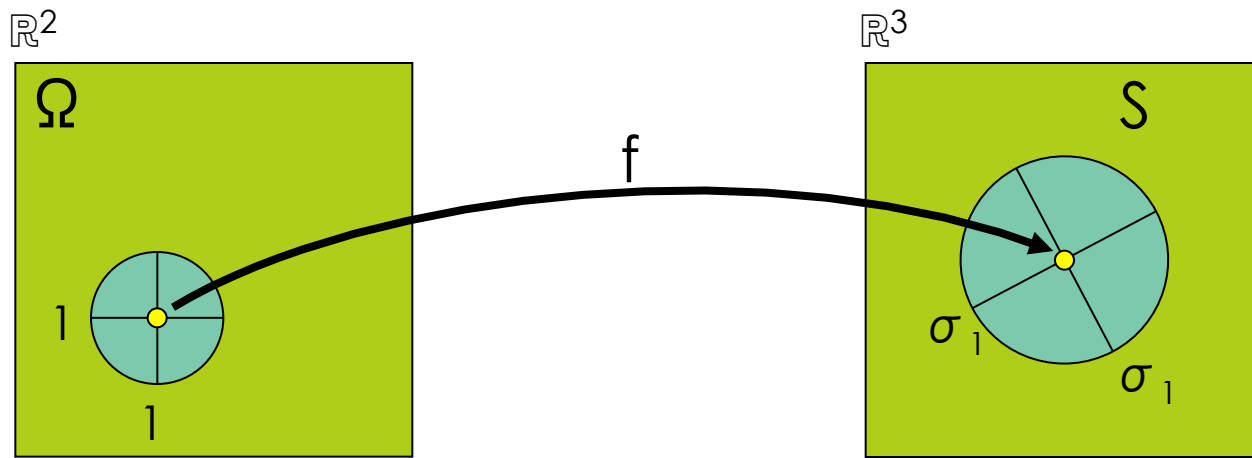
□  $\sigma_1 = \sigma_2 = 1$



□ preserves **areas**, **angles** and **lengths**

# Conformal Mapping

□  $\sigma_1 / \sigma_2 = 1$

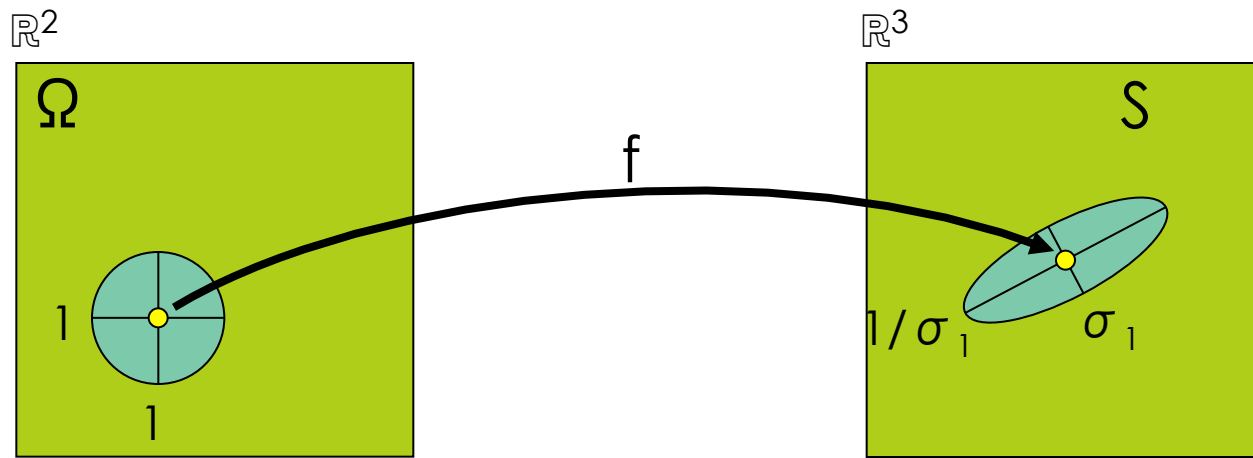


□ preserves **angles**



# Equiareal Mapping

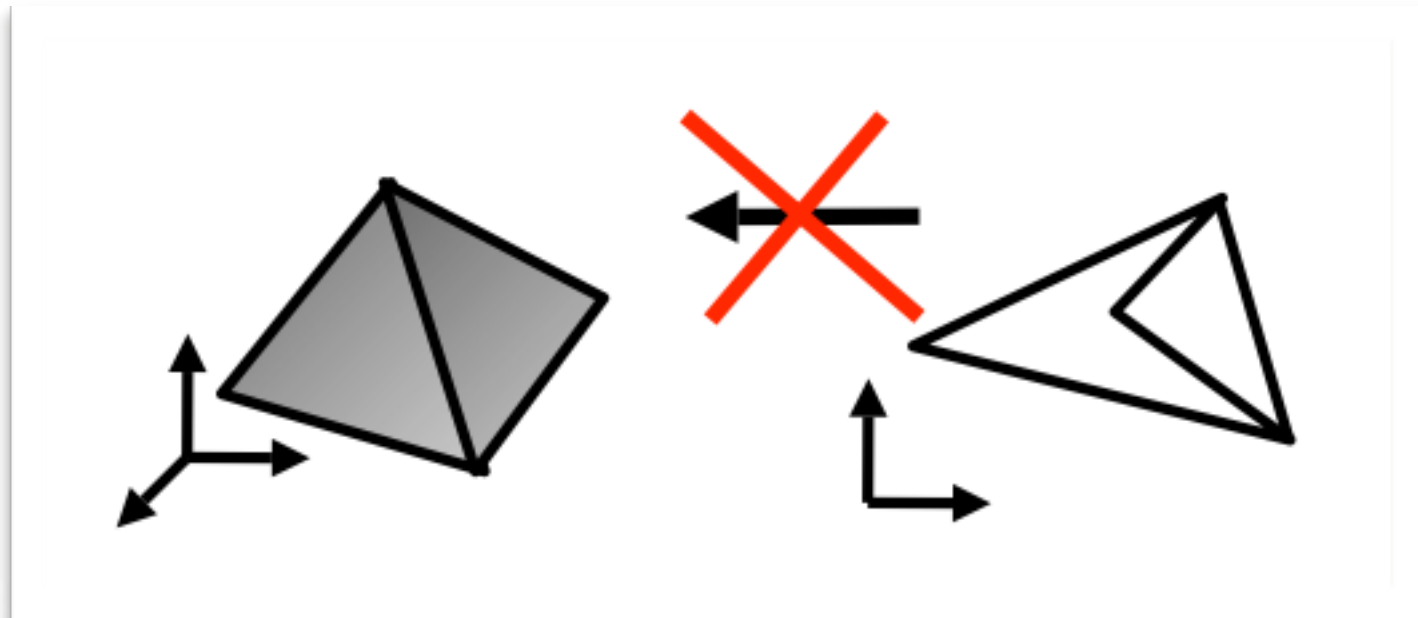
□  $\sigma_1 \cdot \sigma_2 = 1$



□ preserves **areas**

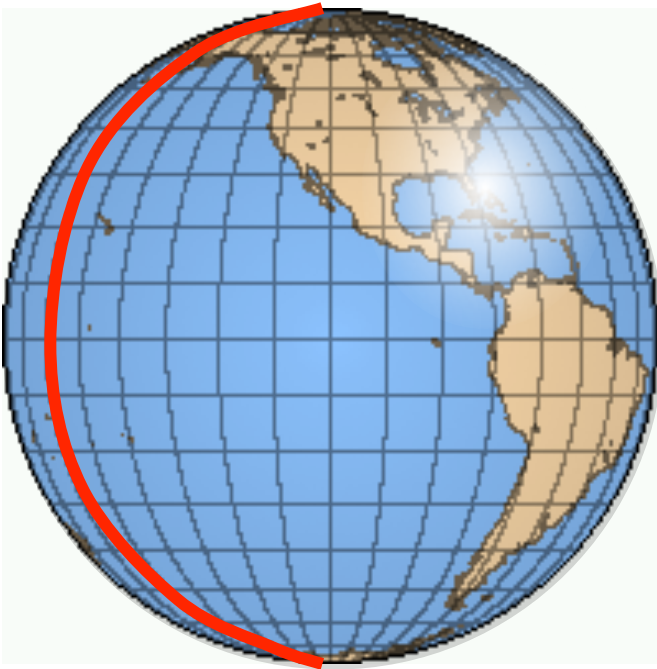
# Bijectivity

- Parametrization map must be bijective  $\Leftrightarrow$  triangles in parametric domain do not overlap (no triangle flips)

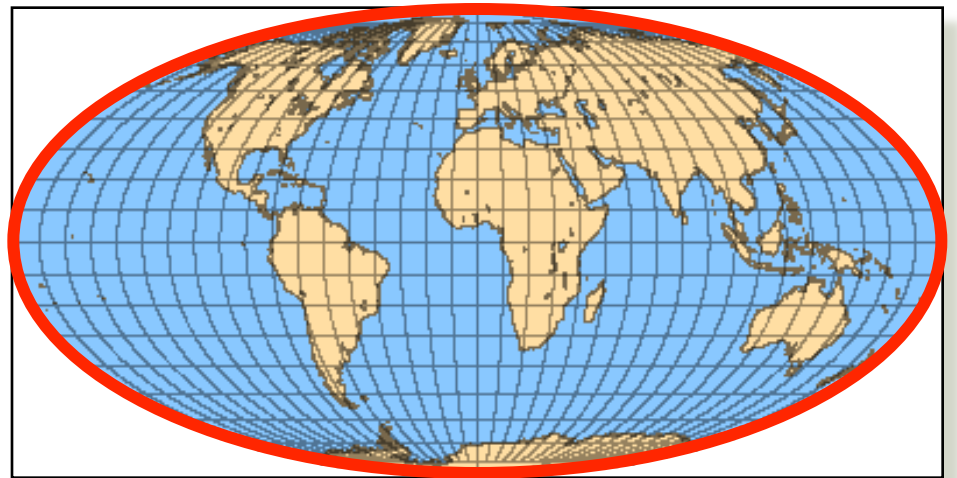


# Cuts 1

- Clearly needed for closed surfaces



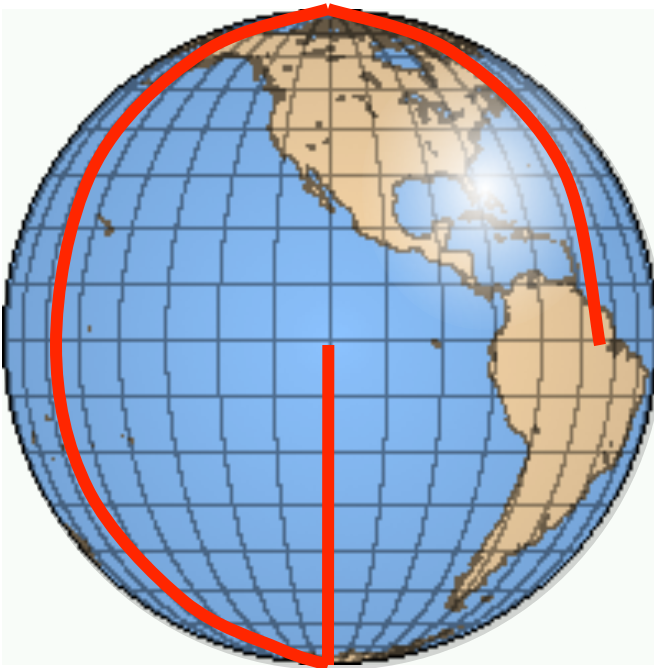
sphere in 3D



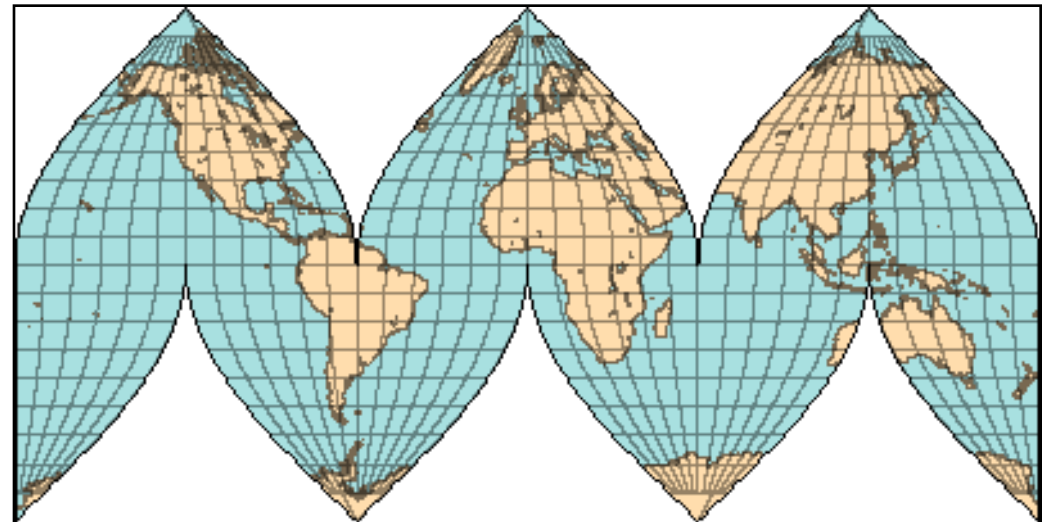
2D surface disk

# Cuts 2

- ▣ Usually more cuts -> less distortion



sphere in 3D



2D surface

# Cuts 3: closed surfaces

■ How many cuts?



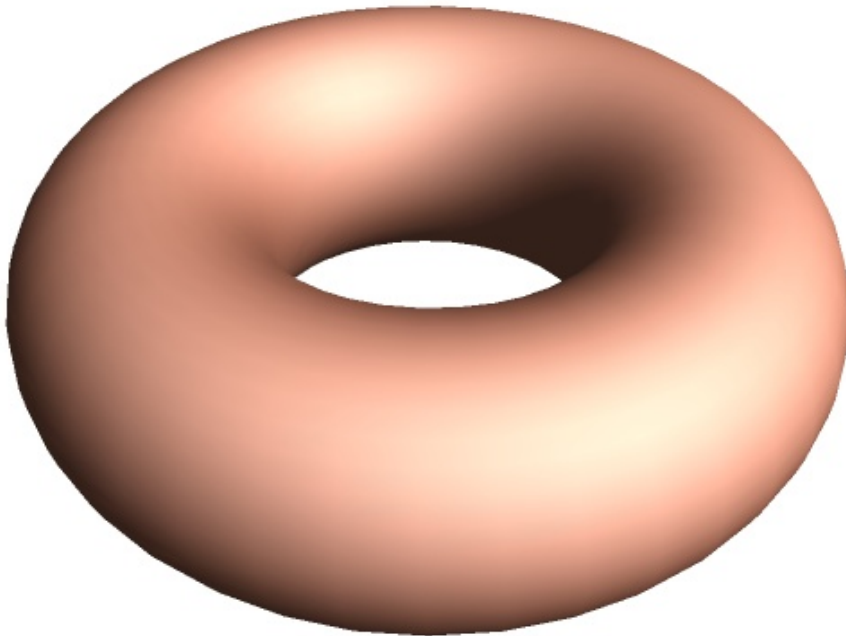
for a genus 0 surface ?



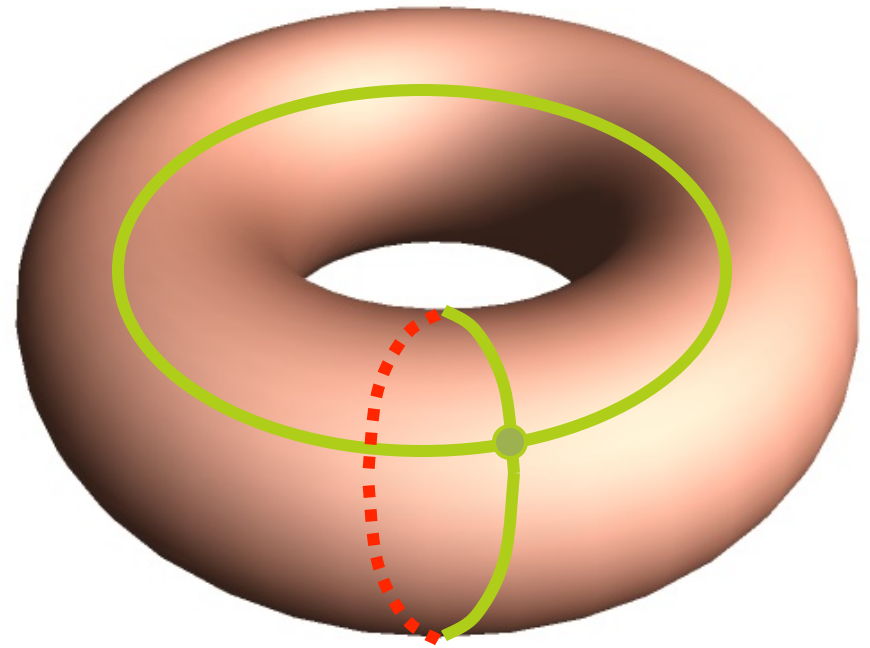
any tree of cuts

# Cuts 3: closed surfaces

■ How many cuts?



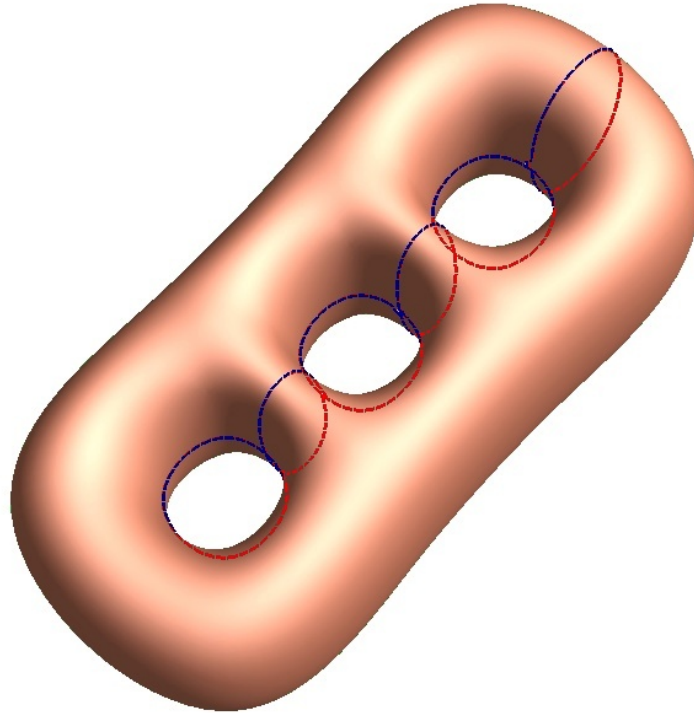
for a genus 1 surface ?



two looped cuts

# Cuts 3: closed surfaces

- How many cuts?

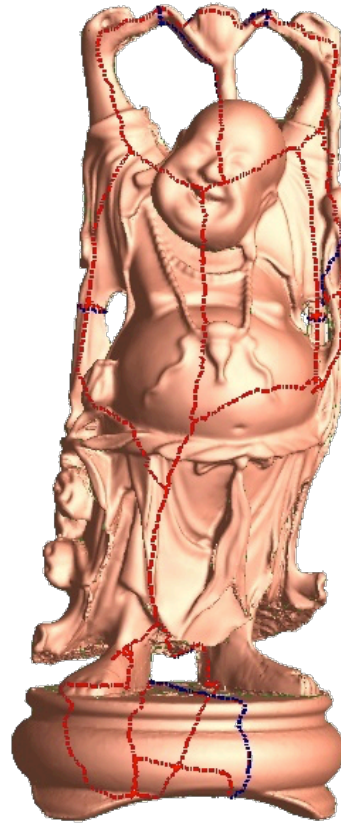


for a genus 3 surface ?

6 looped cuts

# Cuts 3: closed surfaces

- How many cuts?



genus 6

for a genus  $n$  surface ?

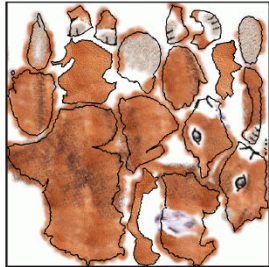
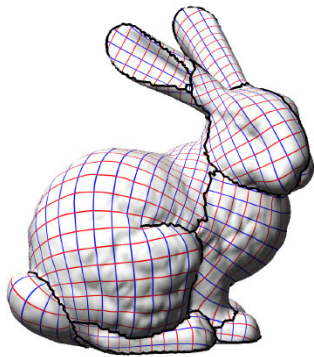
$2n$  looped cuts



# Generic Cut Strategies

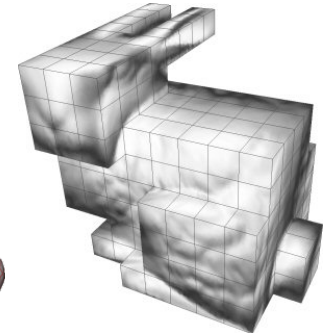
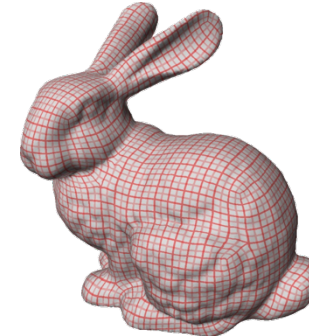
## ■ Texture Mapping

### UNSTRUCTURED CUTS



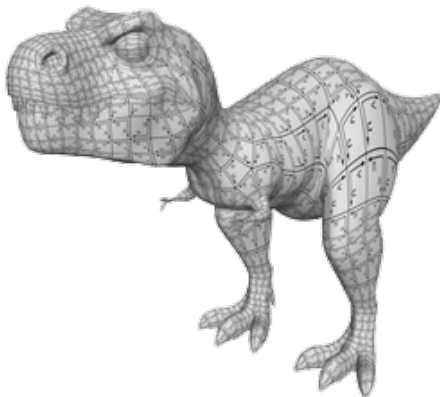
Lévy, et AL.: *Least squares conformal maps for automatic texture atlas generation*

### IMPLICIT



Tarini, et AL.: *PolyCube Maps*

### PER QUAD



Brent Burley et al : *Ptex: Per-Face Texture Mapping for Production Rendering*

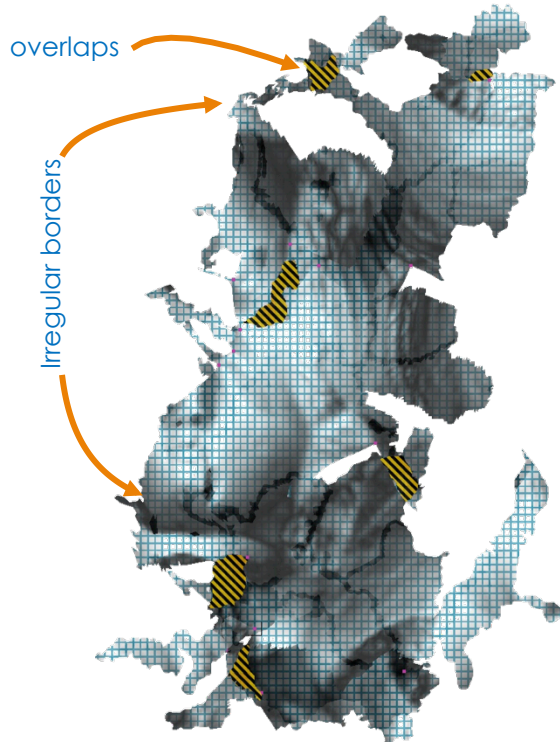
### REGULAR CUTS



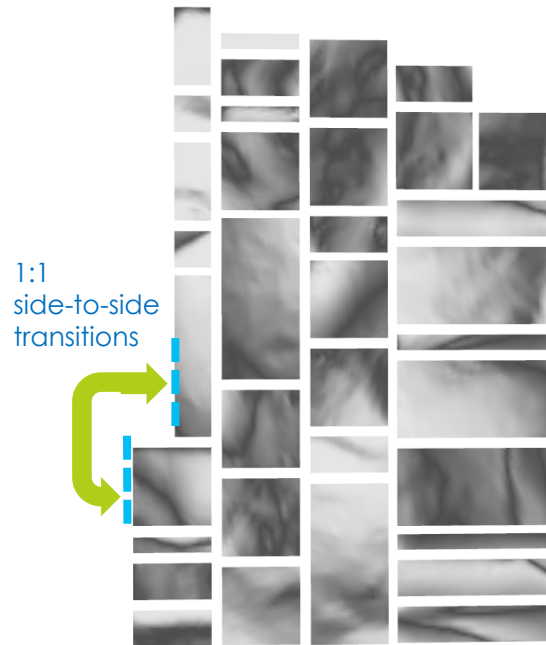
Pietroni, et AL.: *Almost isometric mesh parameterization through abstract domains*

# What is a good Cut?

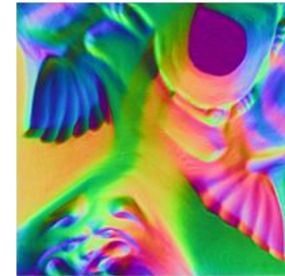
## □ Simplicity of parametric domain



Pietroni, et AL.: *Global Parametrization of Range Image Sets*



Tarini, et AL.: *Simple Quad Domains for Field Aligned Mesh Parametrization*



Implicit transitions



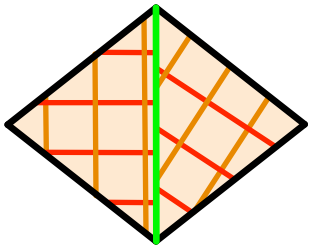
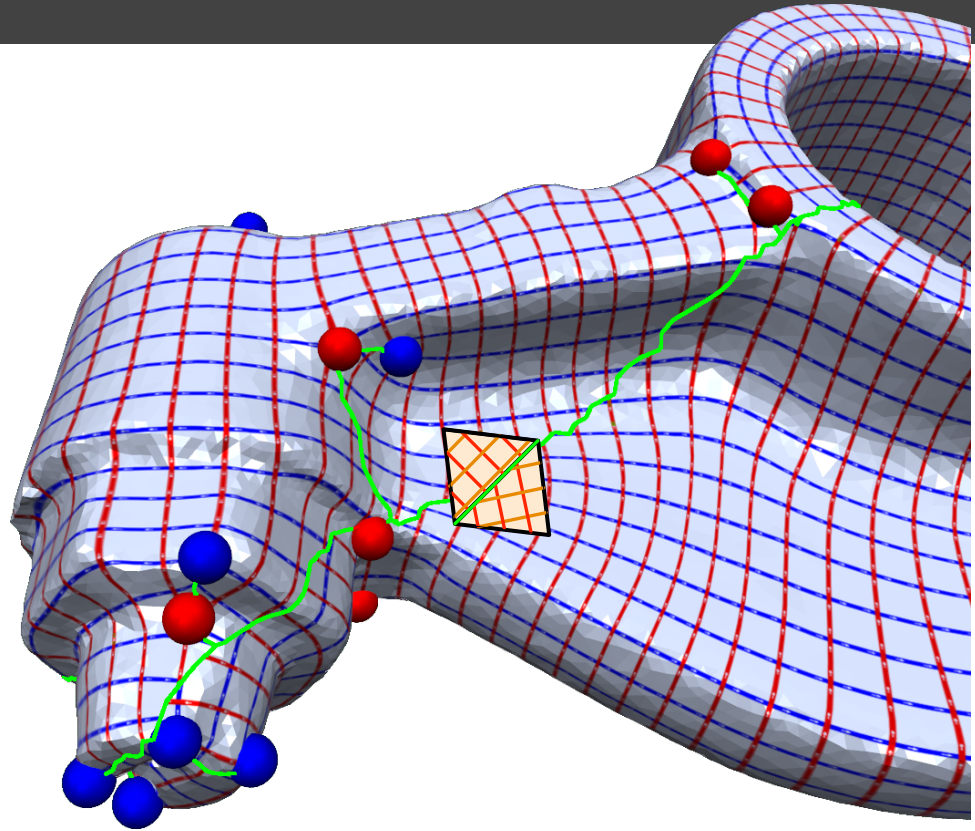
Gu, et AL.: *Goemetry Images*

Versatility

Simplicity

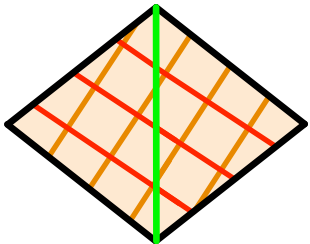
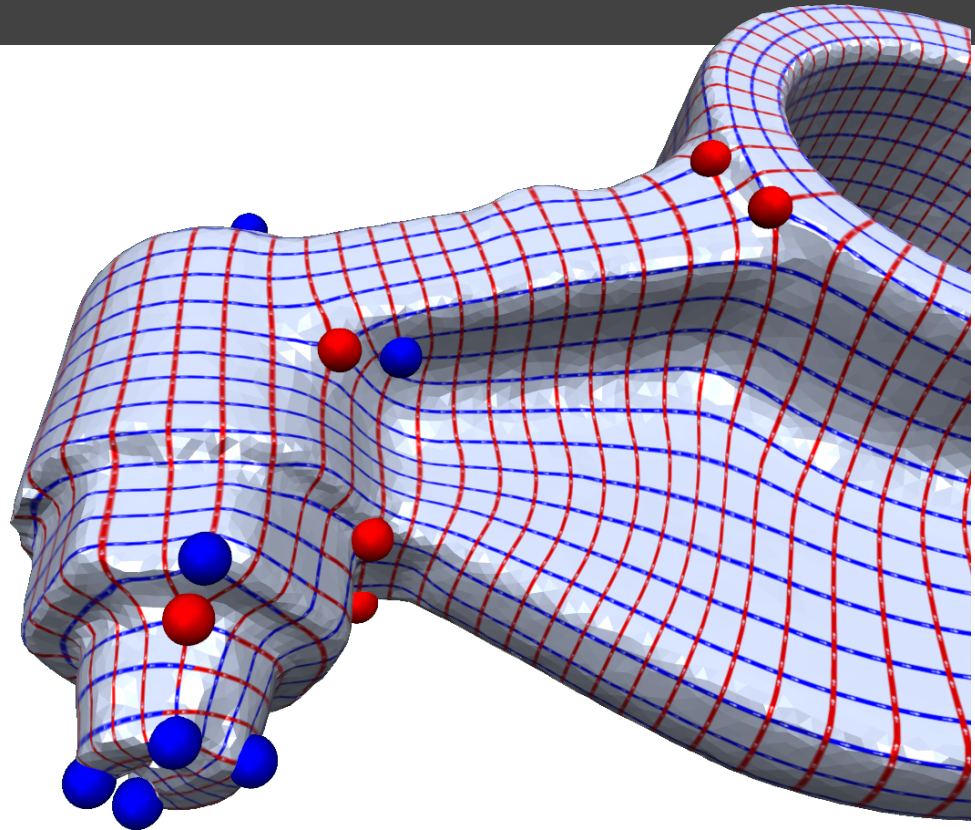
# Globally Smoothness

- Tangent directions varies smoothly across seams



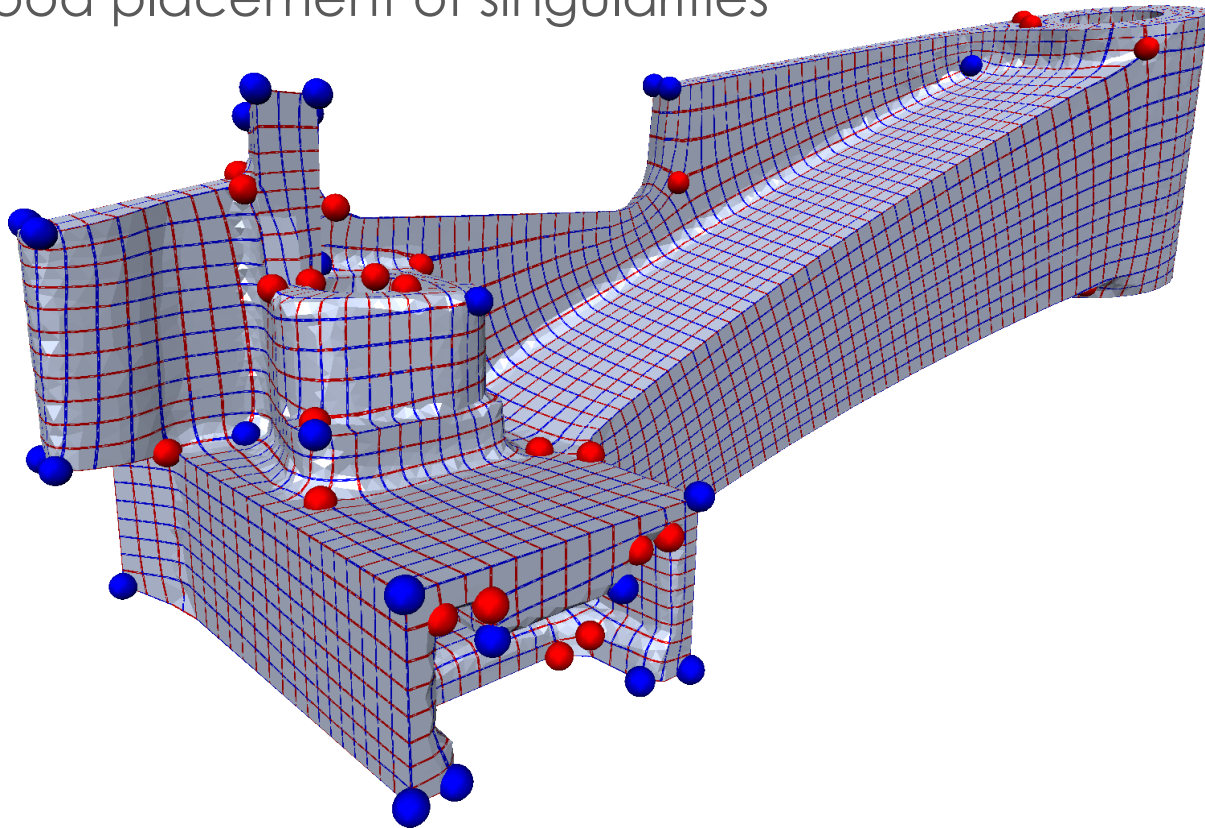
# Globally Smoothness

- Tangent directions varies smoothly across seams



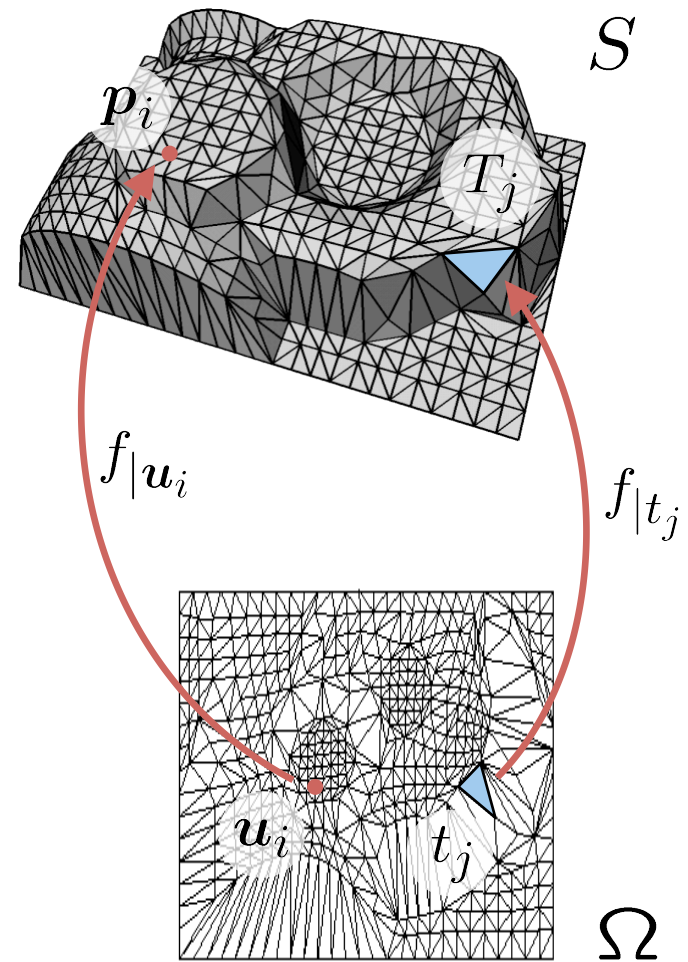
# Feature Alignment

- Useful for quadrangulation
- Need good placement of singularities



# Details: Parametrization

- triangle mesh  $S \subset \mathbb{R}^3$ 
  - vertices  $\mathbf{p}_1, \dots, \mathbf{p}_{n+b}$
  - Triangles  $T_1, \dots, T_m$
- parameter mesh  $\Omega \subset \mathbb{R}^2$ 
  - parameter points  $\mathbf{u}_1, \dots, \mathbf{u}_{n+b}$
  - parameter triangles  $t_1, \dots, t_m$
- parameterization  $f : \Omega \rightarrow S$ 
  - piecewise linear map  $f(t_j) = T_j$



# Parametrization (VCG)

## ▣ triangle mesh

### ▣ vertices

### ▣ Triangles

```
MyTrimesh m;  
MyTrimesh::VertexType *v=&m.vert[0];  
MyTrimesh::CoordType pos=v->P();
```

## ▣ parameter mesh

```
MyTrimesh m;  
MyTrimesh::FaceType *f=&m.face[0];  
MyTrimesh::VertexType *v=f->V(0);
```

```
class MyTriFace;  
class MyTriVertex;  
struct TriUsedTypes: public vcg::UsedTypes<. . .>;  
  
class MyTriVertex:public vcg::Vertex<TriUsedTypes, ...vcg::vertex::TexCoord2d>{};  
class MyTriFace:public vcg::Face<TriUsedTypes, ...vcg::face::WedgeTexCoord2d>{};  
  
class MyTriMesh: public vcg::tri::TriMesh< std::vector<MyTriVertex>,std::vector<MyTriFace > >
```

## ▣ Per vertex UV

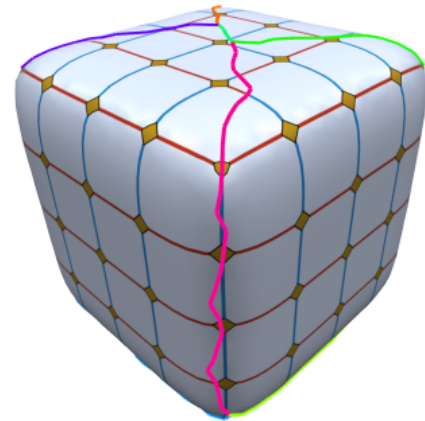
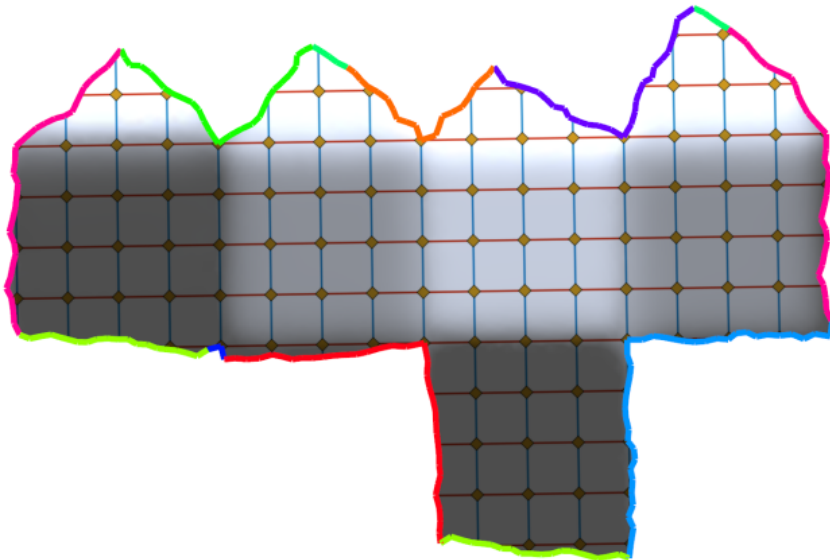
```
MyTrimesh m;  
MyTrimesh::VertexType *v=&m.vert[0];  
vcg::Point2d uv=v->T().P();
```

## ▣ Per Wedge UV

```
MyTrimesh m;  
MyTrimesh::FaceType *f=&m.face[0];  
vcg::Point2d uv=f->T(0).P();
```

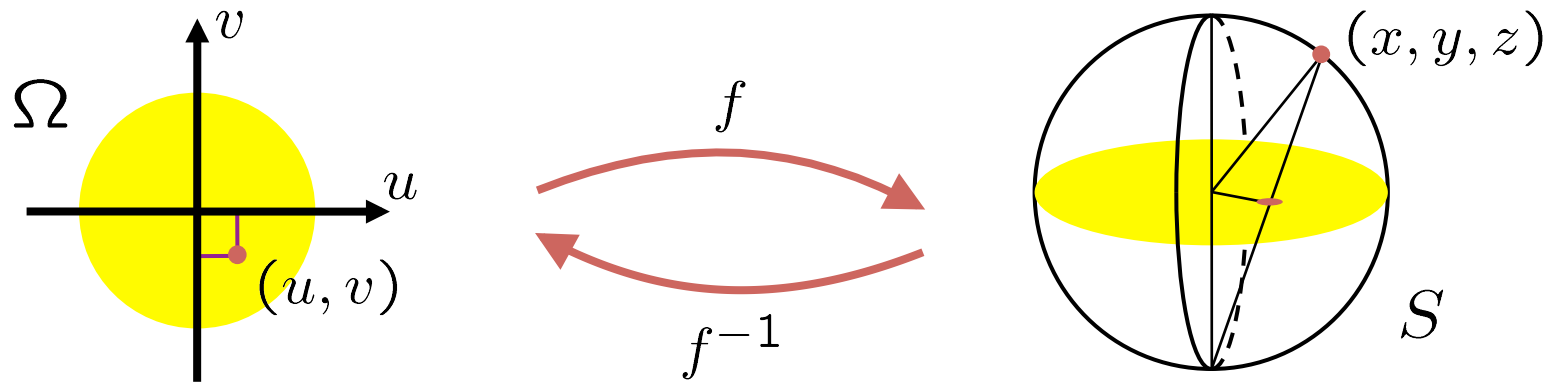
# Parametrization (VCG)

- ▣ Per Wedge = per triangle per vertex
- ▣ Why ?
- ▣ Needed to handle seams





# Parametrization: Stereo Projection



$$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, z \geq 0\}$$

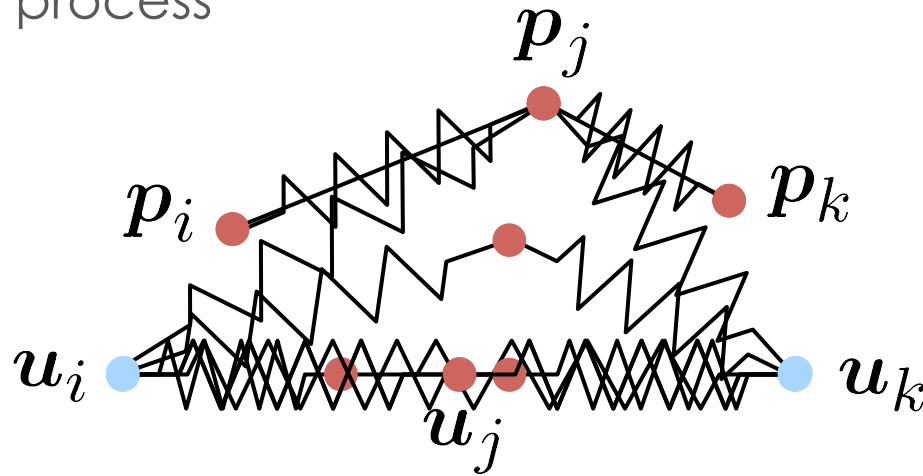
$$\Omega = \{(u, v) \in \mathbb{R}^2 : u^2 + v^2 \leq 1\}$$

$$f^{-1}(x, y, z) = \left( \frac{x}{1+z}, \frac{y}{1+z} \right)$$

$$f(u, v) = \left( \frac{2u}{1+u^2+v^2}, \frac{2v}{1+u^2+v^2}, \frac{1-u^2-v^2}{1+u^2+v^2} \right)$$

# Parametrization: Mass-Spring

- replace **edges** by **springs**
- Position of vertices  $p_0..p_n$
- UV Position of vertices  $u_0..u_n$
- relaxation** process



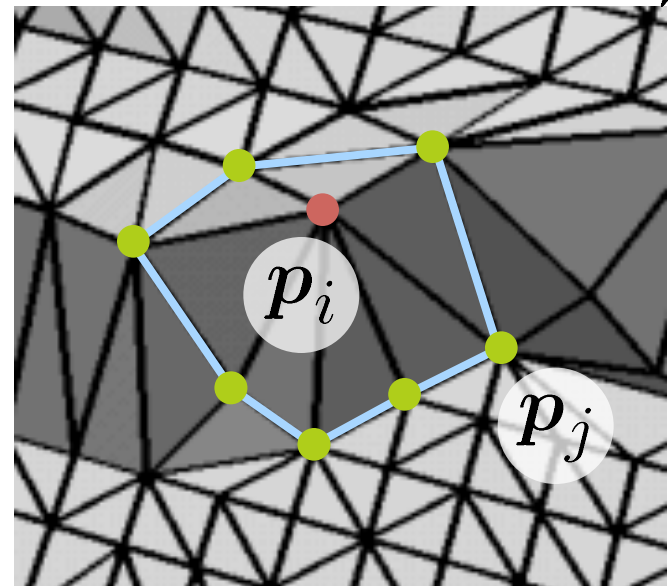
# Energy Minimization

- energy of spring between  $p_i$  and  $p_j$ :  $\frac{1}{2}D_{ij}s_{ij}^2$
- spring constant (stiffness)  $D_{ij} > 0$
- spring length (in parametric space)  $s_{ij} = \|\mathbf{u}_i - \mathbf{u}_j\|$
- total energy

$$E = \sum_{(i,j) \in \mathcal{E}} \frac{1}{2}D_{ij}\|\mathbf{u}_i - \mathbf{u}_j\|^2$$

- partial derivative

$$\frac{\partial E}{\partial \mathbf{u}_i} = \sum_{j \in N_i} D_{ij}(\mathbf{u}_i - \mathbf{u}_j)$$



$S$

# Linear System

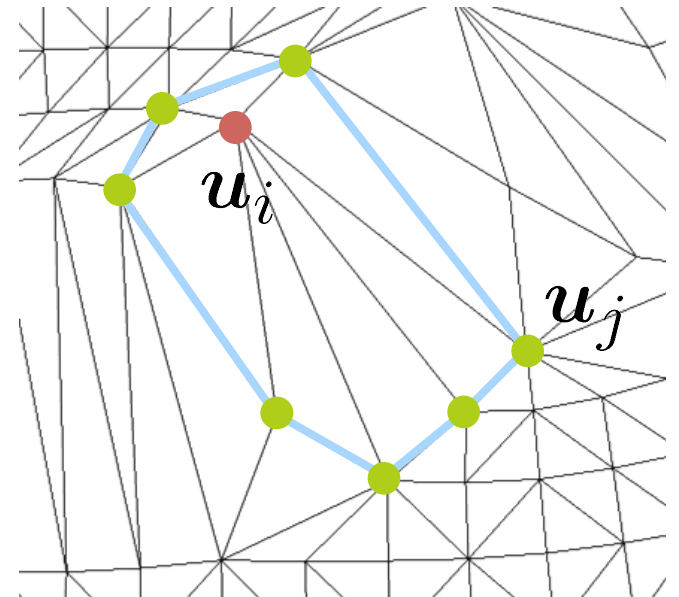
- $u_i$  is expressed as a **convex combination** of its neighbours  $u_j$

$$u_i = \sum_{j \in N_i} \lambda_{ij} u_j$$

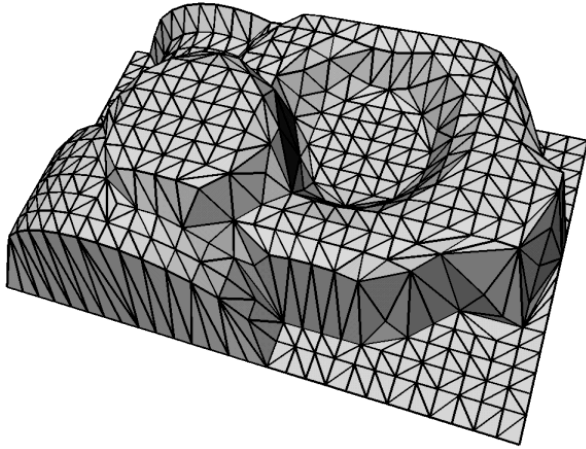
- With weights

$$\lambda_{ij} = D_{ij} / \sum_{k \in N_i} D_{ik}$$

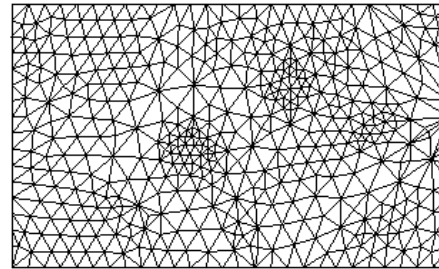
- LEAD to Linear System!



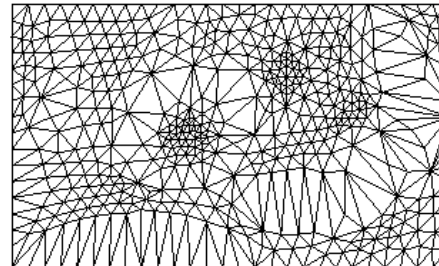
# Which Weights?



□ uniform spring constants

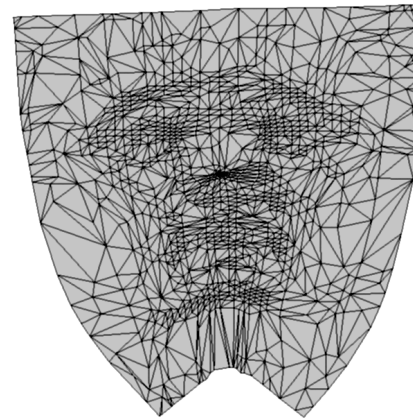
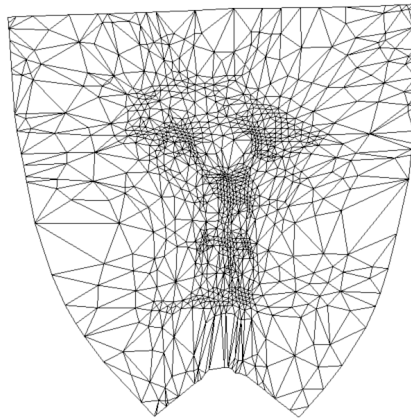
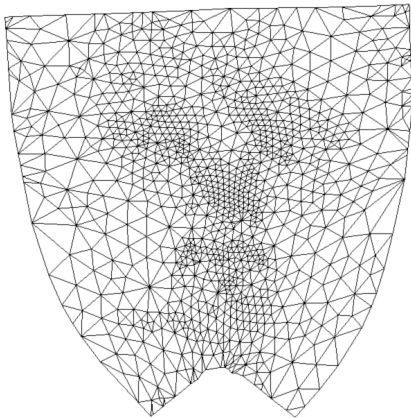


□ Proportional to 3D distance



# Which Weights?

- **NO** linear reproduction
- Planar mesh are distorted



# Which Weights?

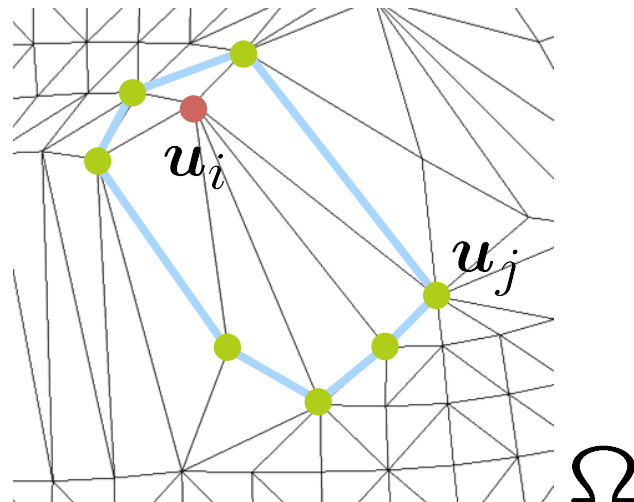
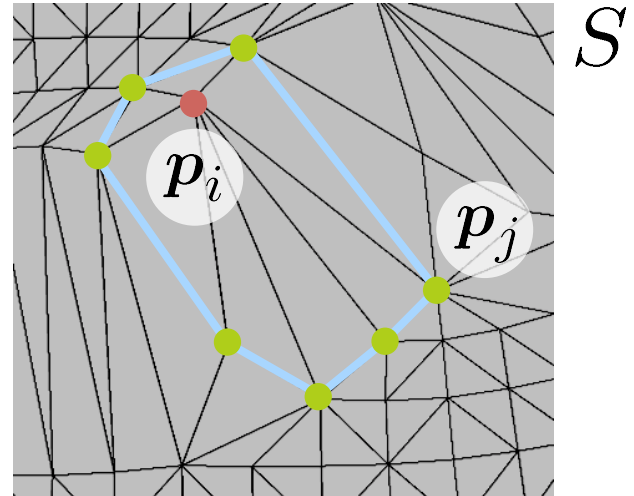
- suppose  $\mathcal{S}$  to be is planar
- specify weights  $\lambda_{ij}$  such that

$$\mathbf{p}_i = \sum_{j \in N_i} \lambda_{ij} \mathbf{p}_j$$

- Then solving

$$\mathbf{u}_i = \sum_{j \in N_i} \lambda_{ij} \mathbf{u}_j$$

- Reproduces  $\mathcal{S}$



# Which Weights?

- Wachspress coordinates

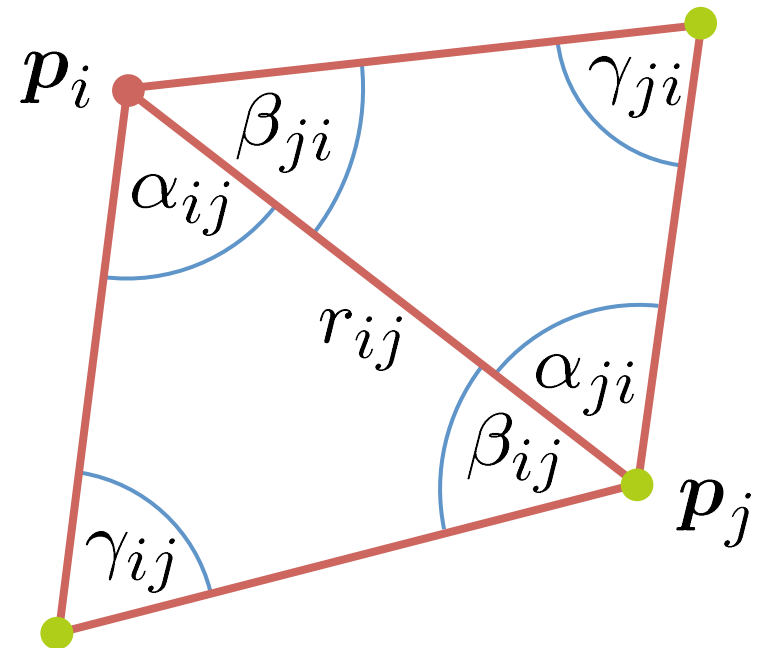
$$w_{ij} = \frac{\cot \alpha_{ji} + \cot \beta_{ij}}{r_{ij}^2}$$

- discrete harmonic coordinates

$$w_{ij} = \cot \gamma_{ij} + \cot \gamma_{ji}$$

- mean value coordinates

$$w_{ij} = \frac{\tan \frac{\alpha_{ij}}{2} + \tan \frac{\beta_{ji}}{2}}{r_{ij}}$$



normalization

$$\lambda_{ij} = \frac{w_{ij}}{\sum_{k \in N_i} w_{ik}}$$



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# Parametrization & Remeshing

Nico Pietroni ([nico.pietroni@isti.cnr.it](mailto:nico.pietroni@isti.cnr.it))

ISTI – CNR