Multi-modal Registration of Visual Data

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Overview

- Introduction and Background
- Features Detection and Description (2D case)
- Features Detection and Description (3D case)
- Image-geometry registration
- Recent Advances and Applications

Overview

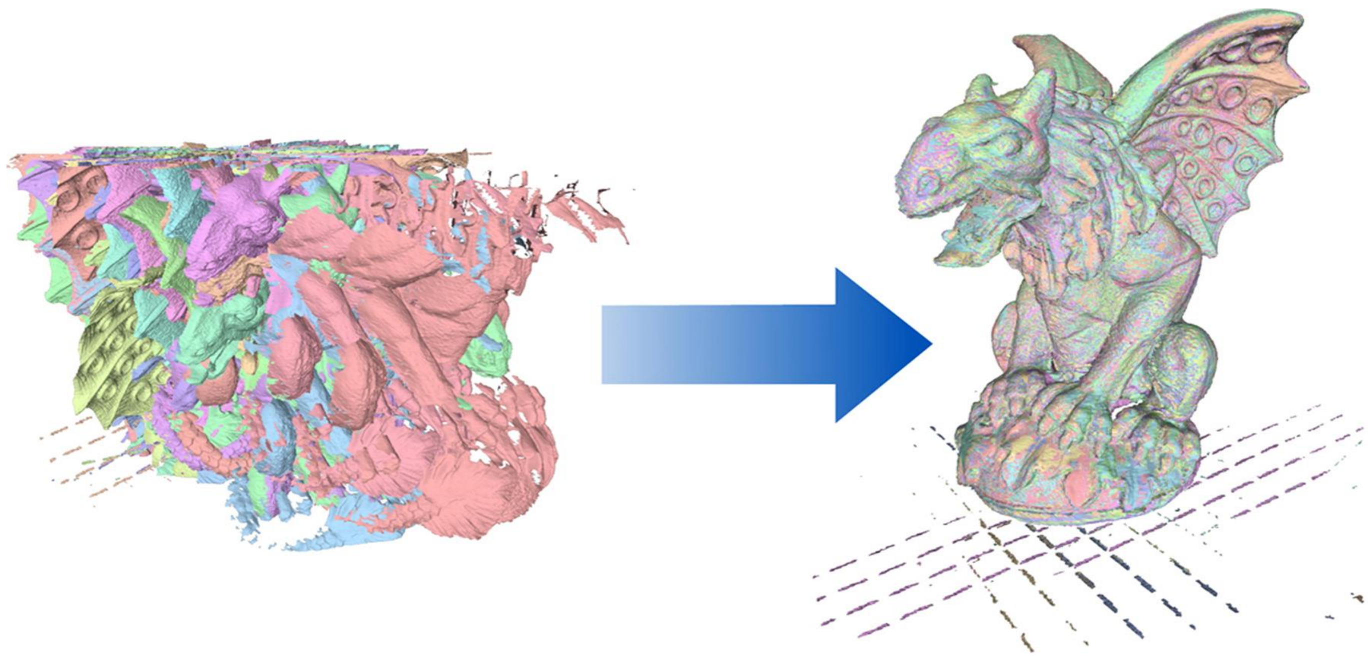
- Introduction and Background
- Features Detection and Description (2D case)
- Features Detection and Description (3D case)
  - Problem Statement and Motivations
  - Geometry Processing
    - Parameterization
    - Discrete Differential Geometry
  - Range Maps registration
  - Spin Images
  - MeshDOG and MeshHOG
  - Heat Kernel Signature (HKS)
- Image-geometry registration
- Recent Advances and Applications

Features Detection and Description (3D case)

Problem statement and motivations

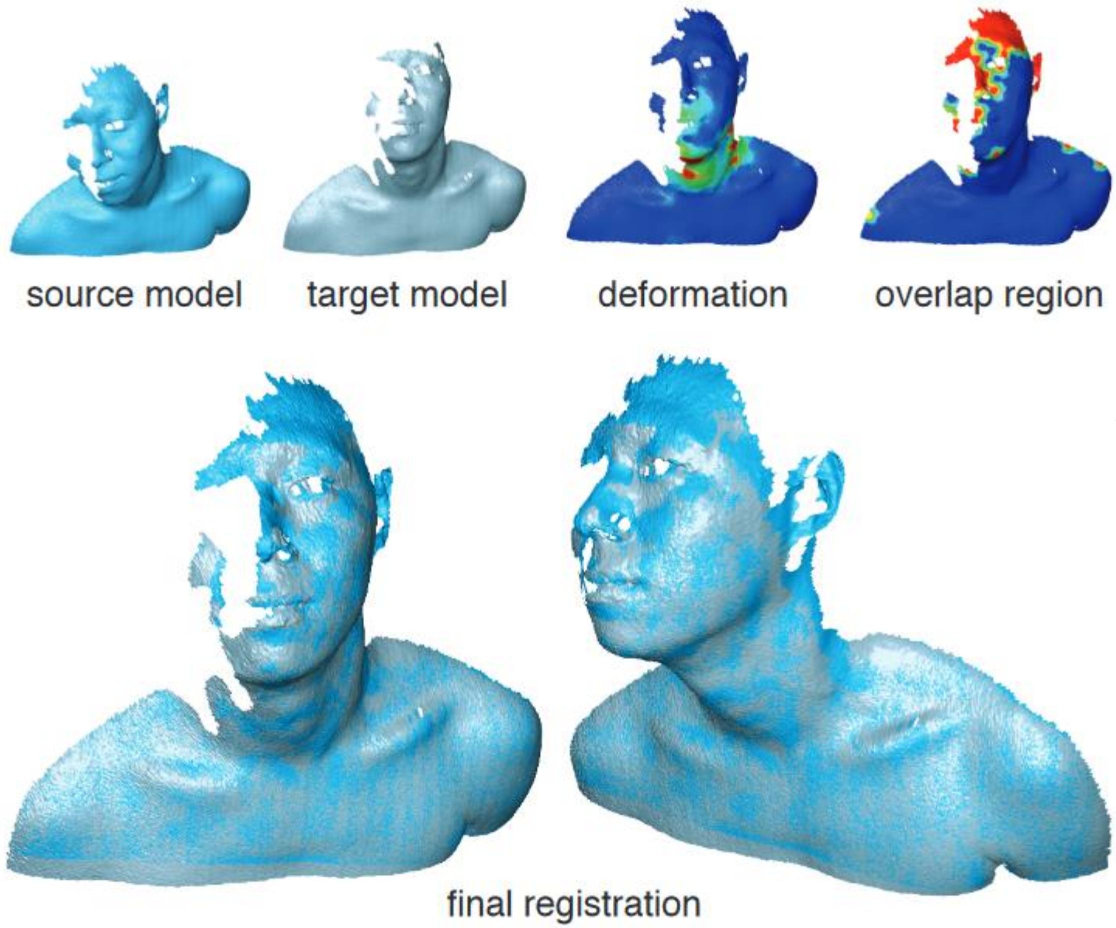
- Correspondences problem:
  – are the two (or more) pieces of geometry *essentially the same*?

Range Maps Registration (rigid transformation)



Problem statement and motivations

- Correspondences problem:
  - are the two (or more) pieces of geometry *similar* ?

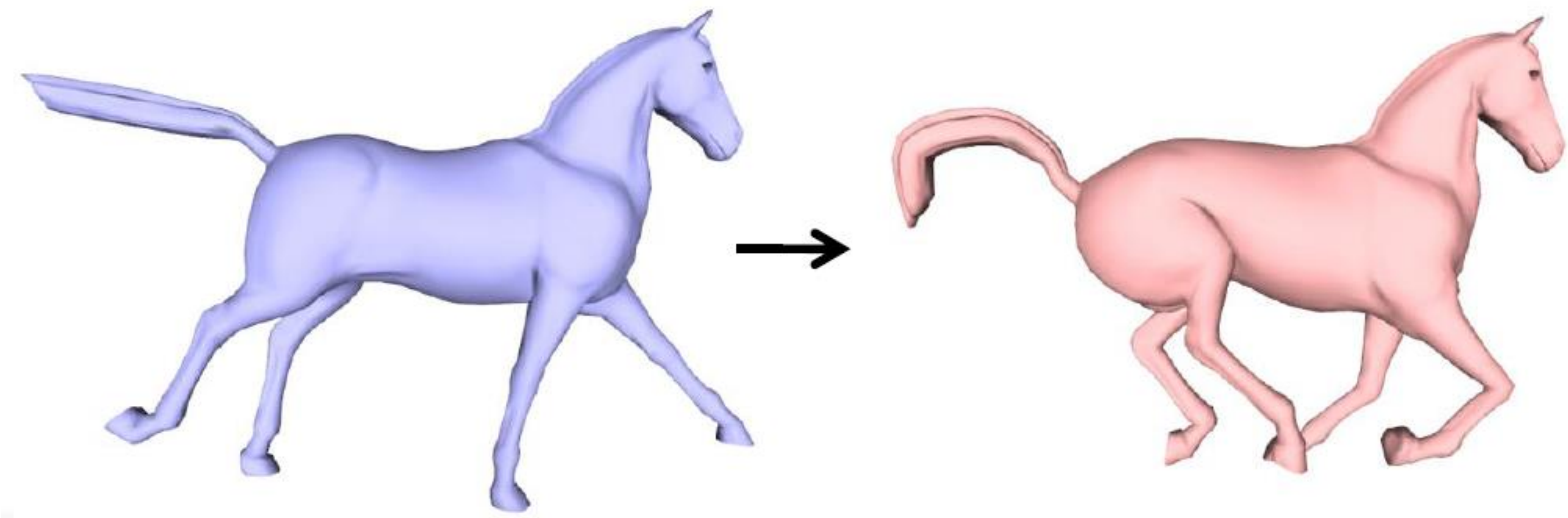


Animation Reconstruction (non-rigid registration)

Image from: Hao Li, Robert Sumner, Mark Pauly, "Global Correspondance Optimization for Non Rigid Registration of Depth Scans", *Symposium on Geometry Processing (SGP08), 2008.*

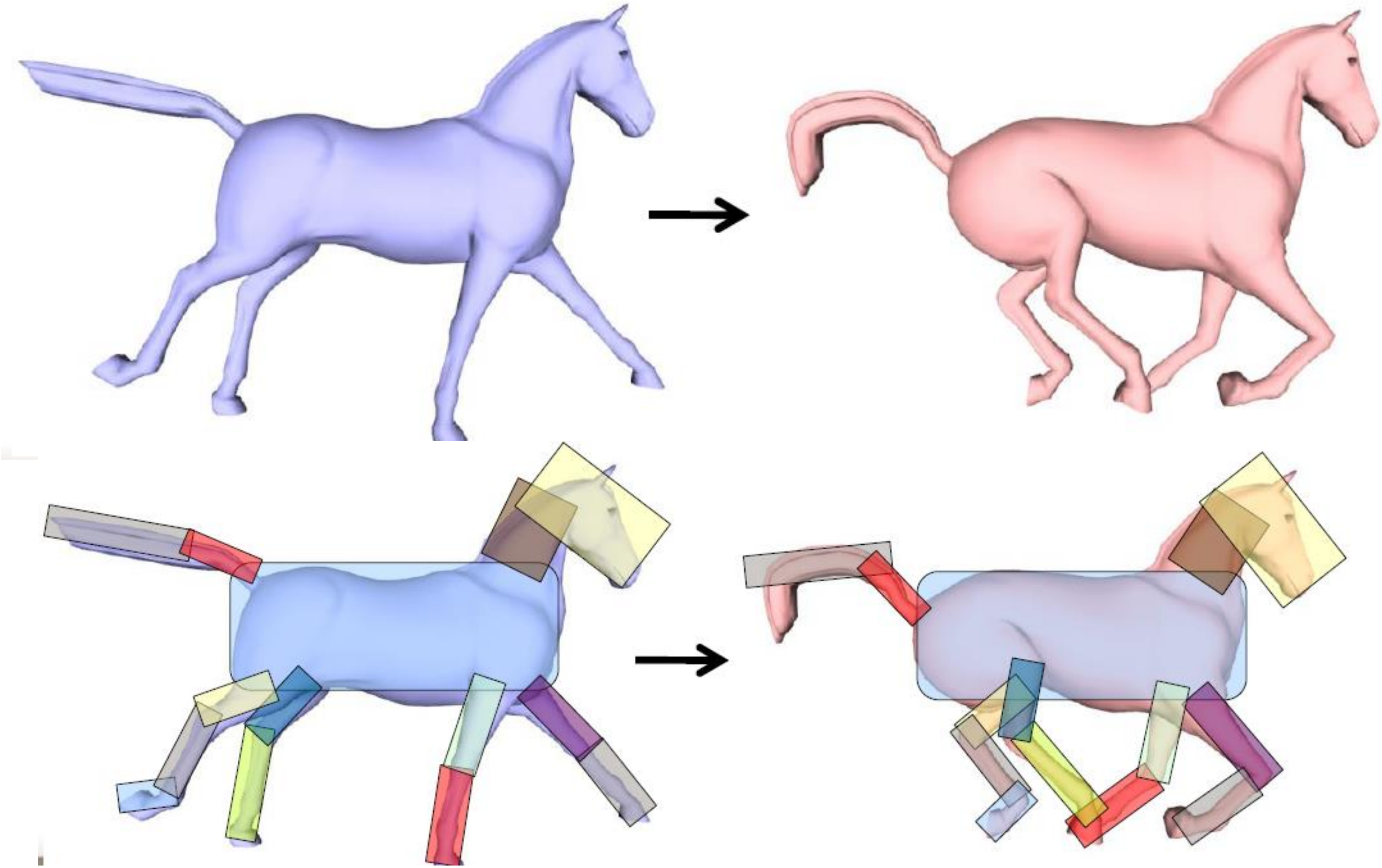
Problem statement and motivations

- Correspondences problem
  – are the two or more pieces of geometry *similar* ?



Articulated Shape Matching
(parts-based rigid transformation)

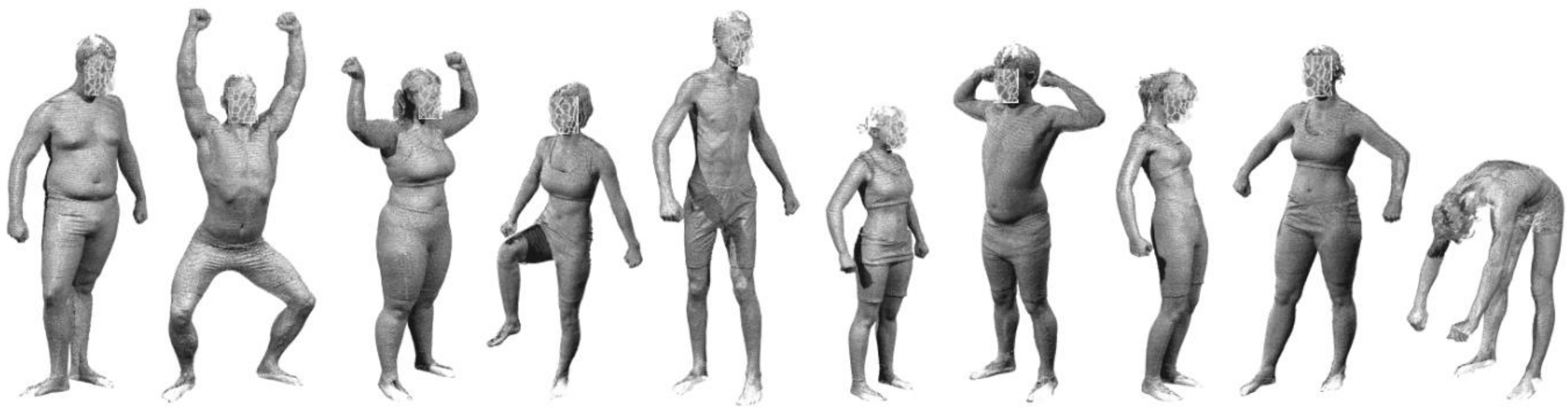
Problem statement and motivations



Images from: W. Chang and M. Zwicker, "Automatic registration for articulated shapes", ***Symposium on Geometry Processing (SGP08)*****, 2008.**

Problem statement and motivations

- Correspondences problem:
  - *Similar 3D models*



Shape Analysis and Synthesis
(e.g. shape priors, editing, recognition)

Image from: Haslet et al. "A Statistical Model of Human Pose and Body Shape",
***Proc. of Eurographics 2009*.**

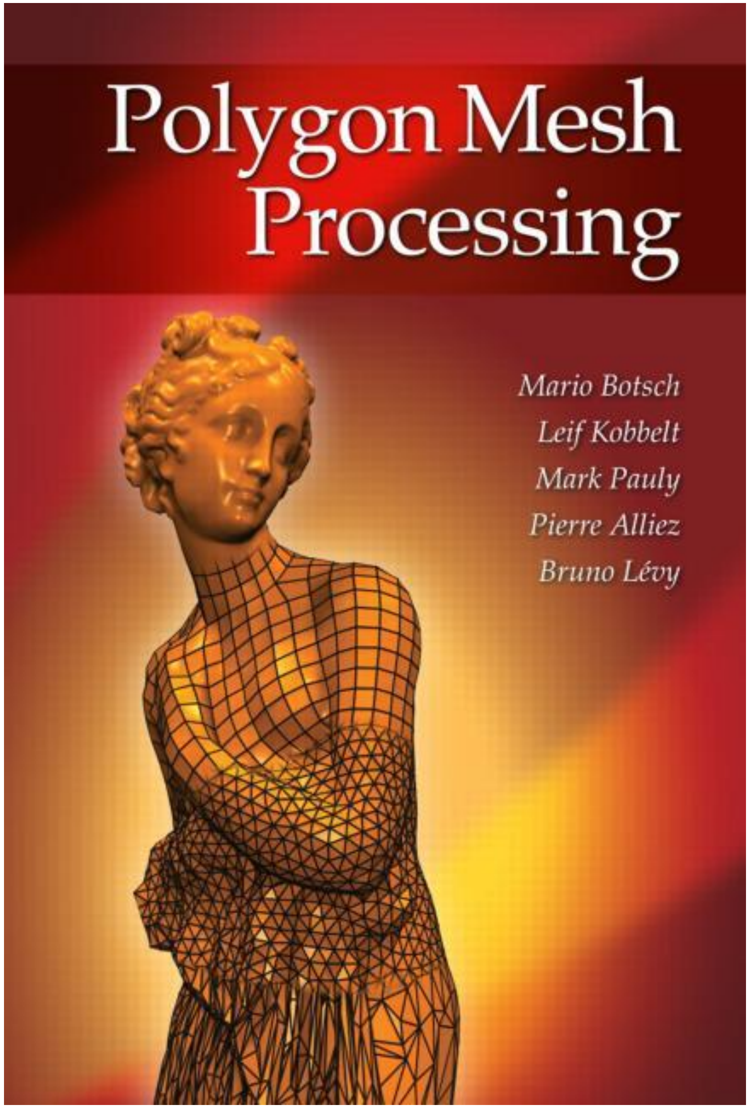
Problem statement and motivations

- Different correspondences problem.
- Different type of data (range maps, point clouds, point clouds with normals, polygonal meshes, geometry+color, etc.).
- These aspects and the specific application should be taken into account.

Geometry Processing

- Representation and Data Structure
- (Discrete) Differential Geometry
- Parameterization
- Remeshing
- Simplification & Approximation
- Model Repair

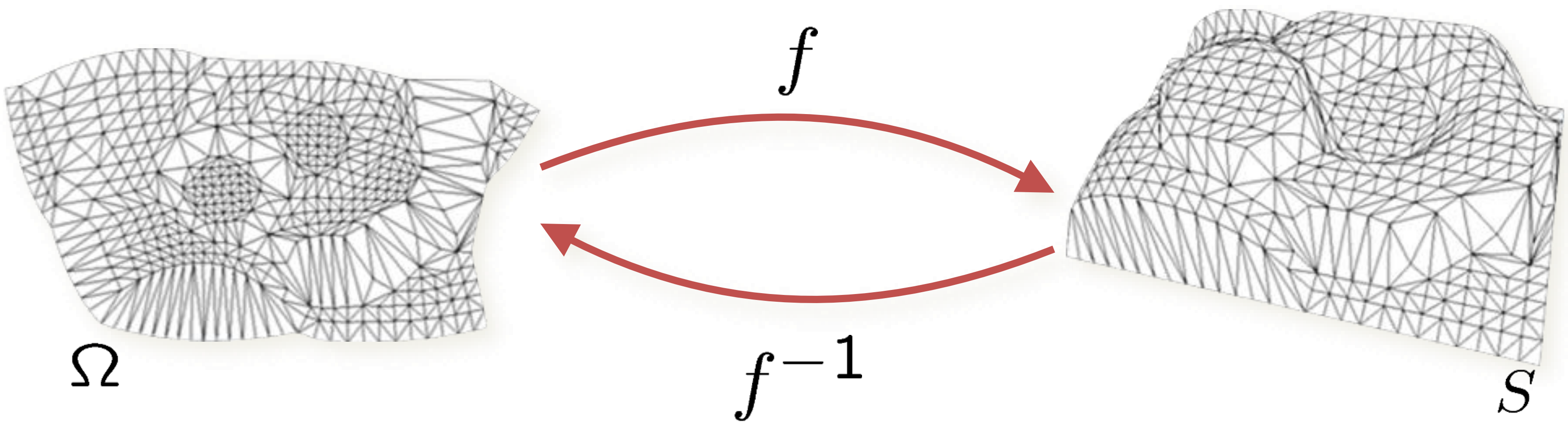
Geometry Processing



Mario Botsch, Leif Kobbelt, Mark Pauly, Pierre Alliez and Bruno Levy
October 7, 2010 by A K Peters/CRC Press
Reference - 250 Pages
ISBN 9781568814261

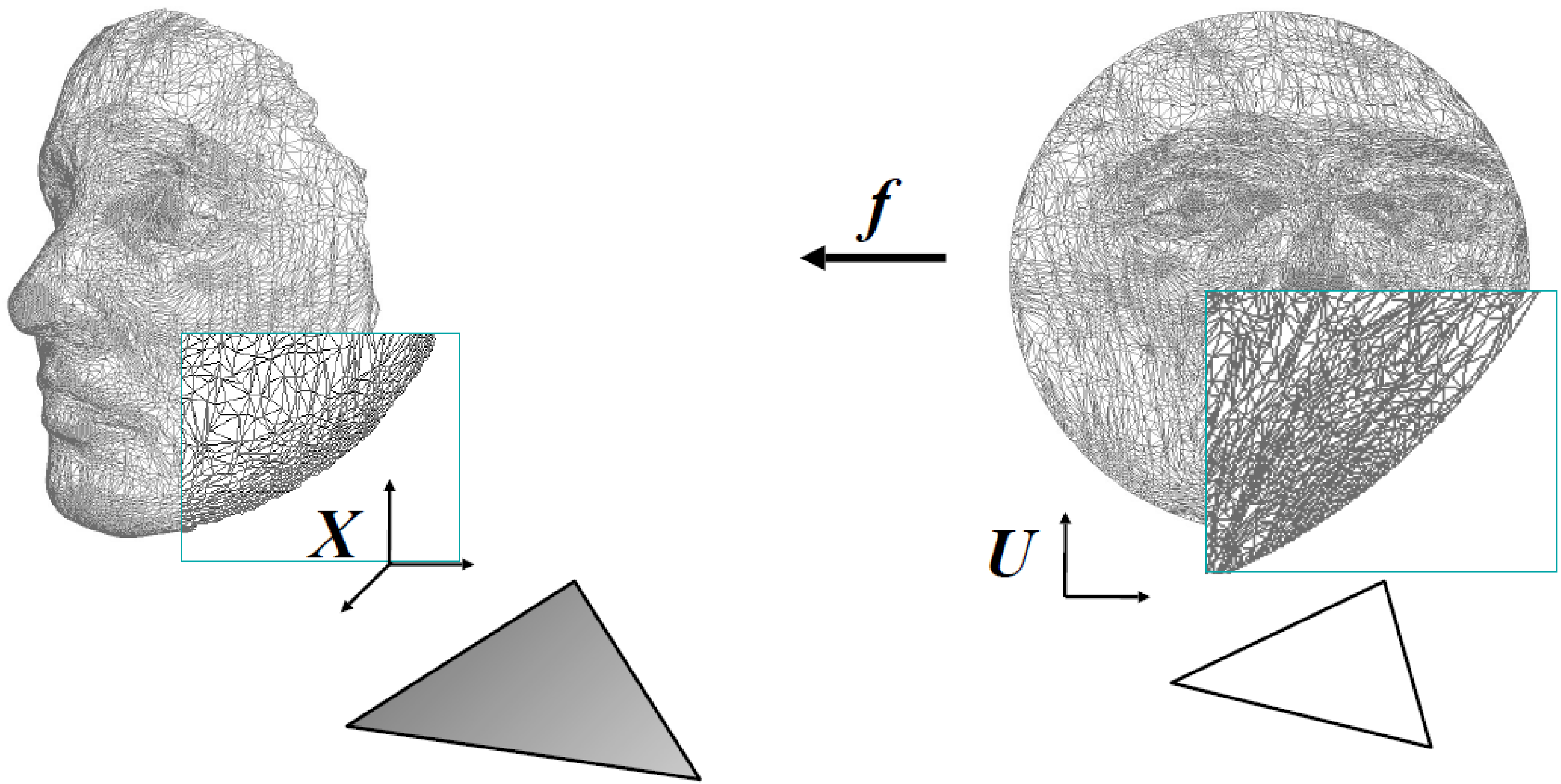
Parameterization

- Surface $S \subset \mathbb{R}^3$
- Planar domain $\Omega \subset \mathbb{R}^2$
- Define a mapping $f : \Omega \rightarrow S$ and $f^{-1} : S \rightarrow \Omega$



Parameterization

- Mapping = 2D mesh with same connectivity

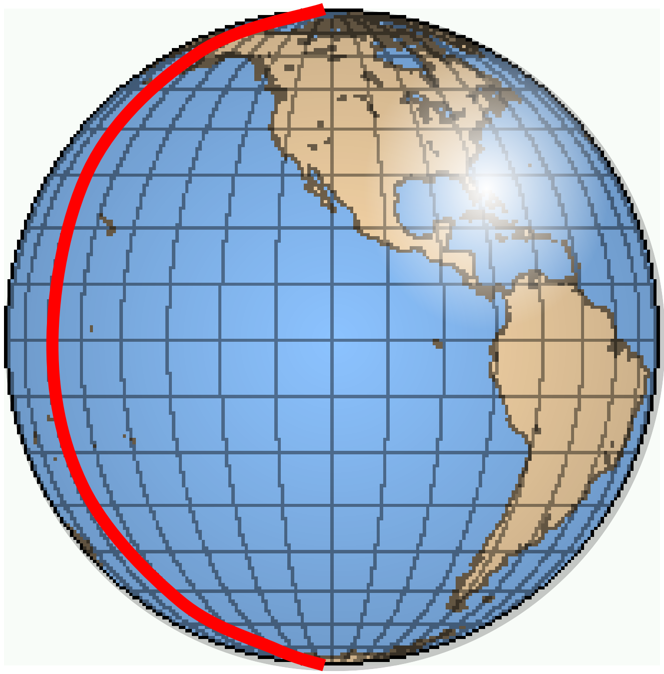


Parameterization

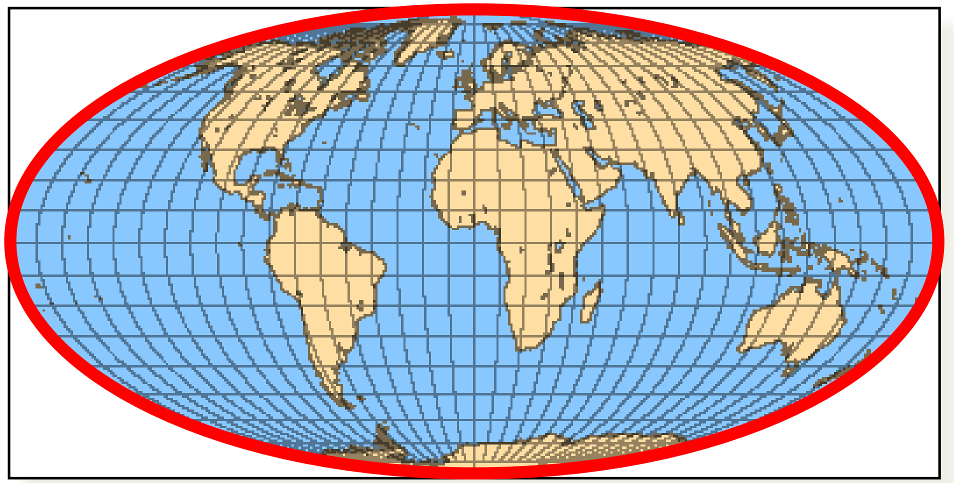
- Parameterization map must be *bijective* → triangles no overlap in the planar domain.
- Some distortions are always introduced:
  - Some parameterizations are angle-preserving (*conformal*)
  - Some parameterizations are area-preserving (*equi-areal*)
  - Some parameterizations preserve area and angle (*isometric*)

Parameterization - cuts

- The surface may need to be cut (!)



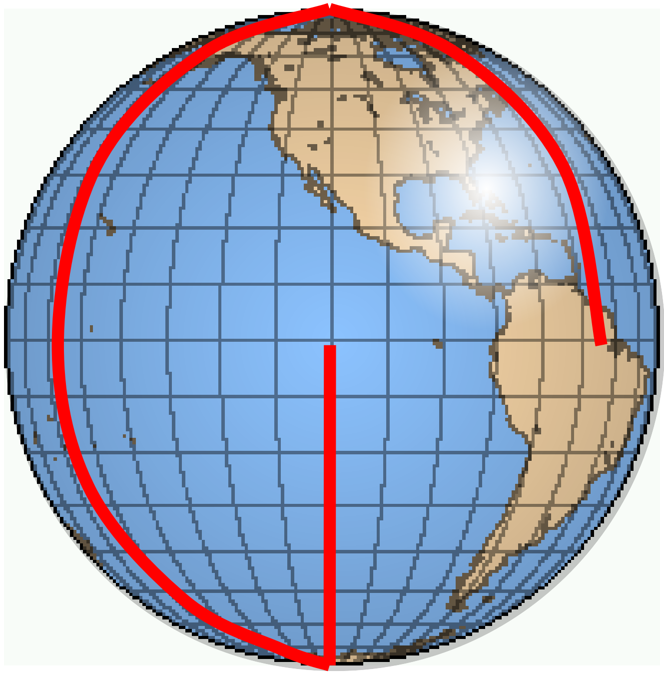
sphere in 3D



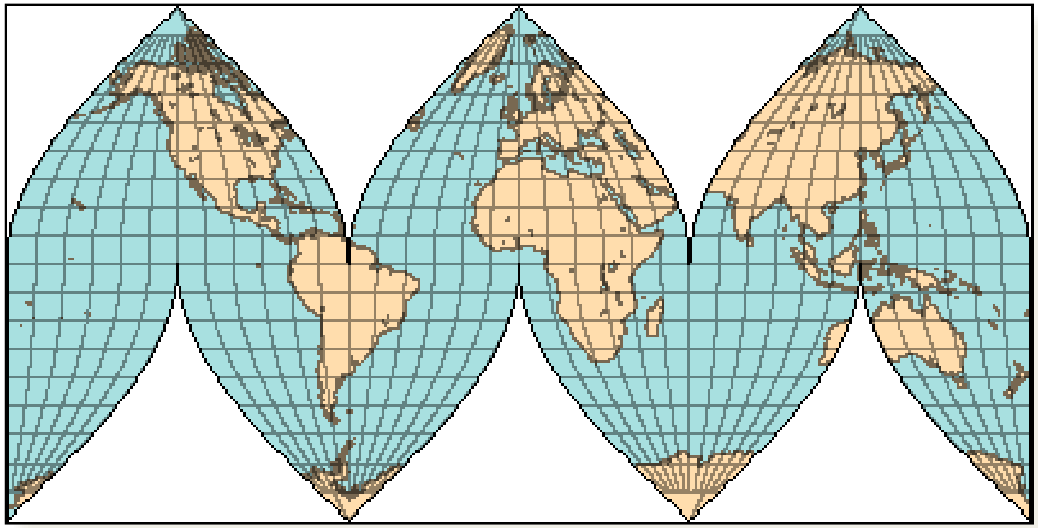
2D surface disk

Parameterization - cuts

- More cuts usually produce less distortions.



sphere in 3D



2D surface

Parameterization – example

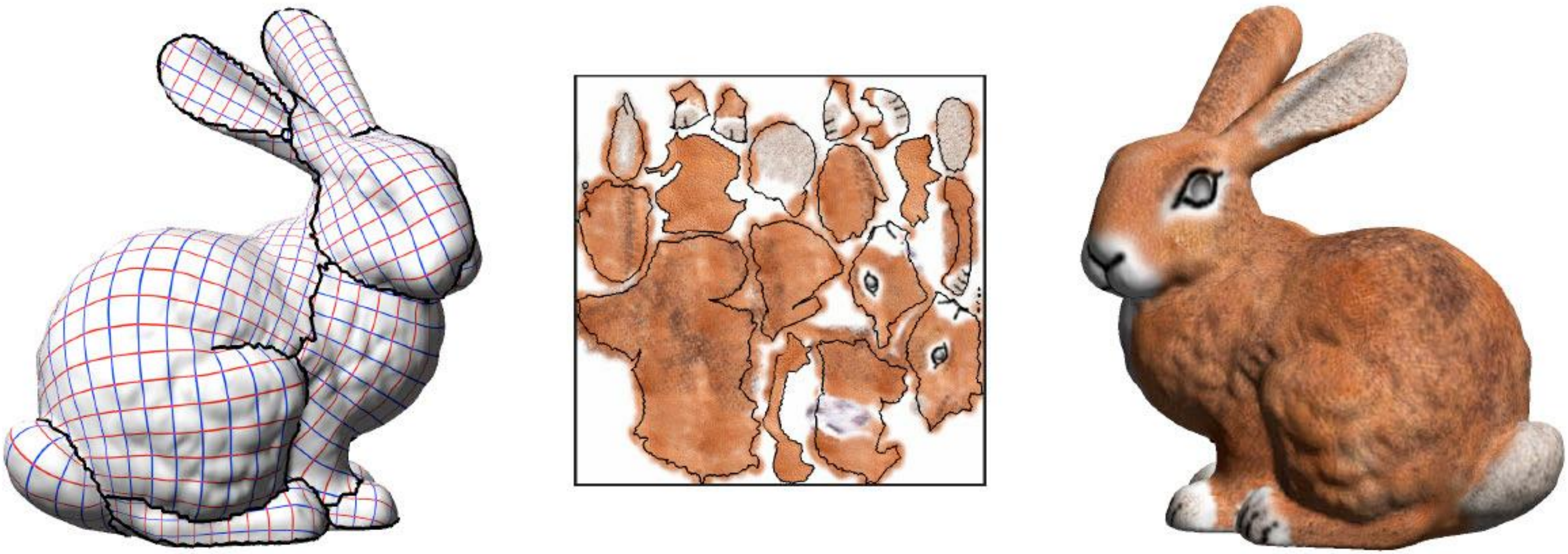
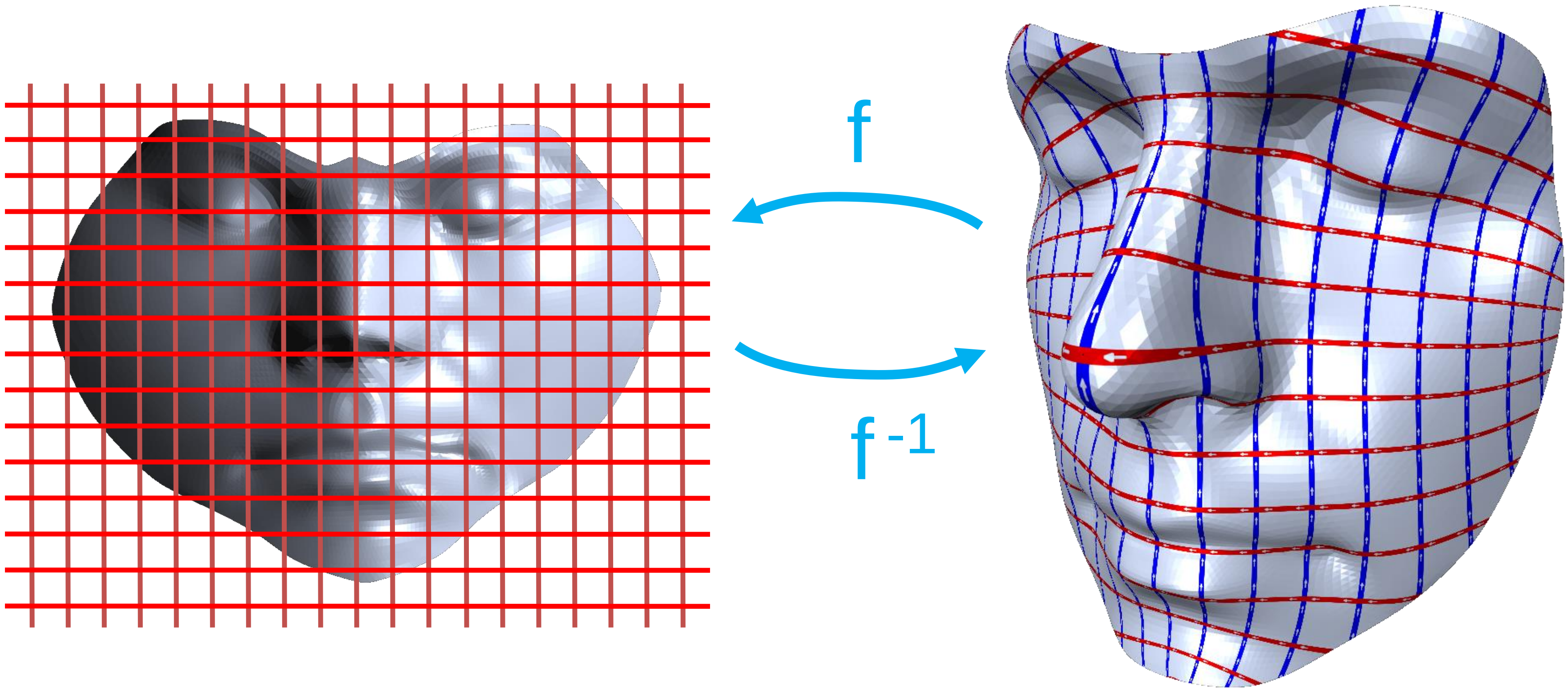


Image from: Lévy, Petitjean, Ray, and Maillot, "Least squares conformal maps for automatic texture atlas generation", *SIGGRAPH 2002*.

Parameterization – Remeshing



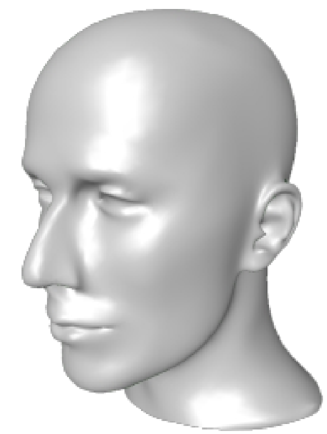
Differential Geometry

- Continuous surface

$$\mathbf{x}(u,v) = \left(\begin{array}{c} x(u,v) \\ y(u,v) \\ z(u,v) \end{array} \right), \ (u,v) \in \mathbb{R}^2$$

- Normal vector

$$\mathbf{n} = (\mathbf{x}_u \times \mathbf{x}_v)/\|\mathbf{x}_u \times \mathbf{x}_v\|$$

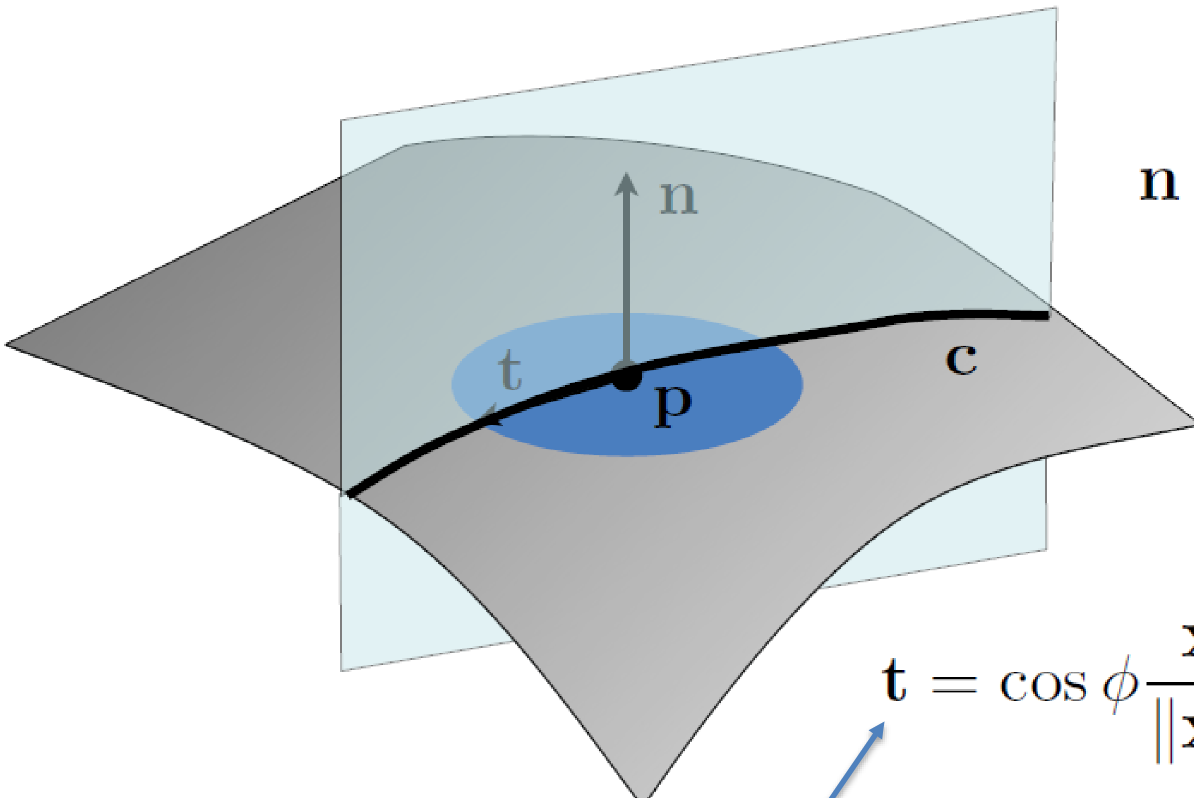


  – assuming regular parameterization, i.e.

$$\mathbf{x}_u \times \mathbf{x}_v \neq \mathbf{0}$$

Differential Geometry

- Normal Curvature



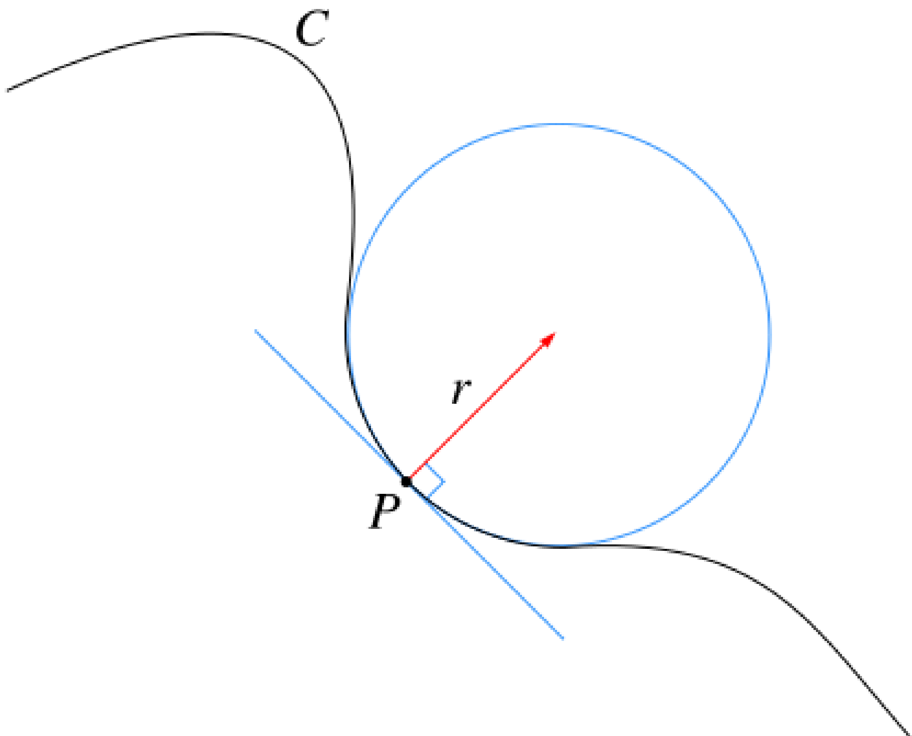
$$\mathbf{n} = \frac{\mathbf{x}_u \times \mathbf{x}_v}{\|\mathbf{x}_u \times \mathbf{x}_v\|}$$

$$\mathbf{t} = \cos\phi \frac{\mathbf{x}_u}{\|\mathbf{x}_u\|} + \sin\phi \frac{\mathbf{x}_v}{\|\mathbf{x}_v\|}$$

Tangent vector

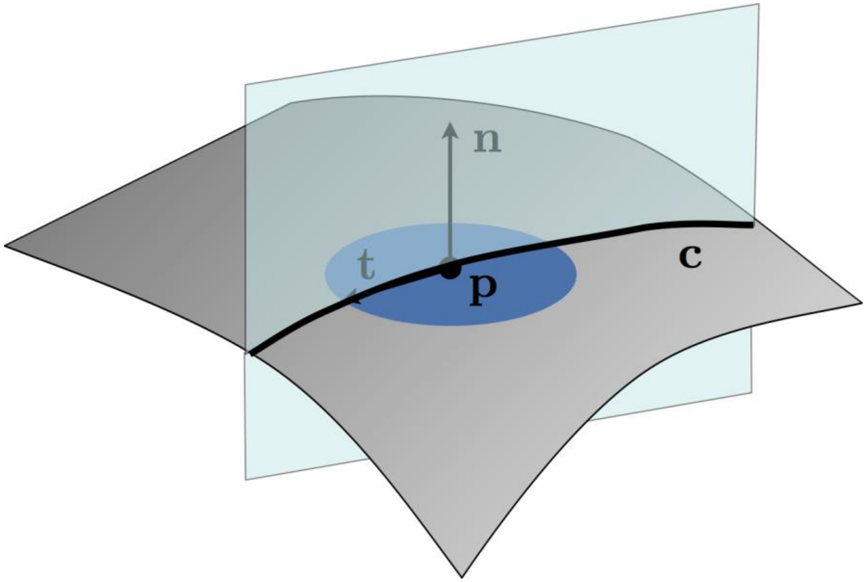
Curvature (2D)

- The curvature of *C* at *P* is the reciprocal of the radius of osculating circle at point *P*.



Principal Curvatures

- For each $\boldsymbol{t}$, we have a value of curvature.
- The two directions, $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$, where the value of curvature is respectively the *maximum* and *minimum* are said to be the directions of principal curvature.

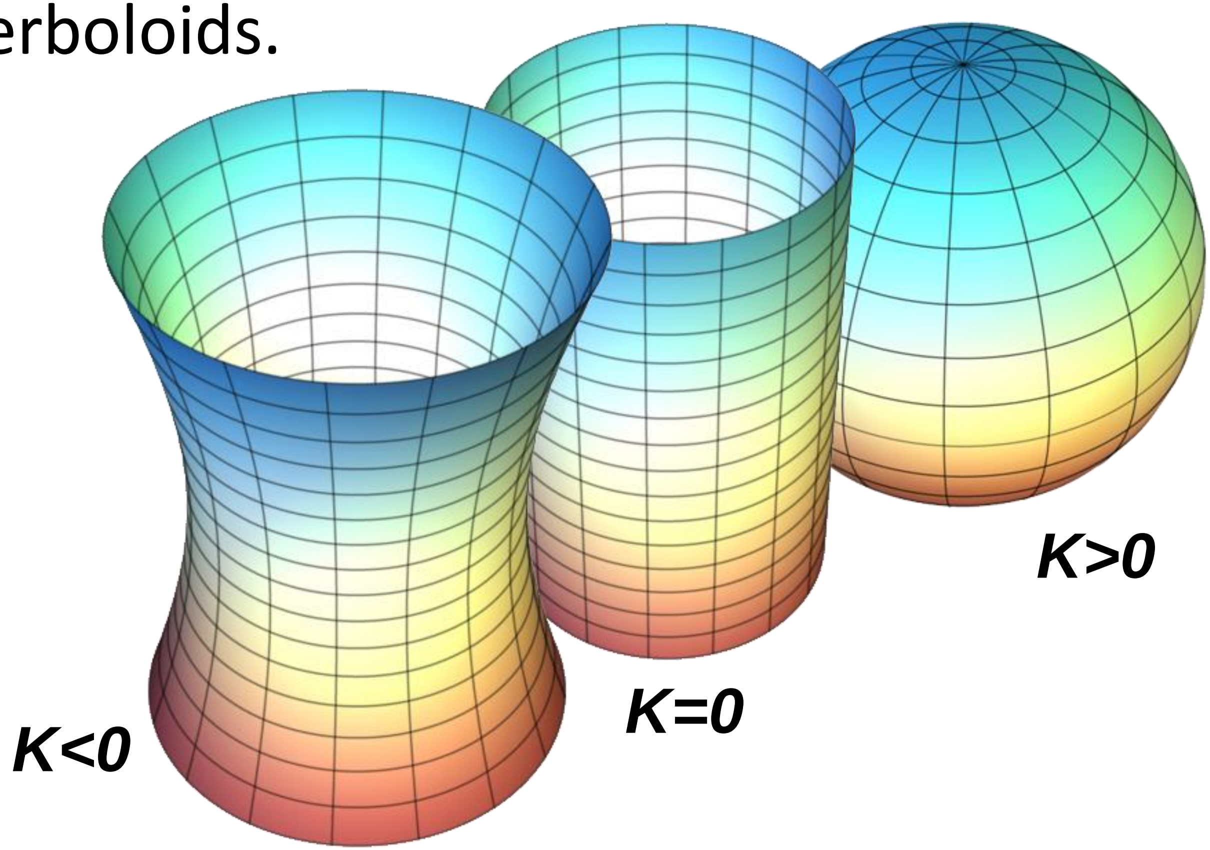


Differential Geometry

- Principal Curvatures
  - maximum curvature $\kappa_1 = \max_{\phi} \kappa_n(\phi)$
  - minimum curvature $\kappa_2 = \min_{\phi} \kappa_n(\phi)$
- Euler Theorem: $\kappa_n(\bar{\mathbf{t}}) = \kappa_n(\phi) = \kappa_1 \cos^2 \phi + \kappa_2 \sin^2 \phi$
- Mean Curvature $H = \dfrac{\kappa_1 + \kappa_2}{2} = \dfrac{1}{2\pi} \int_0^{2\pi} \kappa_n(\phi) d\phi$
- Gaussian Curvature $K = \kappa_1 \cdot \kappa_2$

Surface Characterization with Gaussian Curvature

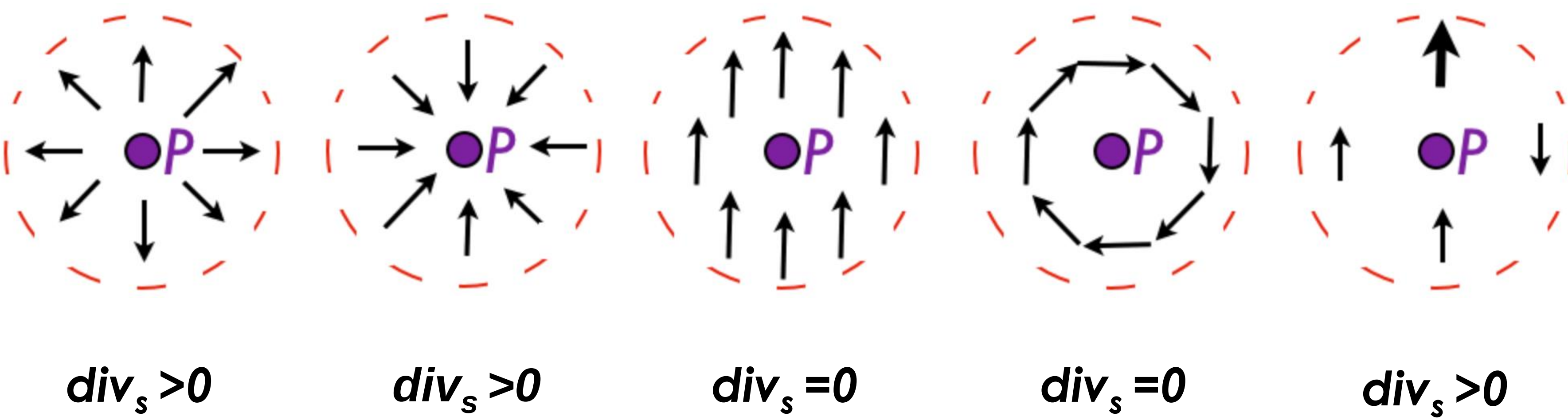
- ***K > 0*** → when the surface is elliptical.
- ***K = 0*** → locally flat.
- ***K < 0*** → for hyperboloids.



Mean Curvature

- Measure the *divergence* of the normal in a local neighborhood of the surface.
- The **divergence** is an operator that measures how much a vector field originate from or converge upon a given point.
- The Laplacian operator is the divergence of the gradient:

Divergence Operator



Laplace Operator

Laplace operator

gradient operator

2nd partial derivatives

$$\Delta f = \mathrm{div} \nabla f = \sum_i \frac{\partial^2 f}{\partial x_i^2}$$

function in Euclidean space

divergence operator

Cartesian coordinates

Laplace-Beltrami Operator

Laplace-
Beltrami

gradient
operator

$$\Delta_{\mathcal{S}} f = \mathrm{div}_{\mathcal{S}}\, \nabla_{\mathcal{S}} f$$

function on
manifold $\mathcal{S}$

divergence
operator

Laplace-Beltrami Operator

Laplace-Beltrami

gradient operator

mean curvature

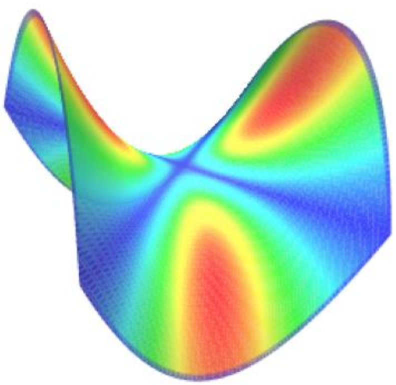
$$\Delta_{\mathcal{S}} \mathbf{x} = \mathrm{div}_{\mathcal{S}} \, \nabla_{\mathcal{S}} \mathbf{x} = -2H\mathbf{n}$$

coordinate function

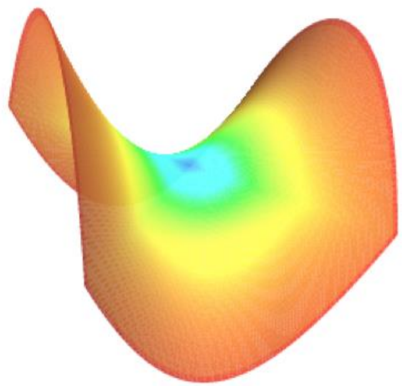
divergence operator

surface normal

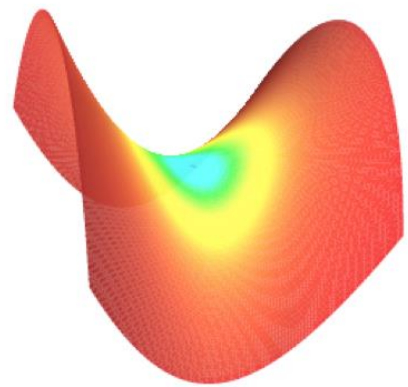
Mean and Gaussian Curvature



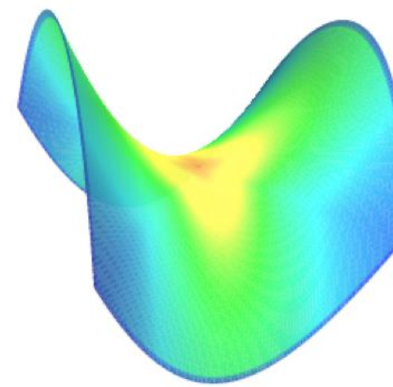
Mean (*H*)



Gaussian (*K*)



Min (k_2)



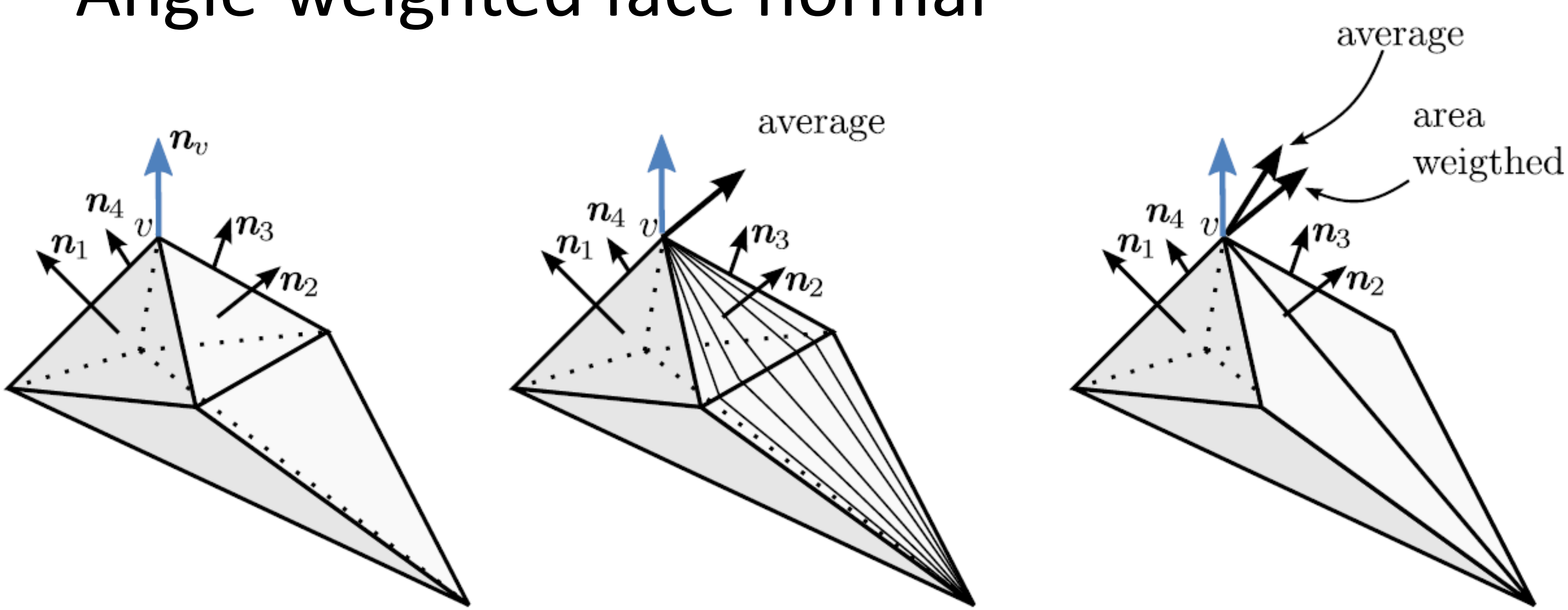
Max (K_1)

Discrete Differential Geometry

- Meshes are a piecewise linear approximation of the corresponding (smooth) surface.
- Surface normal ?
- Curvature ?
- Laplace operator ?

Normal estimation

- Vertex normal → average the face normal of the incidents face
- Area-weighted face normal
- Angle-weighted face normal



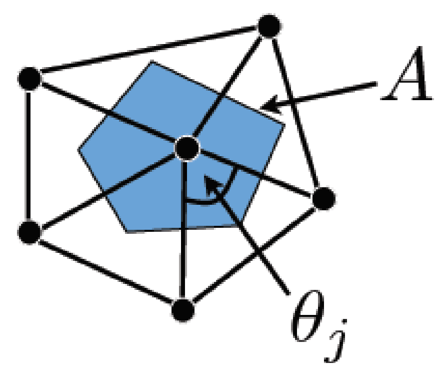
Discrete Curvatures

- Mean curvature

$$H = \|\Delta_{\mathcal{S}} \mathbf{x}\|$$

- Gaussian curvature

$$G = (2\pi - \sum_j \theta_j)/A$$



- Principal curvatures

$$\kappa_1 = H + \sqrt{H^2 - G}$$

$$\kappa_2 = H - \sqrt{H^2 - G}$$

Discrete Laplace-Beltrami

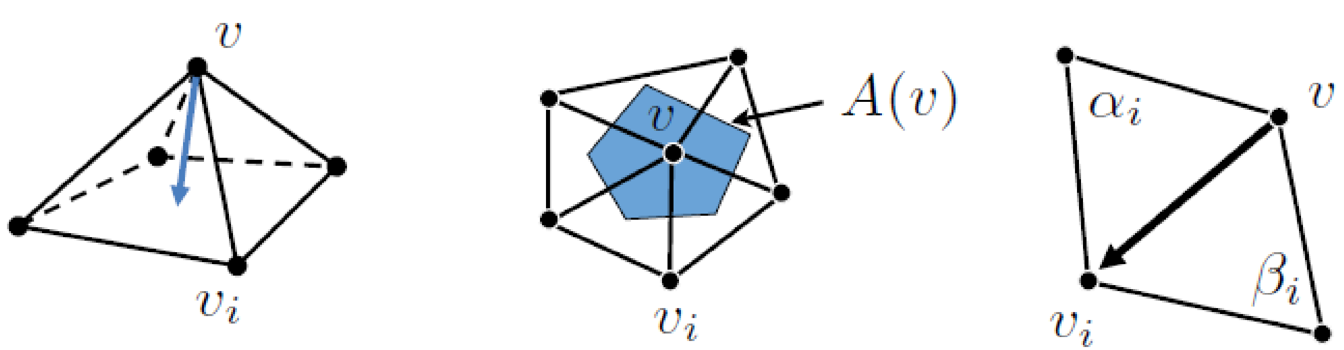
Assuming uniform discretization

$$\Delta_{uni} f(v) := \frac{1}{|\mathcal{N}_1(v)|} \sum_{v_i \in \mathcal{N}_1(v)} \left(f(v_i) - f(v) \right)$$

Discrete Laplace-Beltrami

More general formulation

$$\Delta_{\mathcal{S}} f(v) := \frac{2}{A(v)} \sum_{v_i \in \mathcal{N}_1(v)} \left(\cot \alpha_i + \cot \beta_i\right)\left(f(v_i) - f(v)\right)$$



Range Maps

- We do not need a parameterization (!)
- To apply 2D features detectors/descriptors is more natural (but it works?).
- Local vs global methods.
- Pairwise vs multi-view registration.

Range Maps

- Let’s take a look at [1] and [2] (we are interested in the pairwise registration part only)

Range Maps

- Global methods often produce a coarse registration → *fine registration* is required.
- Standard method → ICP* (Iterative Closest Point)
- Many variants exists.

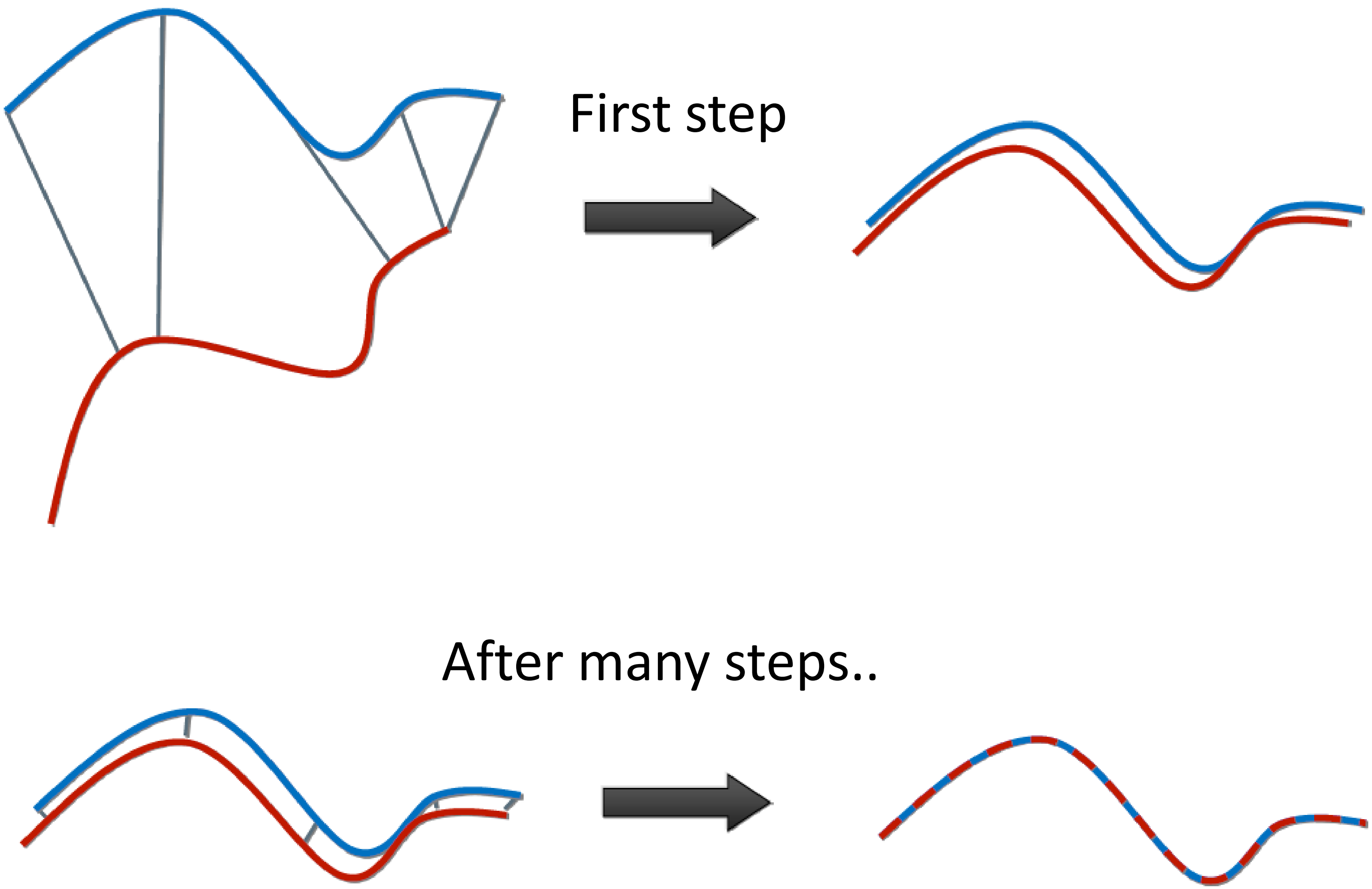
Paul J. Besl and Neil D. MacKay, "A Method for Registration of 3D Shapes",** ***IEEE Tran. PAMI**, Vol. 14(2), Feb, 1992.**

Iterative Closest Points (ICP)

- Select points randomly (e.g. 1000 points)
- Match each point with the closest point on the other surface
- Reject pairs with distance > threshold
- Iterate to minimize:

$$E = \sum |(\mathbf{R}\mathbf{p}_i + \mathbf{t}) - \mathbf{q}|^2$$

Iterative Closest Points (ICP)

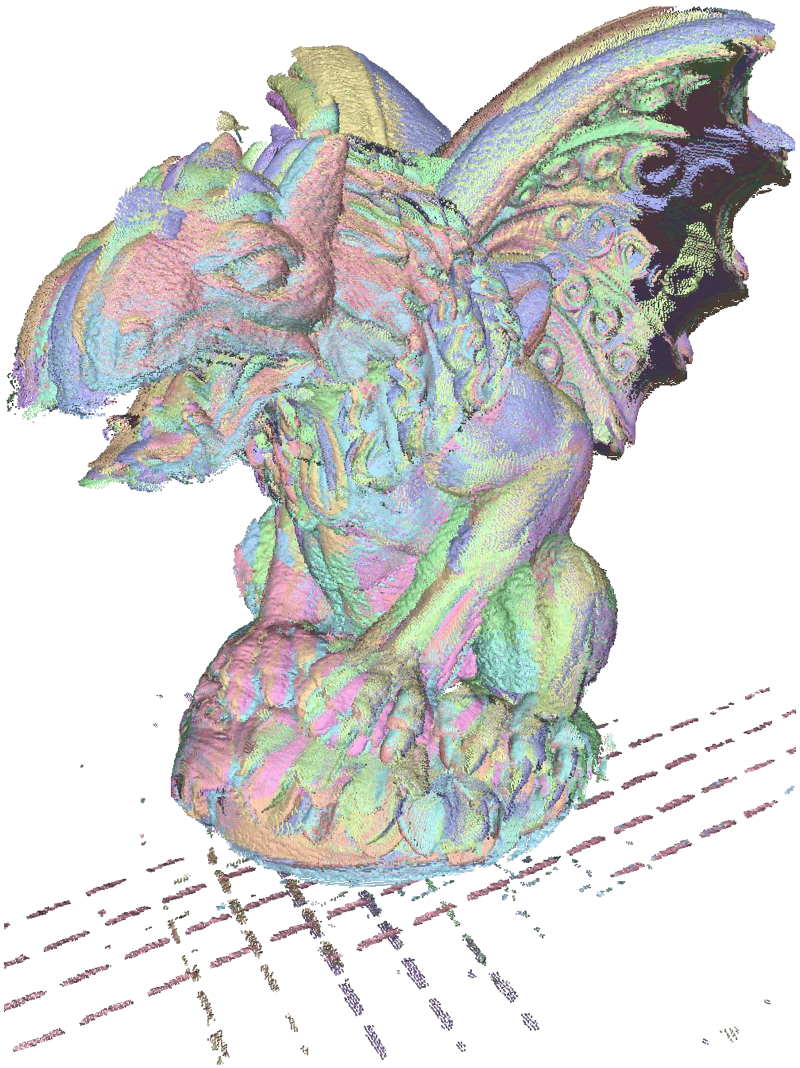


ICP - Variants

- Better Selection (e.g. stable sampling)
- Points Matching
  - Only plausible matching are confirmed (e.g. compatibility of normals)
- Reject certain pairs
- Comparison of many variants in [Rusinkiewicz & Levoy, 3DIM 2001]

S. Rusinkiewicz and M. Levoy, "Efficient Variants of the ICP Algorithms",** ***3DIM 2001.

ICP - example



Before ICP

After ICP

Keypoints Detectors and Descriptors (for meshes/point clouds)

- Many different keypoint detectors and descriptors
  - Curvature-based (fixed-scale, multi-scale)
  - SIFT-inspired
  - Histogram-based (defined on a volume, on a tangent plane, etc.)
  - .. and many others.
- Sometime complex/informative detection/description is not necessary.. (e.g. 4PCS for point cloud registration).

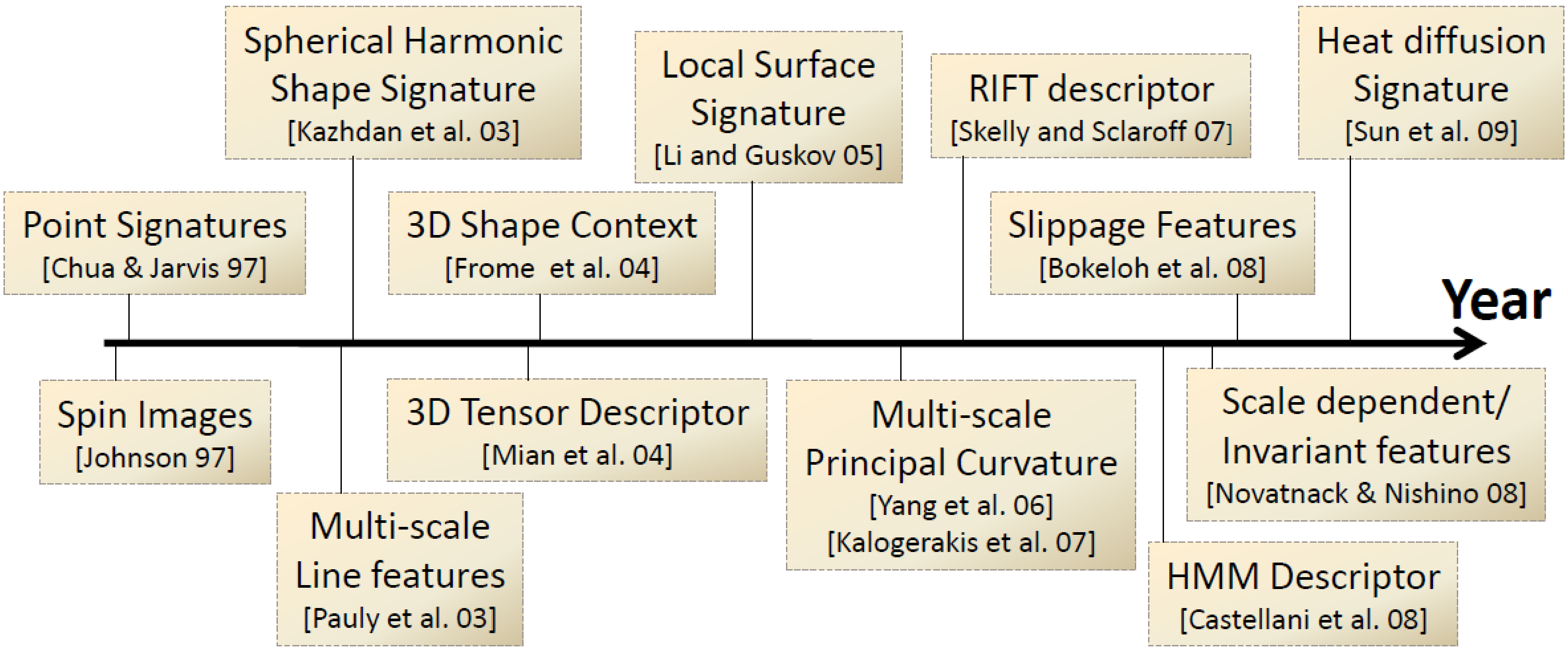
Keypoints Detectors and Descriptors (for meshes/point clouds)

- For an evaluation of keypoint detectors refer to [1].
- For an evaluation of keypoint descriptors refer to [2].

[1] F. Tombari, S. Salti, L. Di Stefano, ”Performance Evaluation of 3D Keypoint Detectors”, *Int. J. of Computer Vision*, Vol. 102(1), pp. 198-220, 2013.

[2] “A Comprehensive Performance Evaluation of 3D Local Feature Descriptors”, *Int. J. of Computer Vision, Vol. 116(1)*, pp. 66-89, 2016.

Surface Descriptors along the Timeline

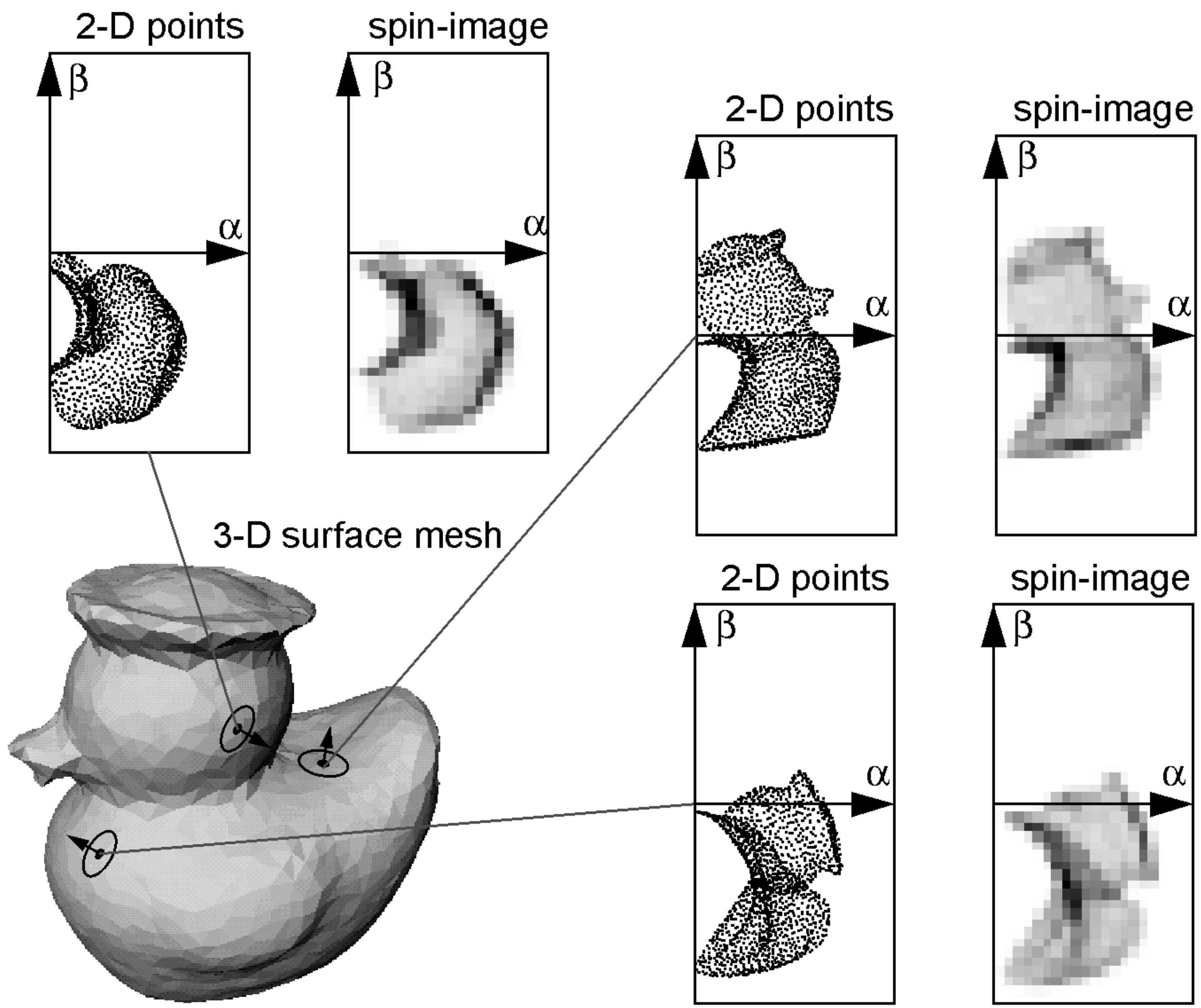


Spin Images

- Originally developed for surface matching.
- It is a dense description.
- Main idea → to associate to each vertex a small “image”.
- It works in *object-centered* coordinate system.

Andrew E. Johnson and M. Hebert, “Using Spin Images for Efficient Object Recognition in Cluttered 3D Scenes”, *IEEE Tran. PAMI*, Vol. 21(5), May, 1999*.*

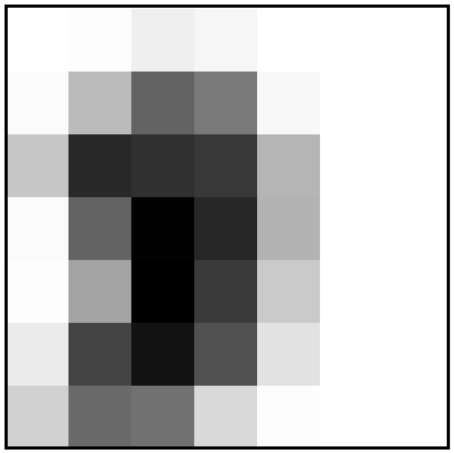
Main Idea Explained



Parameters

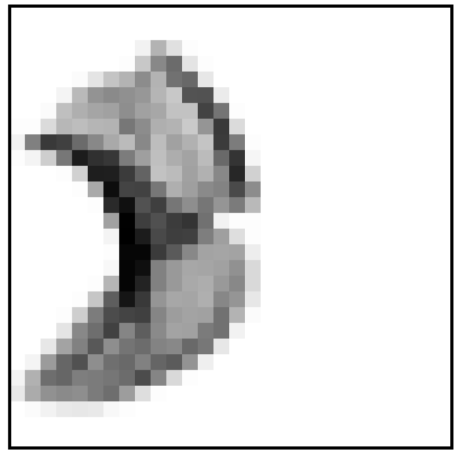
- α (radial coord.) : perpendicular distance to the line through the surface normal
- β (elevation): signed perpendicular distance w.r.t the tangent plane
- *Bin size* : geometric width of the bins in the spin image
- *Image width*: Number of rows and cols in a square spin image
- Support angle: the maximum angle between the normal direction and the normal of the points that are allowed to contribute to the spin image.

Effect of the parameters
Bin Size



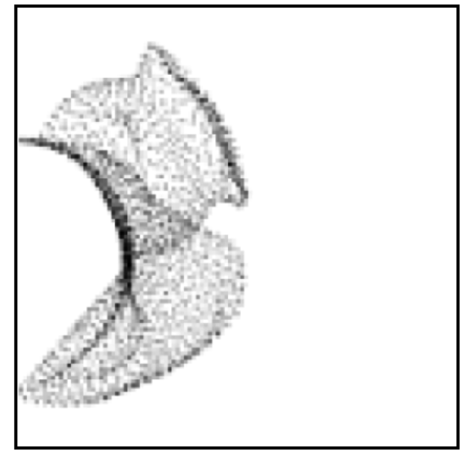
(a)

4x mesh resolution



(b)

≈ mesh resolution

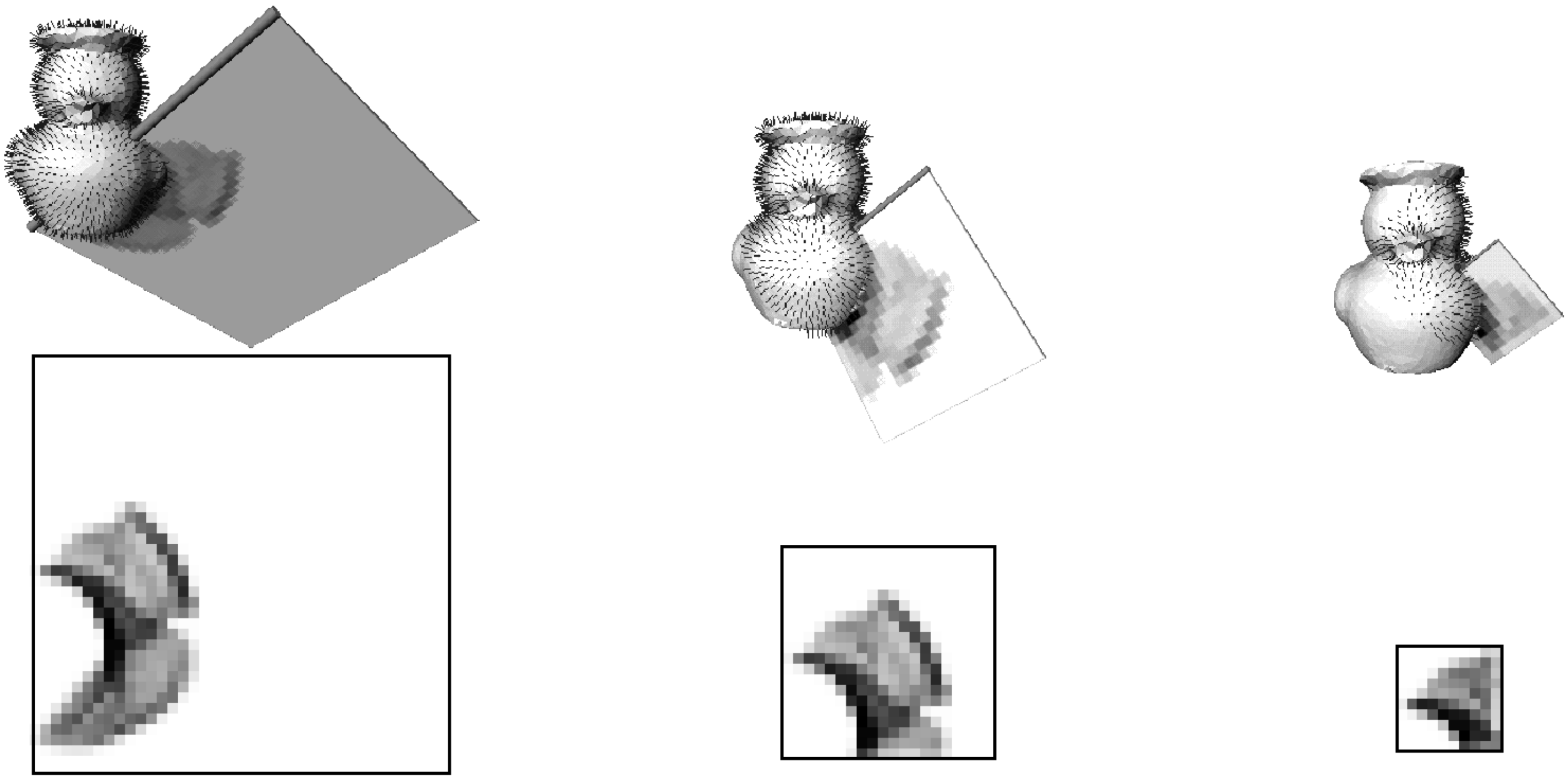


(c)

¼ mesh resolution

Andrew E. Johnson and M. Hebert, "Using Spin Images for Efficient Object Recognition in Cluttered 3D Scenes", ***IEEE Tran. PAMI*****, Vol. 21(5), May, 1999.**

Effect of the parameters
Image width

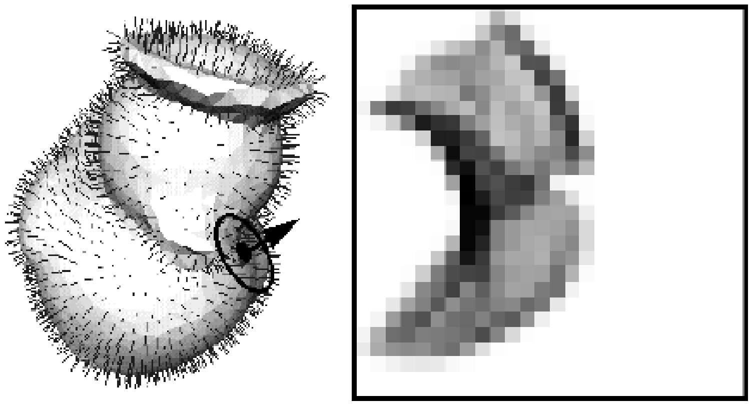


(a) (b) (c)

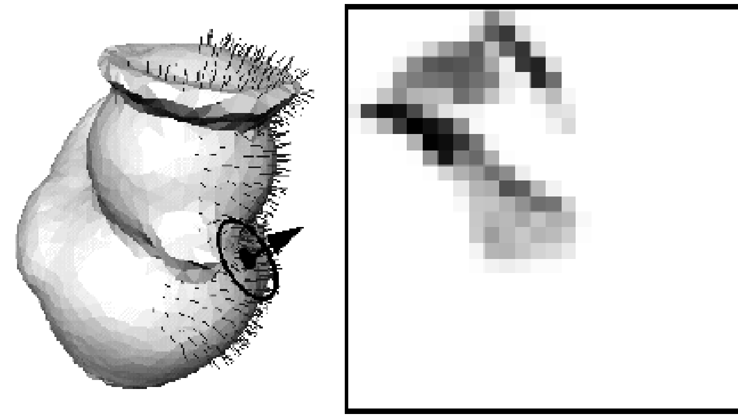
40 pixels **20 pixels** **10 pixels**

Andrew E. Johnson and M. Hebert, "Using Spin Images for Efficient Object Recognition in Cluttered 3D Scenes", ***IEEE Tran. PAMI*****, Vol. 21(5), May, 1999.**

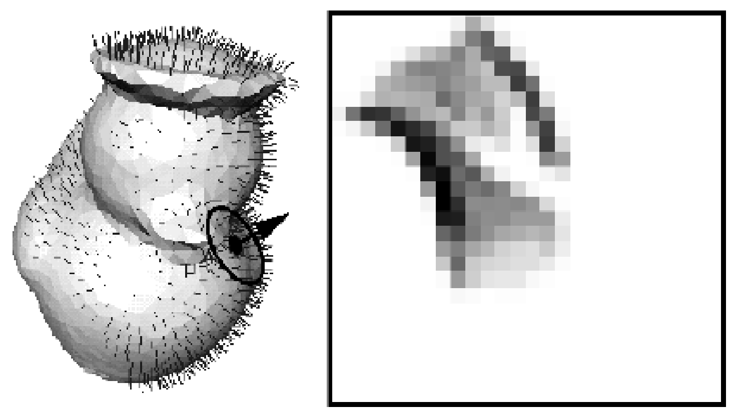
Effect of the parameters
Support angle



180 degree



90 degree



60 degree

Andrew E. Johnson and M. Hebert, "Using Spin Images for Efficient Object Recognition in Cluttered 3D Scenes", *IEEE Tran. PAMI*, Vol. 21(5), May, 1999.

Advantages / Disadvantages

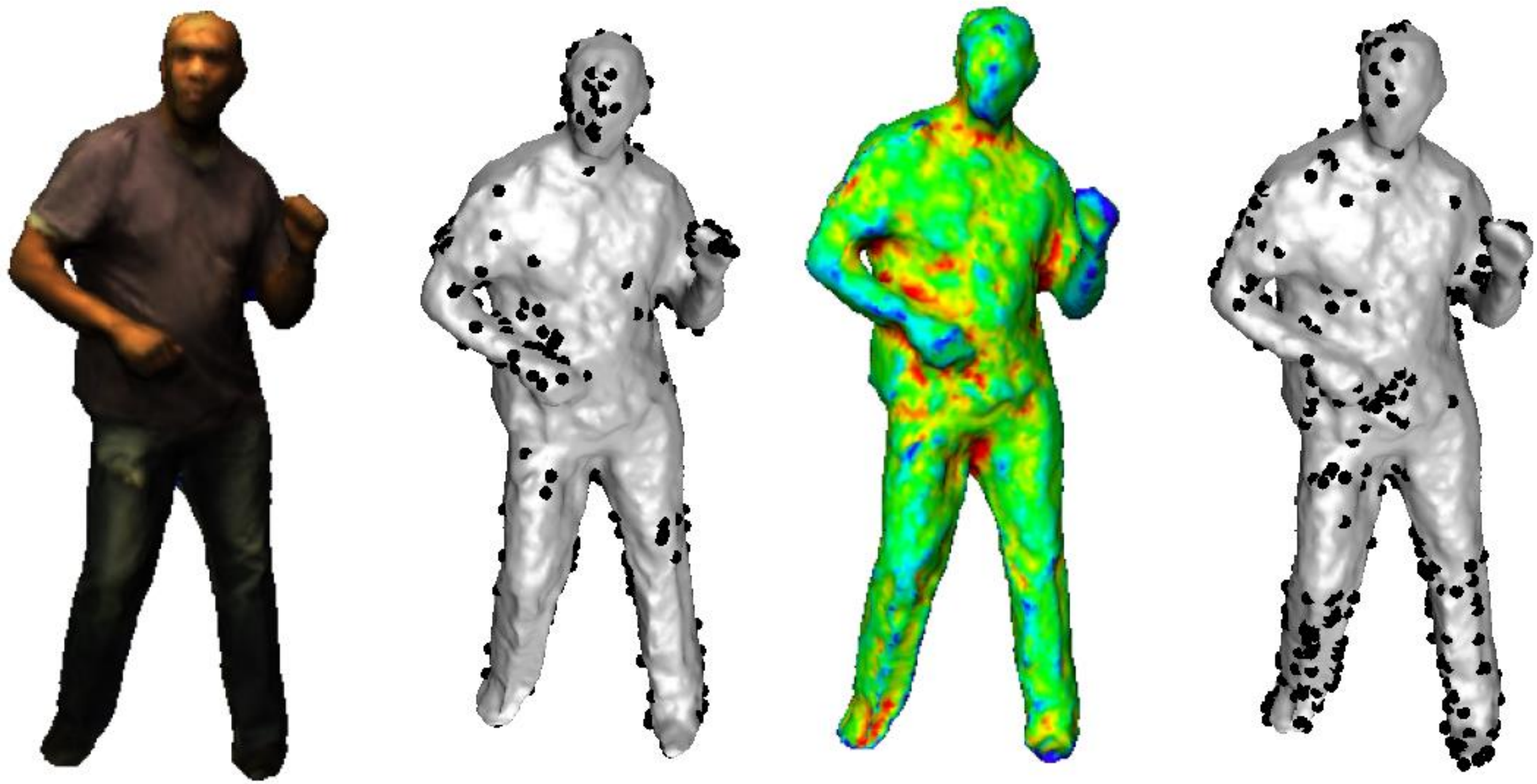
- Sampling of the surfaces → we need an uniform sampling for reliably matching.
- Parameters definition.
- Spin images are able to deal with cluttering/occlusion in complex scene.
- Matching problems
  - False positive (not distinctive in some parts)
  - False negative (sensitive to noise)

MeshDOG / MeshHOG*

- For matching and tracking surface evolving during time (animation reconstruction).
- Detector+descriptor for any scalar field associated with a 2D manifold.
- It extends DOG and HOG.
- The descriptor is able to capture local geometric and/or photometric properties in an elegant fashion.

***A. Zaharescu, E. Boyer, K. Varasani and R. Horaud, "Surface Feature Detection and Description with Applications to Mesh Matching", *CVPR'09*, 2009.**

MeshDOG / MeshHOG

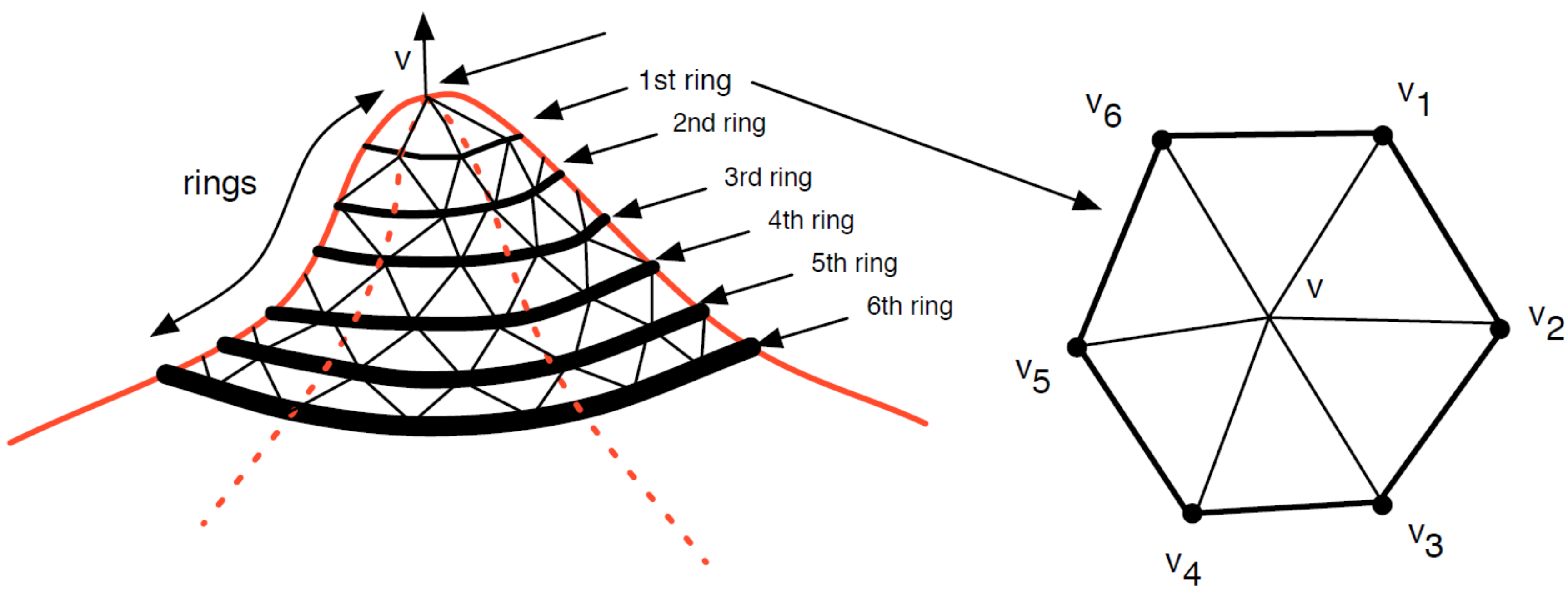


Photometric Information

Mean Curvature

MeshDOG

- Let's consider the *N*-ring of a vertex *v* :



MeshDOG – Definitions

- Directional Derivative:

$$D_{\overrightarrow{u}} f(\mathbf{v}) = \nabla_S f(\mathbf{v}) \cdot \overrightarrow{u}$$

- Discrete Directional Derivative:

$$D_{\overrightarrow{e_{ij}}} f(\mathbf{v}_i) = \frac{1}{||\overrightarrow{v_i v_j}||}(f(\mathbf{v}_j) - f(\mathbf{v}_i))$$

MeshDOG – Definitions

- Discrete Gradient:

$$\nabla_S f(\mathbf{v}_i) = \sum_{v_j \in rg(v_i,1)} \left(w_{ij} D_{\overrightarrow{e_{ij}}} f(\mathbf{v}_i)\right) \overrightarrow{u_{ij}},$$

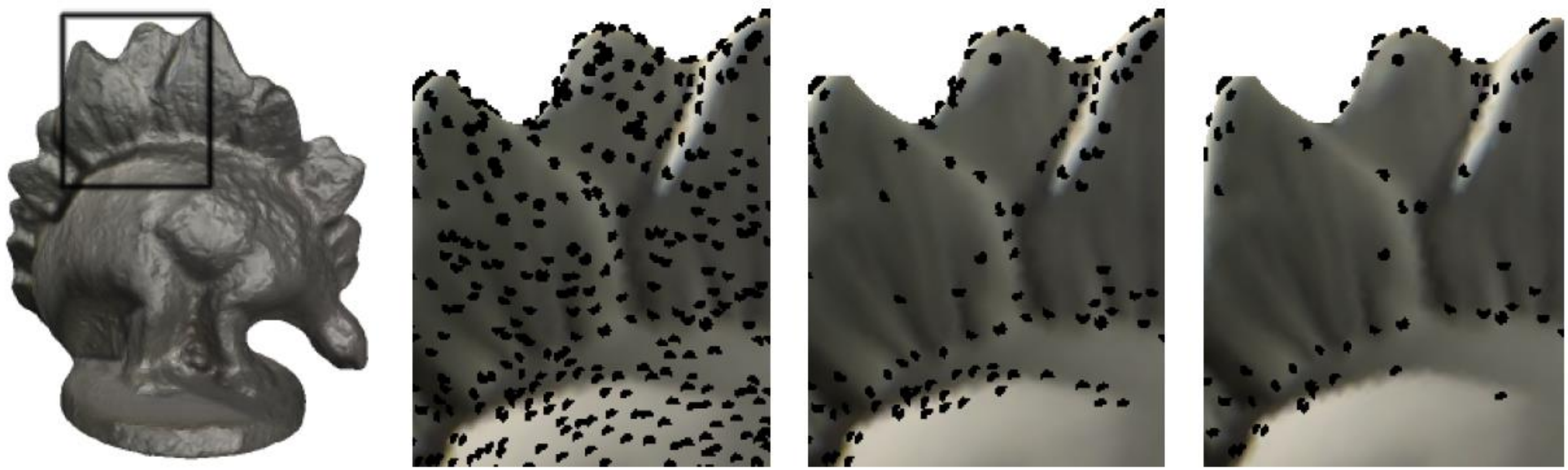
- Discrete Convolution:

$$(f * k)(v_i) = \frac{1}{K} \sum_{v_j \in N_n(v_i)} k(||\overrightarrow{v_i v_j}||) f(\mathbf{v}_j)$$

MeshDOG – Computation

- At this point MeshDOG is computed similarly to the 2D case:
  - $f_0 = f$; $f_1 = f_0 * g$; $f_2 = f_1 * g$; …
  - $DOG_1 = f_1 - f_0$; $DOG_2 = f_2 - f_1$; …
- Keypoints → scale-space maxima (1-ring)
- Only the top 5% magnitude are considered.
- Cornerness is analyzed and non-corners are discarded.

MeshDOG - Example



MeshHOG

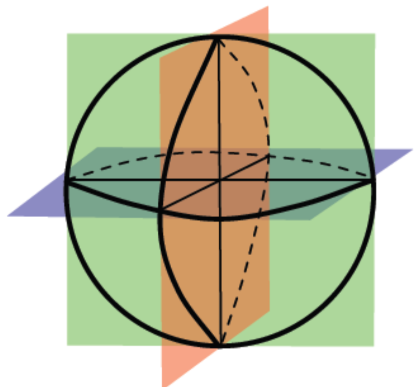
- The descriptor for the vertex *v* is computed using a support region of size *r* (*r*-ring).
- The size of the support region is chosen as a fraction of the total surface area.
- Local Coordinate System is defined as:

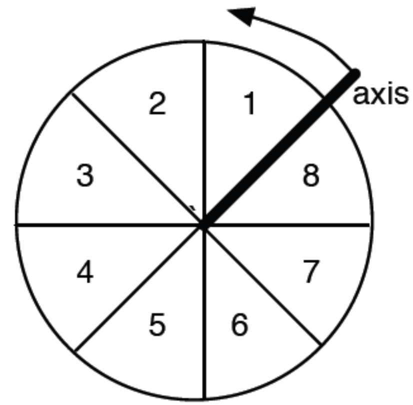
$$\{\overrightarrow{\mathbf{a}}_v, \overrightarrow{\mathbf{n}}_v, \overrightarrow{\mathbf{a}}_v \times \overrightarrow{\mathbf{n}}_v\}$$

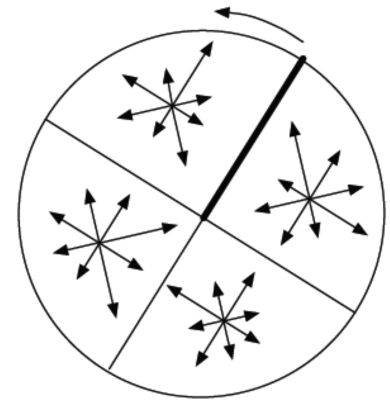
Dominant orientation

Vertex normal

MeshHOG





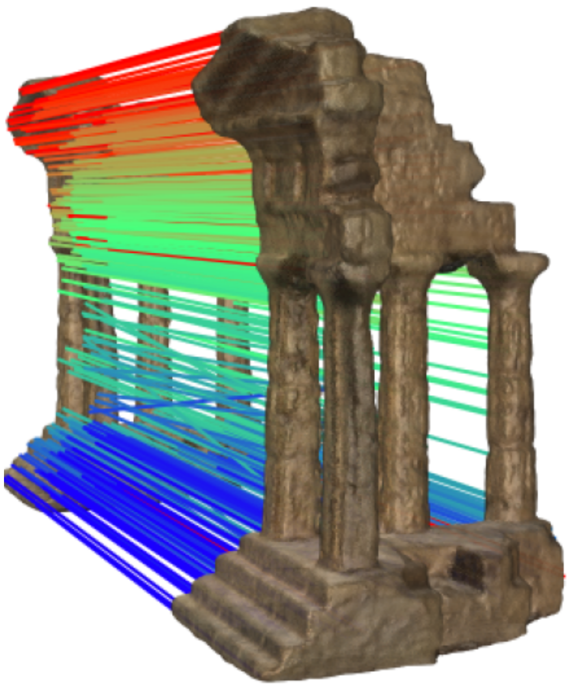


Orthonormal planes where the 3D histogram is projected

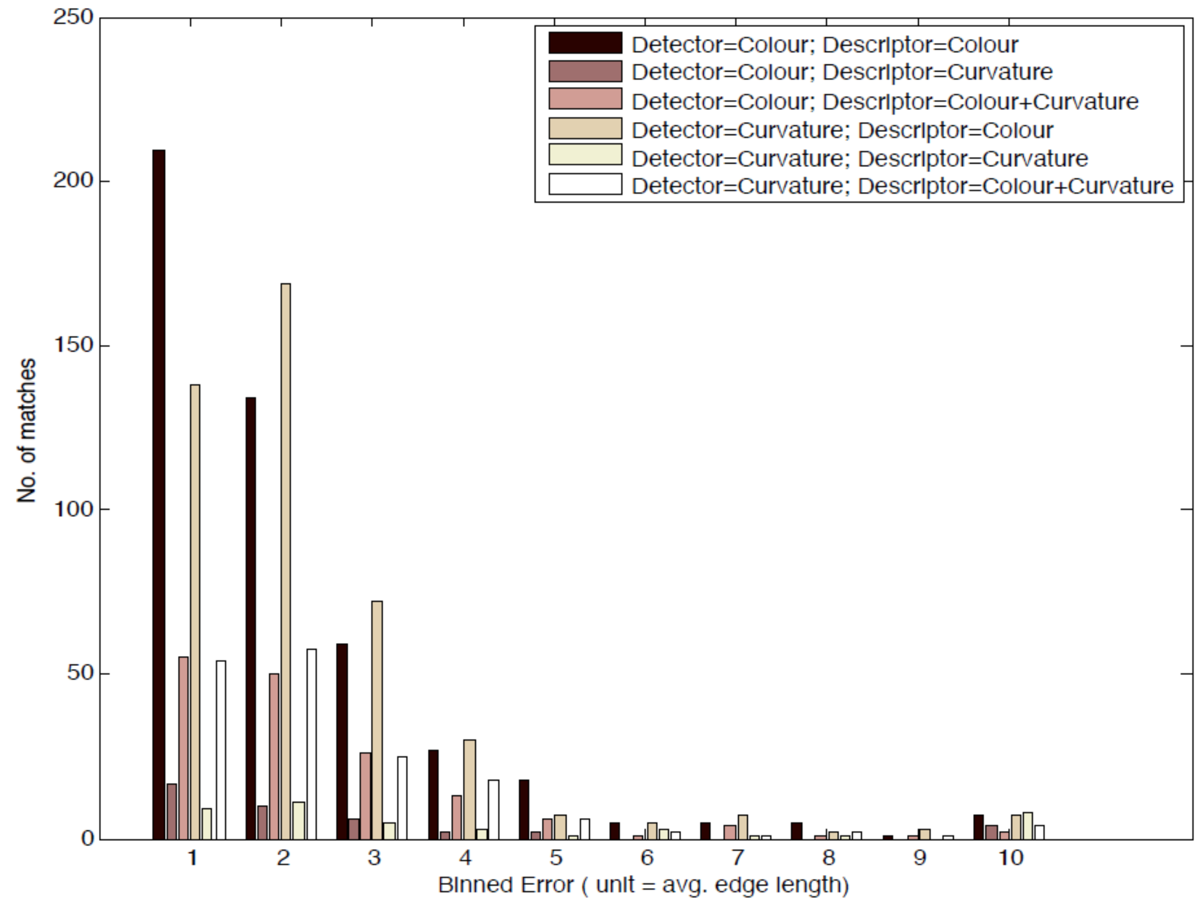
4 Spatial polar slices
8 Orientation slices

Polar Coordinate System for creating the histogram

MeshDOG+MeshHOG – Results



(c) Temple - Matches



(d) Temple - Errors

MeshDOG+MeshHOG - Results

(a) MeshHOG (b) MeshHOG (c) SIFT (d) SIFT

Heat Kernel Signature (HKS)

- *Goal:* find multi-scale signature:
  - Robust (not change if the shape is perturbed)
  - Intrinsic (invariant to isometric transformation)
  - Efficient (easy to compute at multiple scale)
- *The idea is to use as signature the heat diffusion process on a shape* → the restriction of the heat kernel to the temporal domain is used.

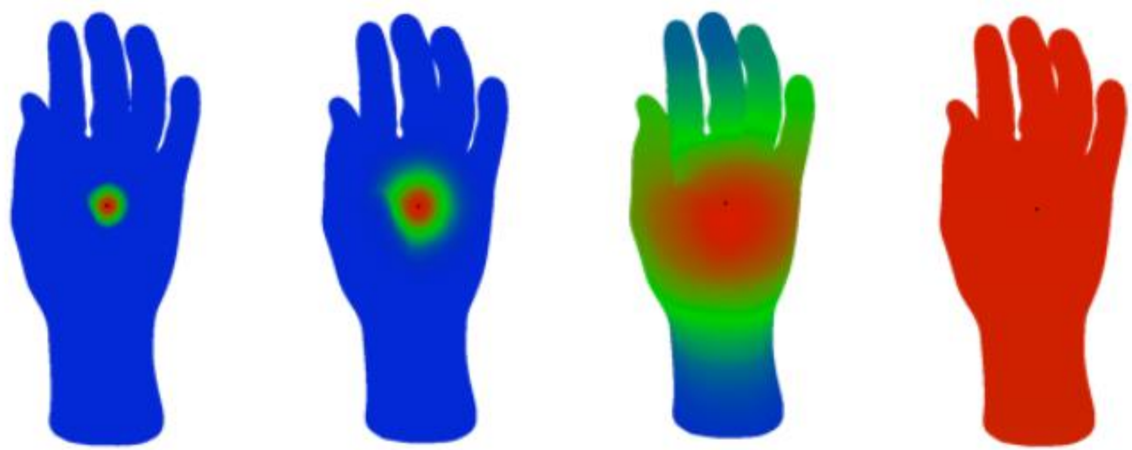
J. Sun, M. Ovsjanikov and L. Guibas, "A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion", ***Symp. on Geometry Processing (SGP'09),*** **2009.**

Heat Diffusion on a Manifold

Heat kernel $k_t(x, y) : \mathbb{R}^+ \times \mathcal{M} \times \mathcal{M} \to \mathbb{R}$

$$f(x,t) = \int_{\mathcal{M}} k_t(x,y) f(y,0) dy$$

$k_t(x, y)$: amount of heat transferred from x to y in time t.



$t = 0.001$ $t = 0.02$ $t = 3$ $t = 40$

Heat Kernel Properties

- *Intrinsic* → invariant under *isometric* transformation (given an isometry *T : M → N)*

$$k_t^M(x,y) = k_t^N(T(x),T(y)) \quad \forall x,y \in M \ \forall t$$

- *Robust* (to local deformations → all the paths are taken into account)
- *Multi-scale* (as *t* increase, heat diffusion influences larger and larger neighborhood)

Heat Kernel Signature

- We would like to use the heat kernel $k_t(x,\cdot)$ as the signature of the point x at scale t but..
  - It is function of the entire manifold !
  - It is difficult to compare two signatures.
- => *The heat kernel is restricted on the temporal domain*.

Heat Kernel Signature

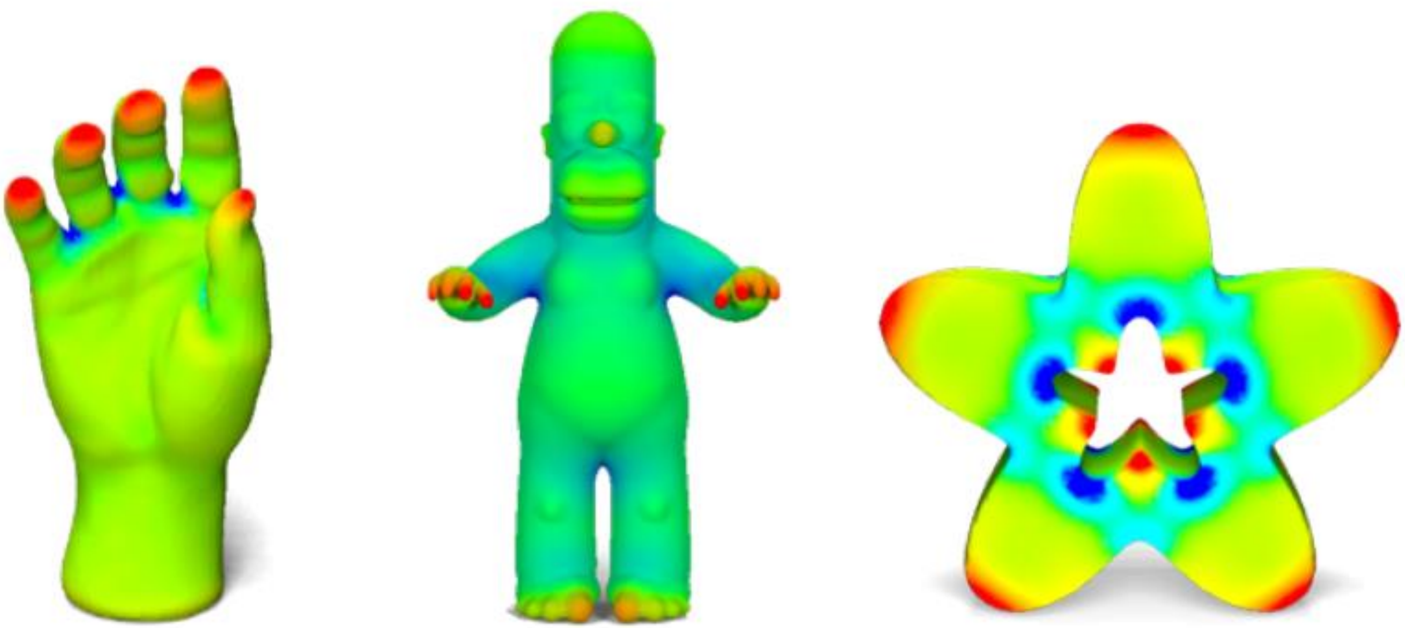
- Given a point *x* on the manifold *M*, the Heat Kernel Signature (HKS) is defined as:

$$HKS(x) : \mathbb{R}^+ \to \mathbb{R}\,,\ HKS(x,t) = k_t(x,x)$$

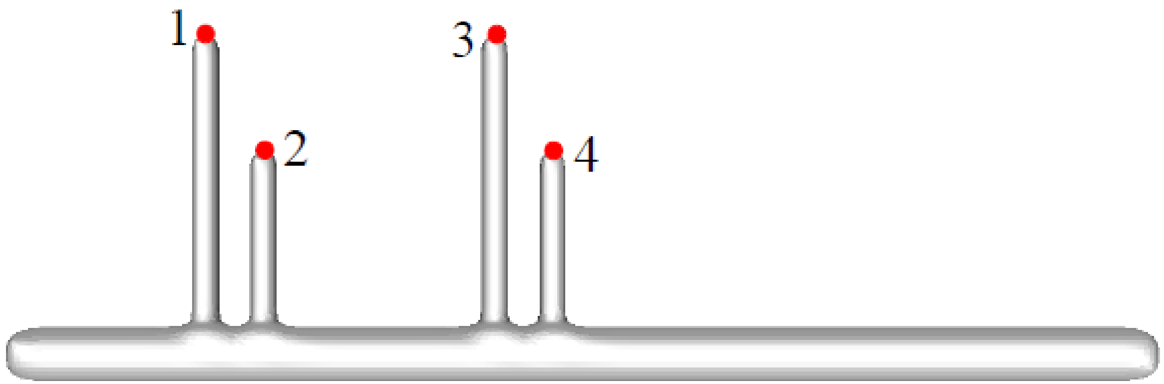
Relation with the Gaussian Curvature

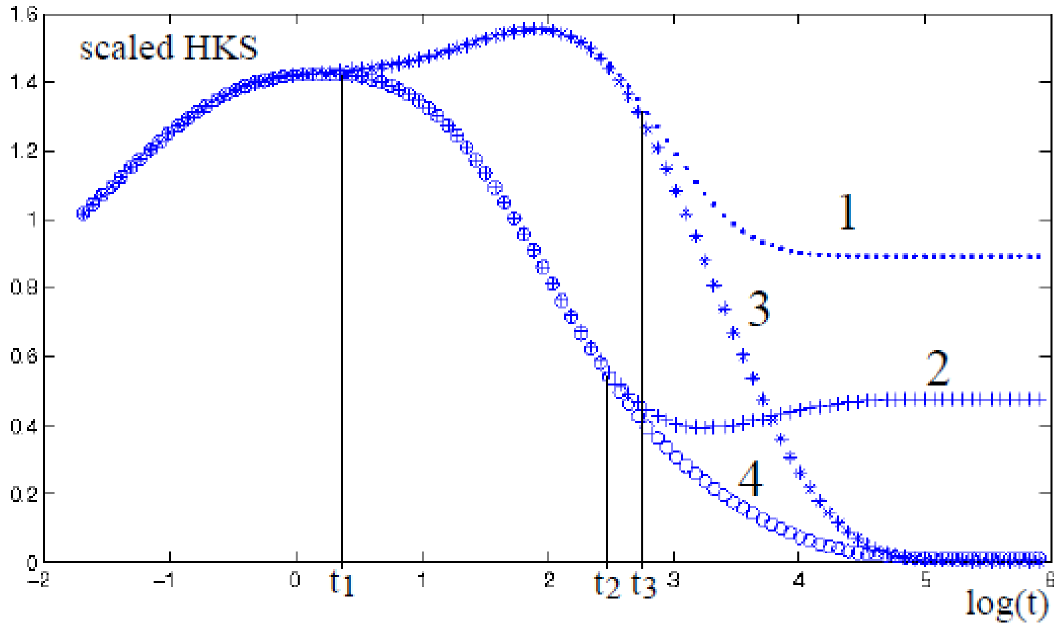
Valid for small values of *t*

$$k_t(x,x) = \frac{1}{4\pi t}\sum_{i=0}^{\infty} a_i t^i \qquad a_0 = 1, a_1 = \frac{1}{6}K$$



HKS – Multi-Scale Properties Illustration





HKS – Computation

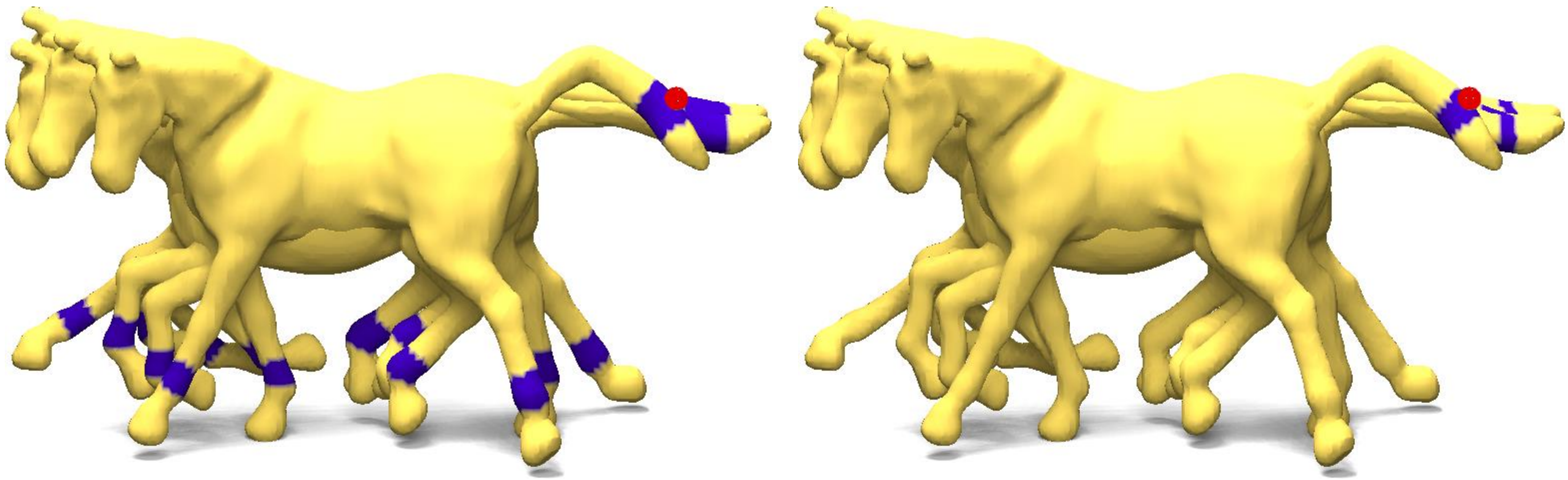
- On a compact manifold:

$$k_t(x,y) = \sum_{i=0}^{N} e^{-t\lambda_i}\phi_i(x)\phi_i(y)$$

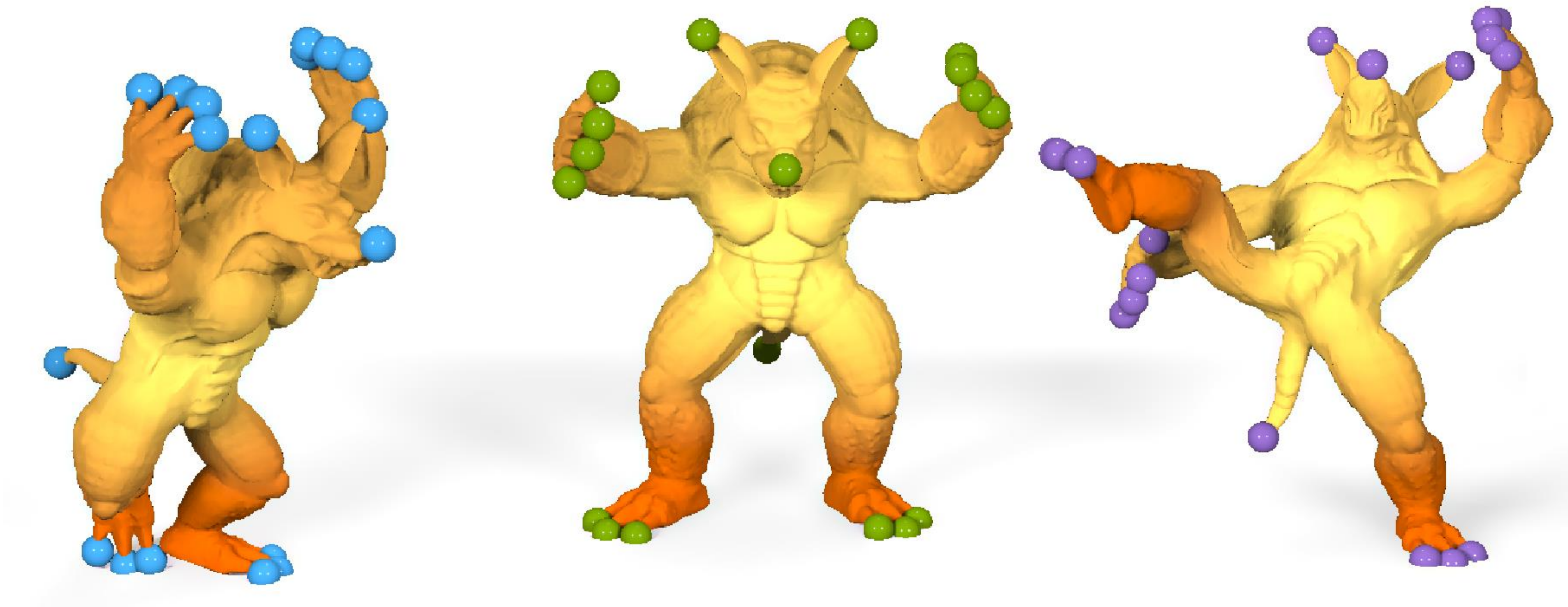
where λ_i, ϕ_i are, respectively, the *i-th* eigenvalue and eigenfunction of the Laplace-Beltrami operator.

- Laplacian operator can be put in matrix form for the entire mesh → use the eigenvalues and eigenvectors of this matrix.

HKS - Results



HKS – Results



Recap

- Different correspondence problems, different data types, different applications → many different solutions to find correspondences on geometric data.
- Notions of geometry processing are fundamental (e.g. discrete curvature).
- Some solutions are
  - Curvature-based
  - Inspired by works about 2D features (e.g. SIFT-inspired)
  - Usually local descriptors are designed by aggregating geometric properties using different type of histograms (defined on a local tangent plane, on a volume, etc.).

Questions ?