# Spatial Search Data Structures

**Corso di dottorato: Geometric Mesh Processing**

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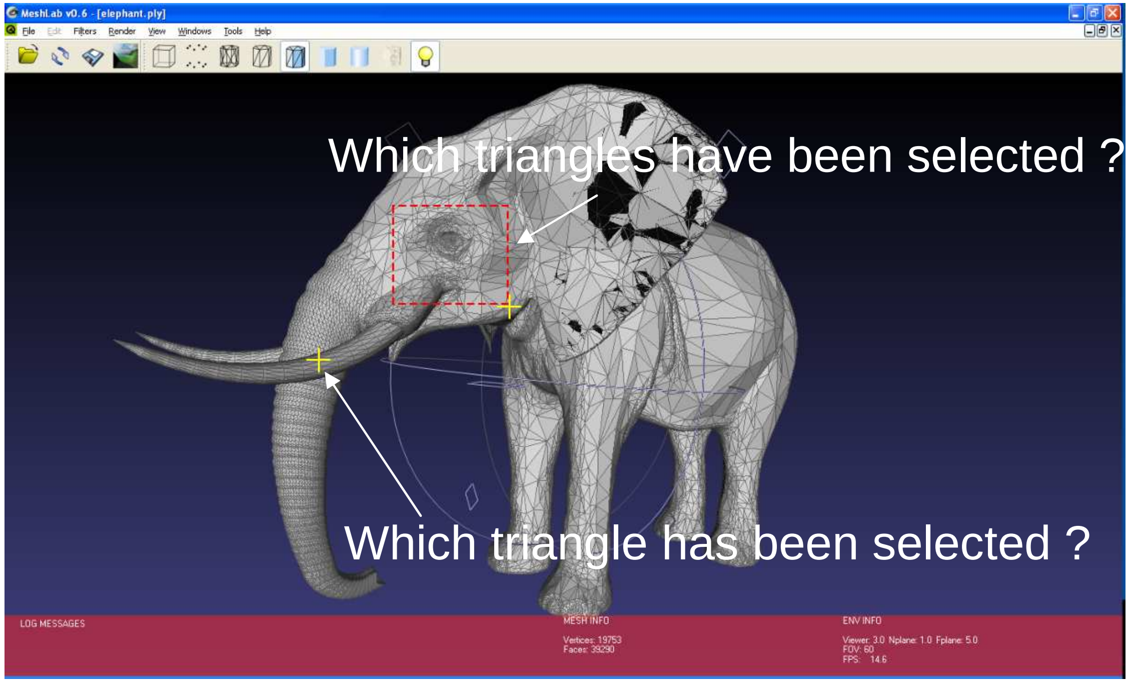
# Problem statement

- Let *m* be a mesh:
  - Which is the mesh element closest to a given point *p*?
  - Which are the elements inside a given region?
  - Which elements are intersected by a given ray *r?*
- Let *m'* be another mesh:
  - Do *m* and *m'* intersect? If so, where?

- A spatial search data structure helps to answer efficiently to these questions

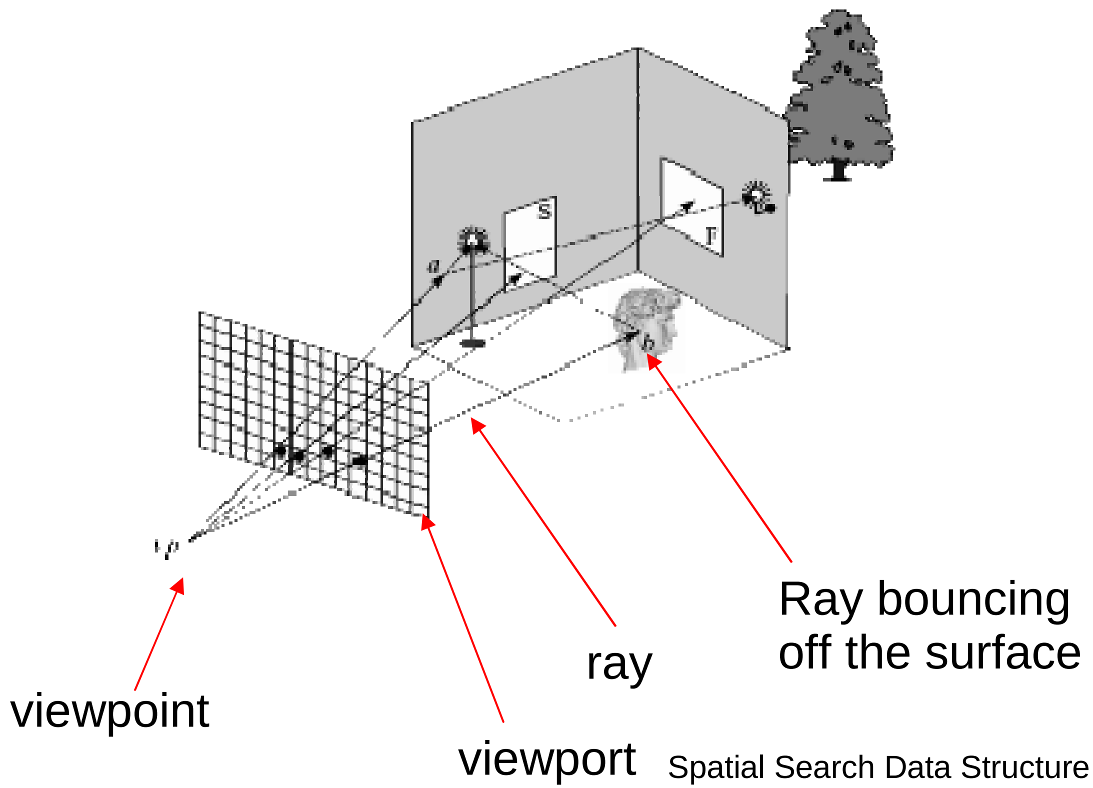
# Motivations




- Picking on a point
- Selecting a region

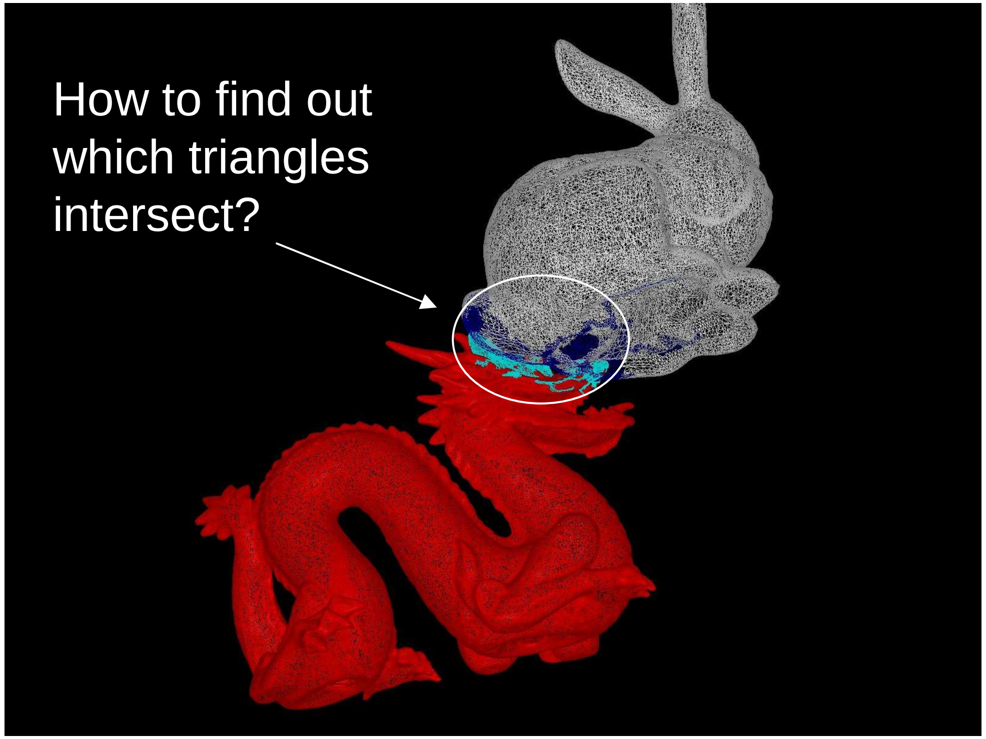
## Motivations$^{cntd}$

- Ray tracing: shoot a ray for each pixel, see what it hits, possibly recur, compute pixel color
- Involves plenty of ray-objects intersections



# Motivations$^{cntd^{cntd}}$

- Collision detection: in dynamic scenes, moving objects can collide.

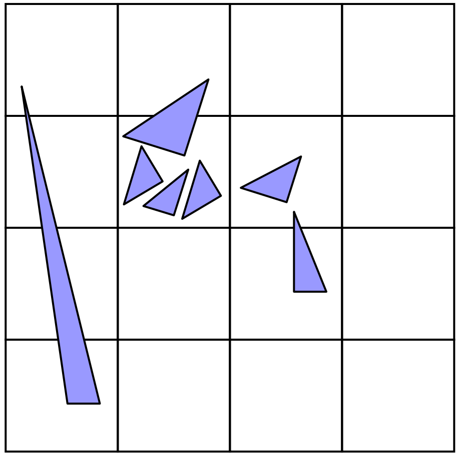


# Motivations$^{cntd^{cntd^{cntd}}}$

- Without any spatial search data structure the solutions to these problems require *O(n)* time, where *n* is the numbers of primitives ( $O(n^2)$ for the collision detection)
- Spatial data structure can make it (average) constant
  - ..or average logarithmic

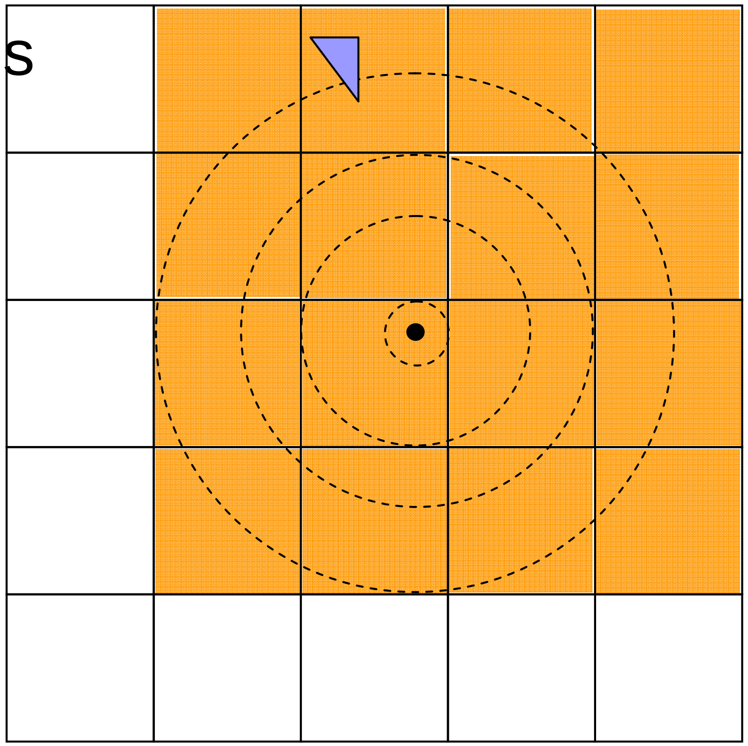
# Uniform Grid (1/4)

- **Description**: the space including the object is partitioned in cubic cells; each cell contains references to “primitives” (i.e. triangles)
- **Construction**.
  Primitives are assigned to:
  - The cell containing their feature point (e.g. barycenter or one of their vertices)
  - The cell**s** spanned by the primitives



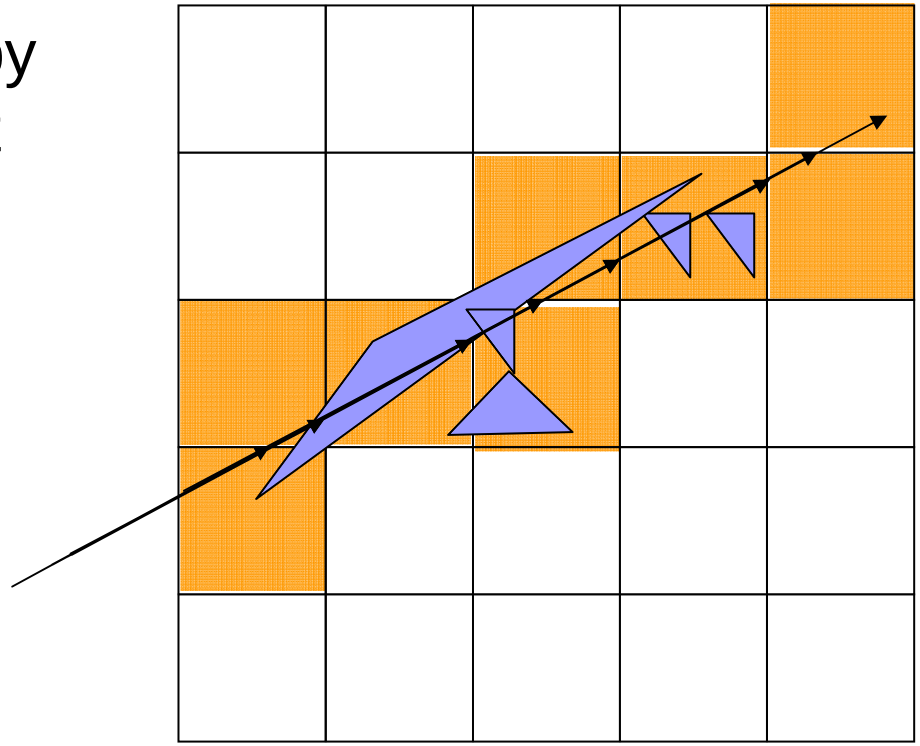
# Uniform Grid (2/4)

- **Closest element** (to point p):
  - Start from the cell containing p
  - Check for primitives inside growing spheres centered at p
  - At each step the ray increases to the border of visited cells
- **Cost**.
  - Worst: O(#cells+n)
  - Average; O(1)



# Uniform Grid (3/4)

- **Intersection with a ray**:
  - Find all the cells intersected by the ray
  - For each intersected cell, test the intersection with the primitives referred in that cell
  - Avoid multiple testing by flagging primitives that have been tested (*mailboxing*)
- **Cost:**
  - Worst: $O(\#cells + n)$
  - Aver: $O(\sqrt[d]{\#cells} + \sqrt[d]{n})$

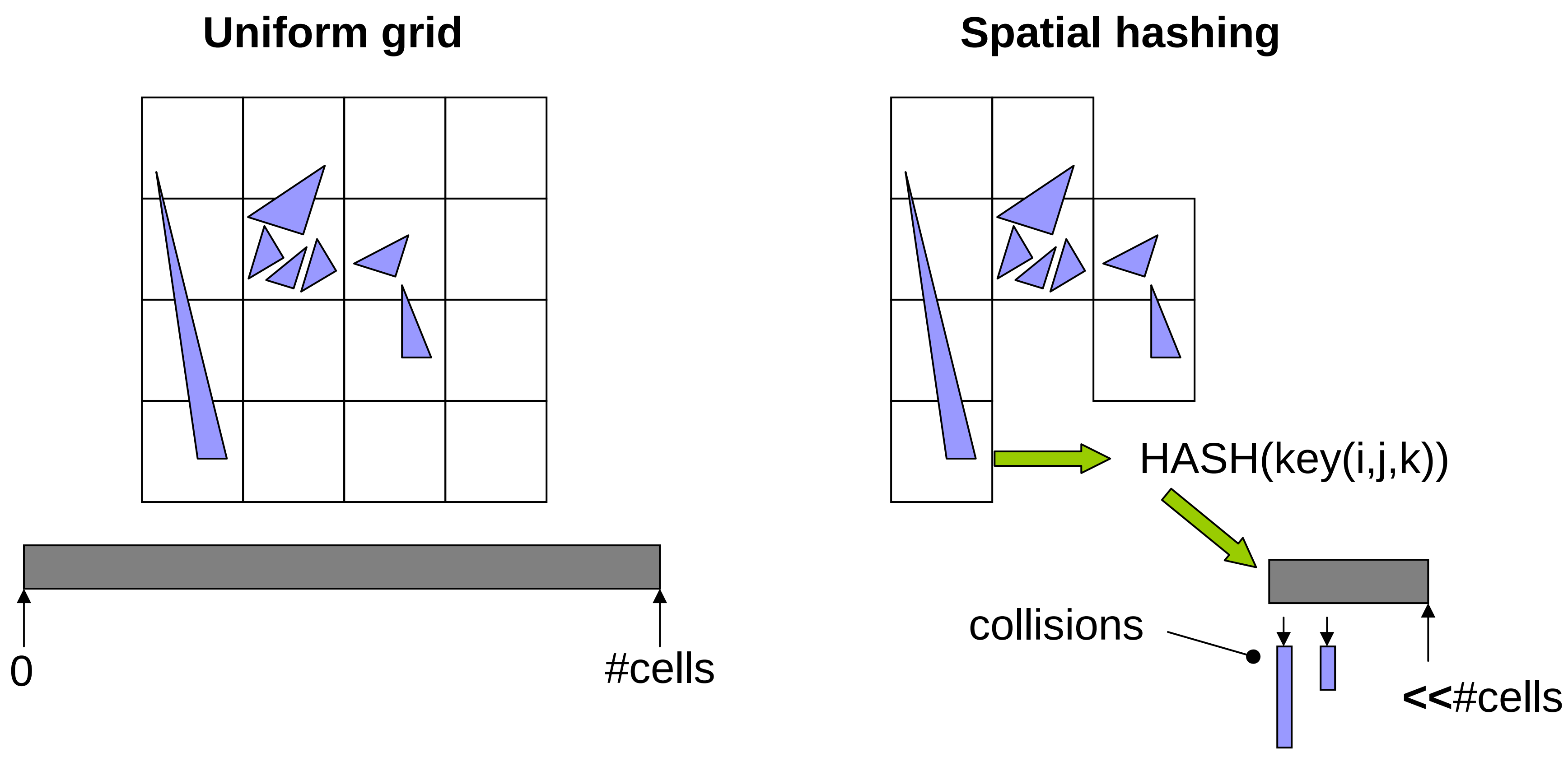


# Uniform Grid (4/4)

- **Memory occupation:** $O(\#cells + n)$
- **Pros:**
  - Easy to implement
  - Fast query
- **Cons**:
  - Memory consuming
  - Performance **very** sensitive to distribution of the primitives.

# Spatial Hashing (1/2)

- The same as uniform grid, except that only non empty cells are allocated

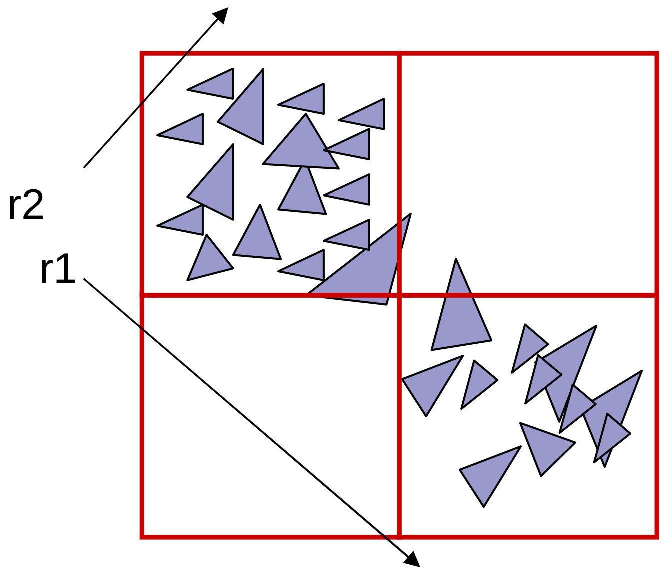


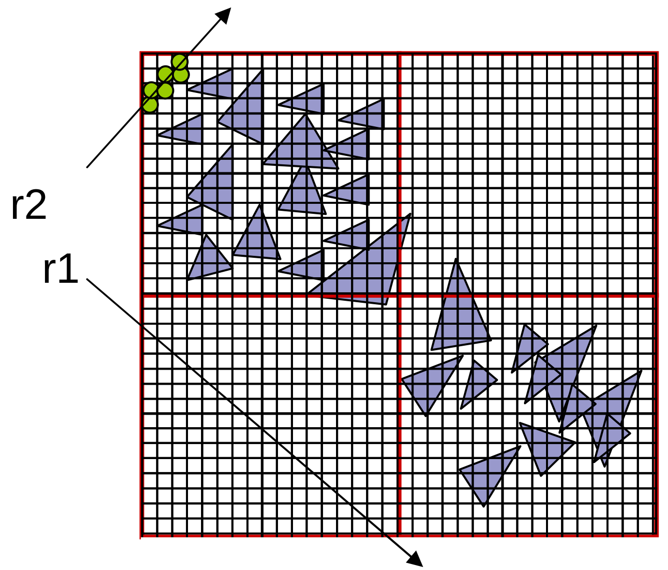
# Spatial Hashing (2/2)

- **Cost:** same as UG, except that in worst case the access to a cell is *O(#cells)* because of collisions
- **Memory occupation:**
  - **Worst. :** $O(\#cells)$
  - **Aver. :** $O\left(\left(\frac{\#cells}{Vol}\right)^{\frac{2}{3}} \cdot S\right) \quad \mathrm{S: surface, Vol: Volume}$
- **Pros:**
  - Easy to implement
  - Fast query if **good hashing** is done
  - Less memory consuming
- **Cons**:
  - Performance **very** sensitive to distribution of the primitives.

# Beyond UG

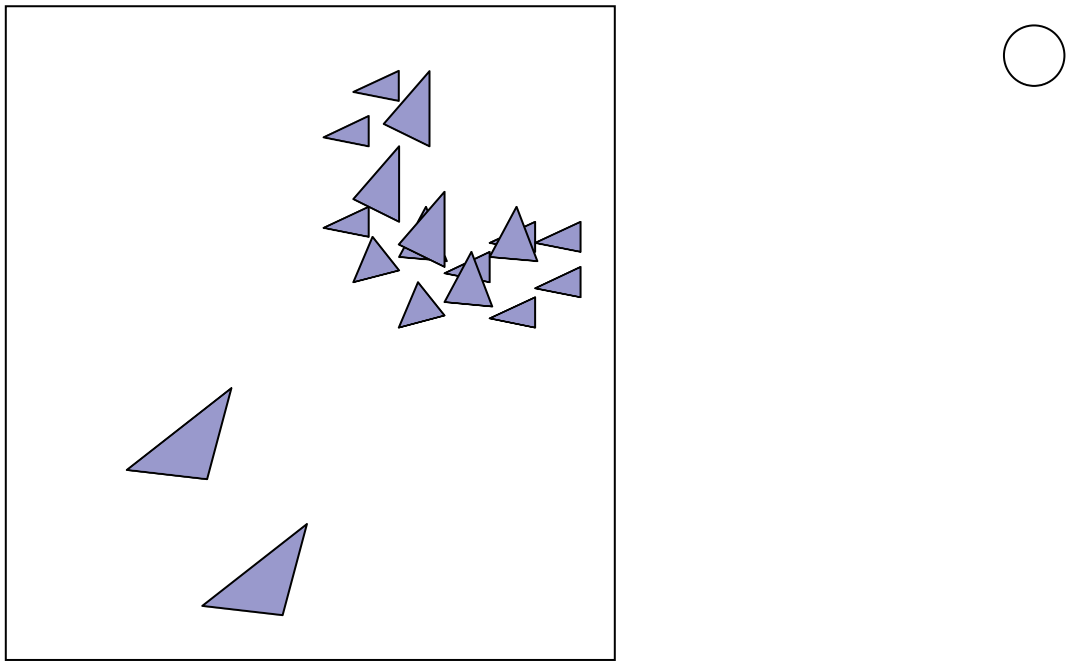
- Uniform grids are **input insensitive**
- What's the best choice for the example below?



# Hierarchical Indexing of Space

- Divide et impera strategies:
  - The space is partitioned in sub regions
  - ..recursively



Spatial Search Data Structure

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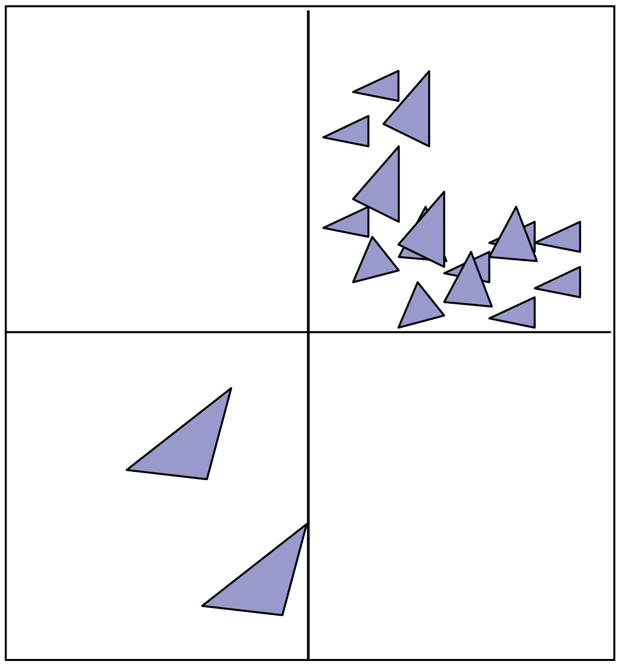

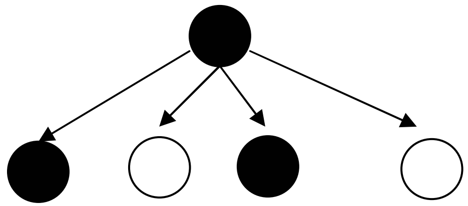

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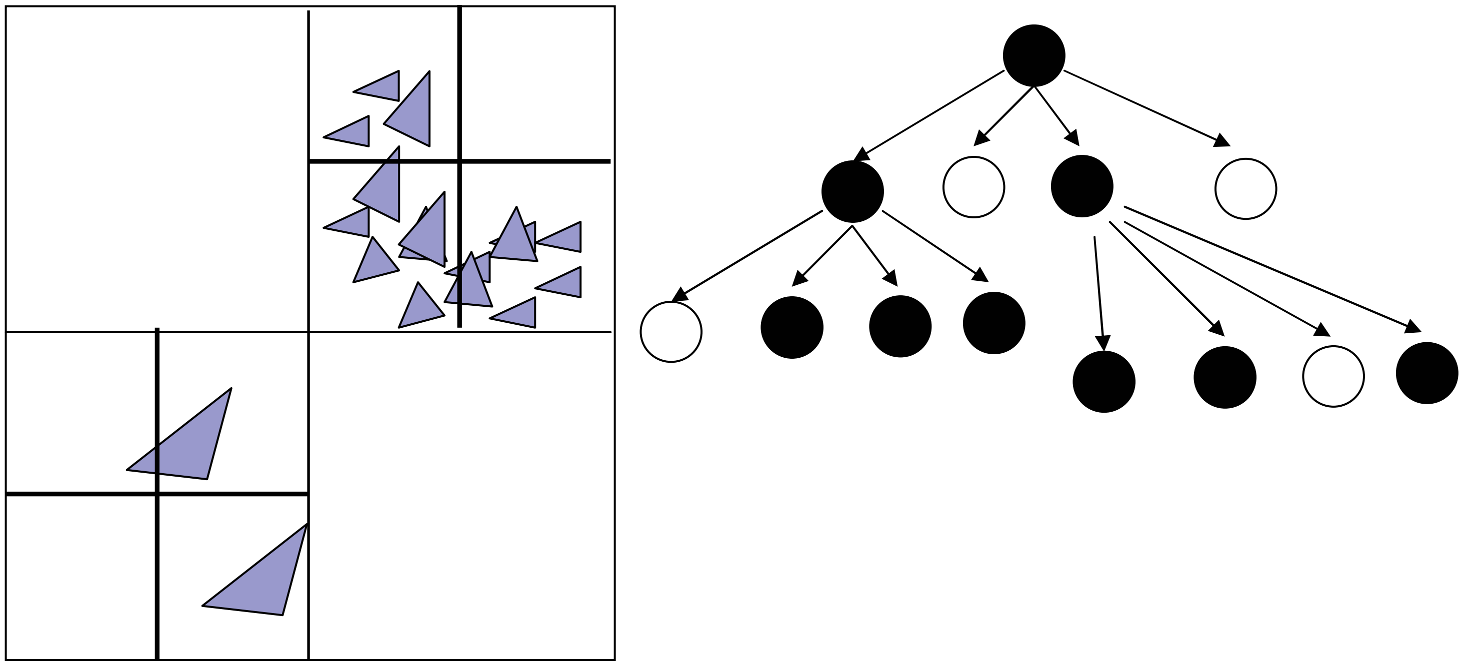

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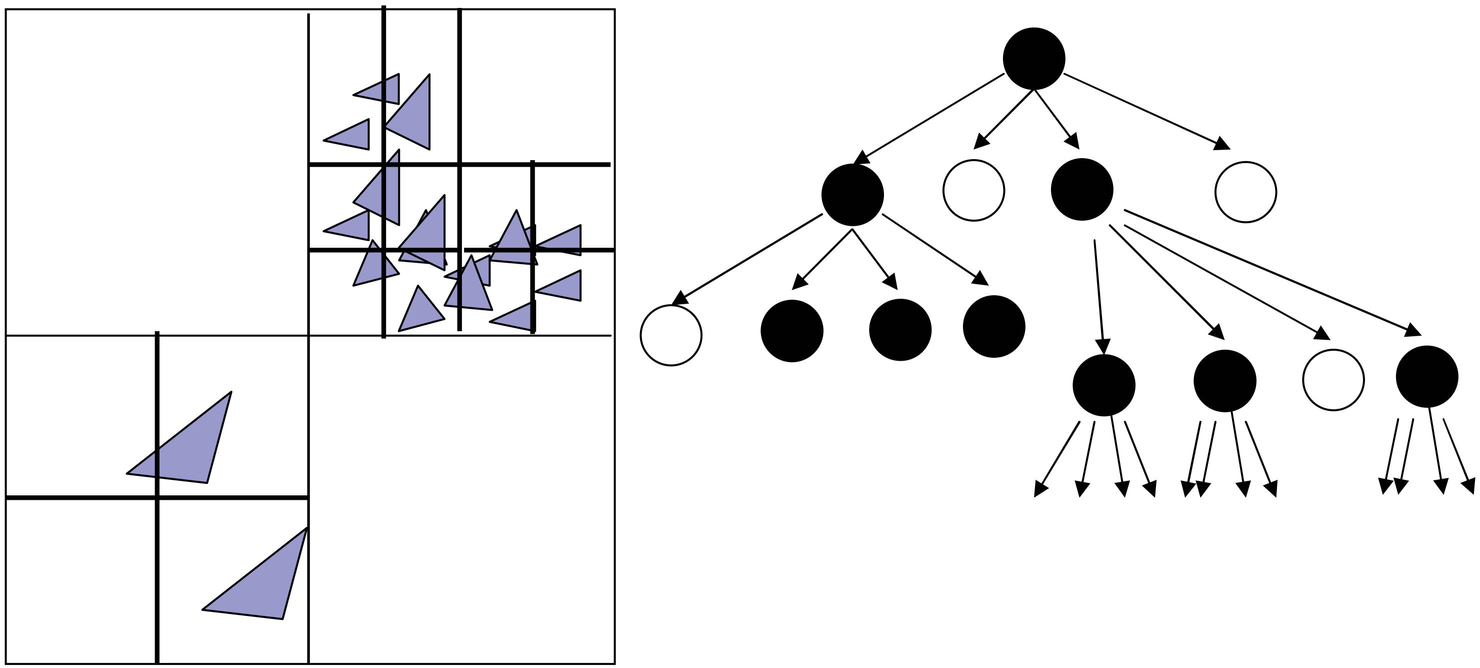


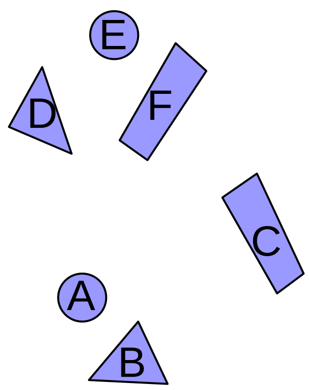
Spatial Search Data Structure

## Basic Facts

- The queries correspond to a visit of the tree
  - The complexity is sublinear in the number of nodes (logarithmic)
  - The memory occupation is linear
- A hierarchical data structure is characterized by:
  - Number of children per node
  - Spatial region corresponding to a node

# Binary Space Partition-Tree (BSP) (1/3)

- **Description**:
  - It's a binary tree obtained by recursively partitioning the space in **two** by a hyperplane
  - therefore a node always corresponds to a **convex region**



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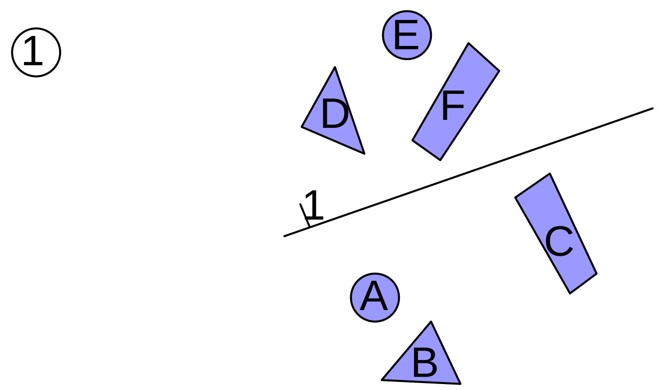

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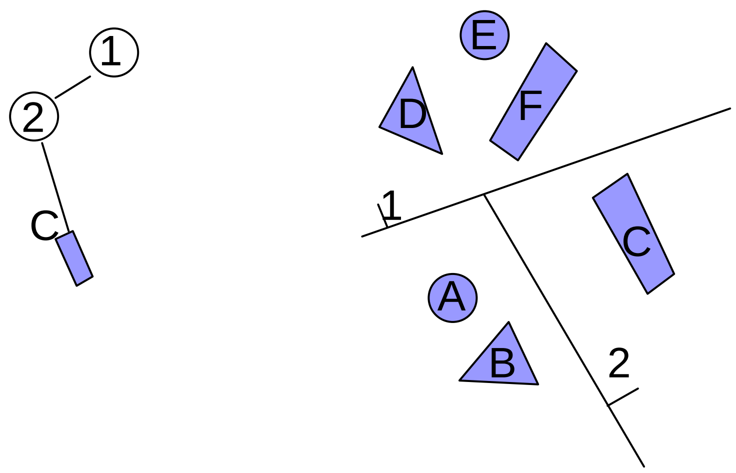

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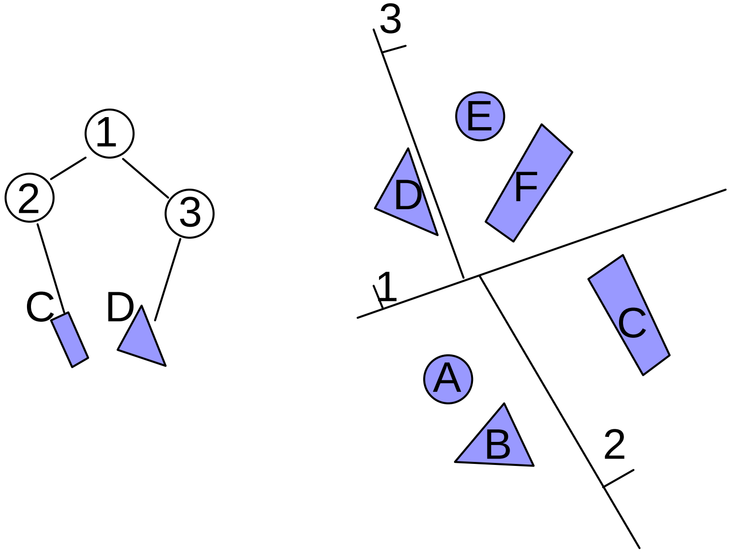

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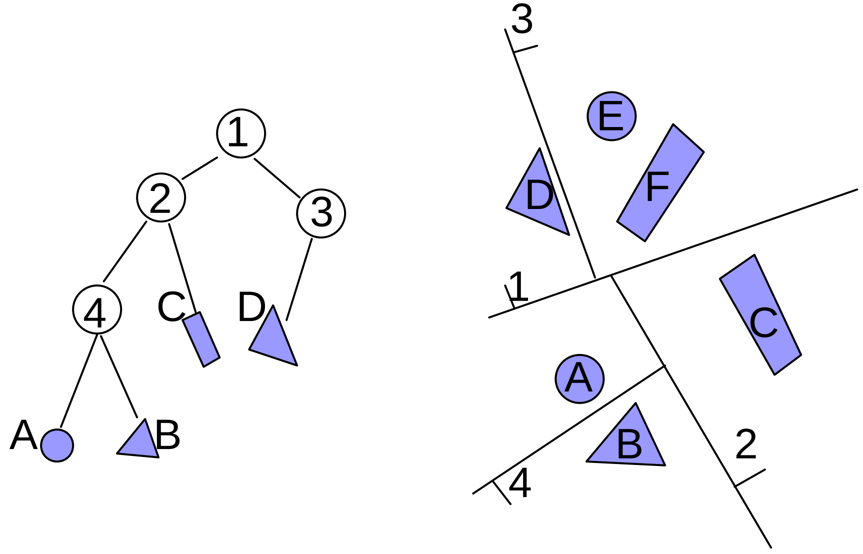

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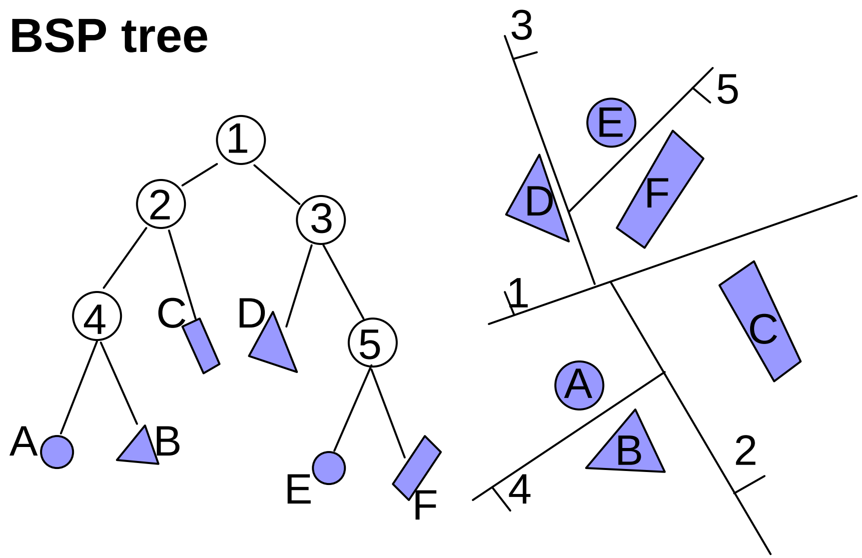
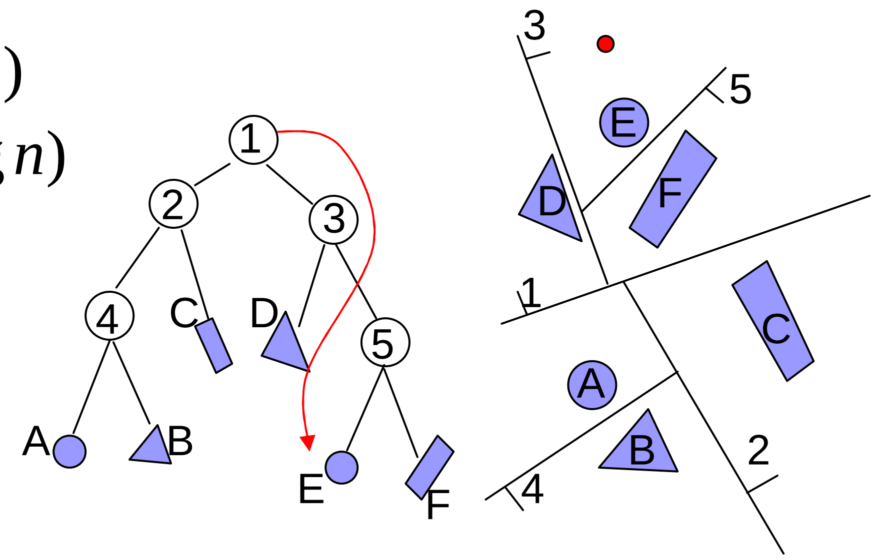


# Binary Space Partition-Tree (BSP) (1/ 3)

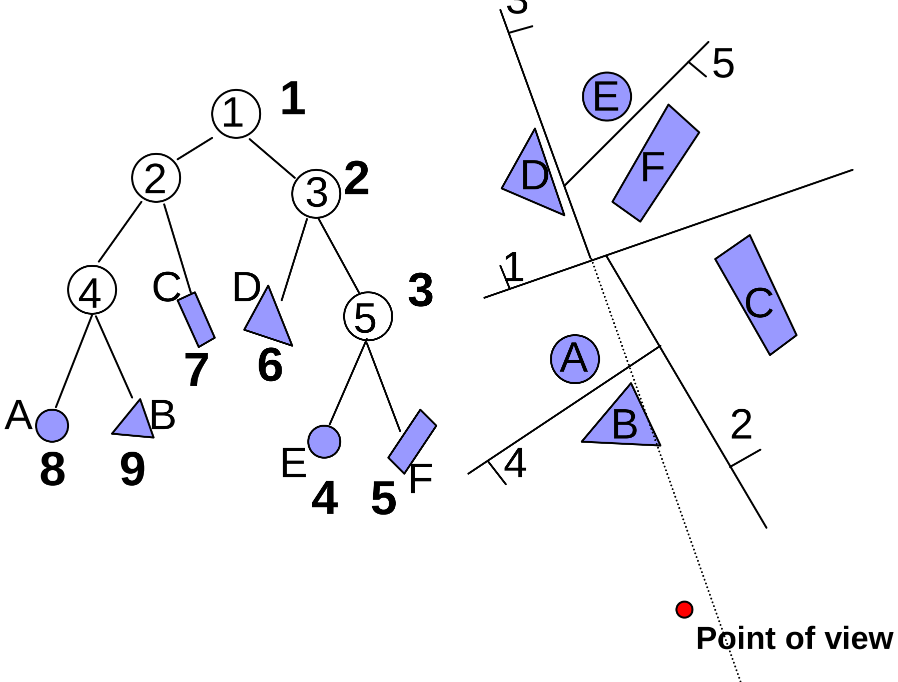
- **Query**: is the point *p* inside a primitive?
  - Starting from the root, move to the child associated with the half space containing the point
  - When in a leaf node, check all the primitives
- Cost:
  - Worst: $O(n)$
  - Aver: $O(\log n)$




**BSP tree**

# BSP-Tree For Rendering

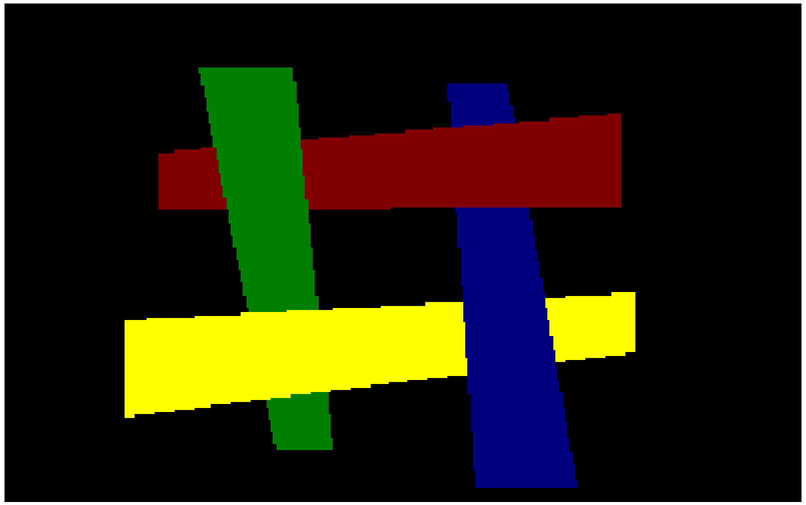
- ordering primitives back-to-front




```
void DrawBackToFront(n,p){
if(IsLeaf(n))
   Draw(n);
if( InNegativeHS(p,n) )
  DrawBackToFront(RightChild(n),p);
else
  DrawBackToFront(LeftChild(n),p);
}
```

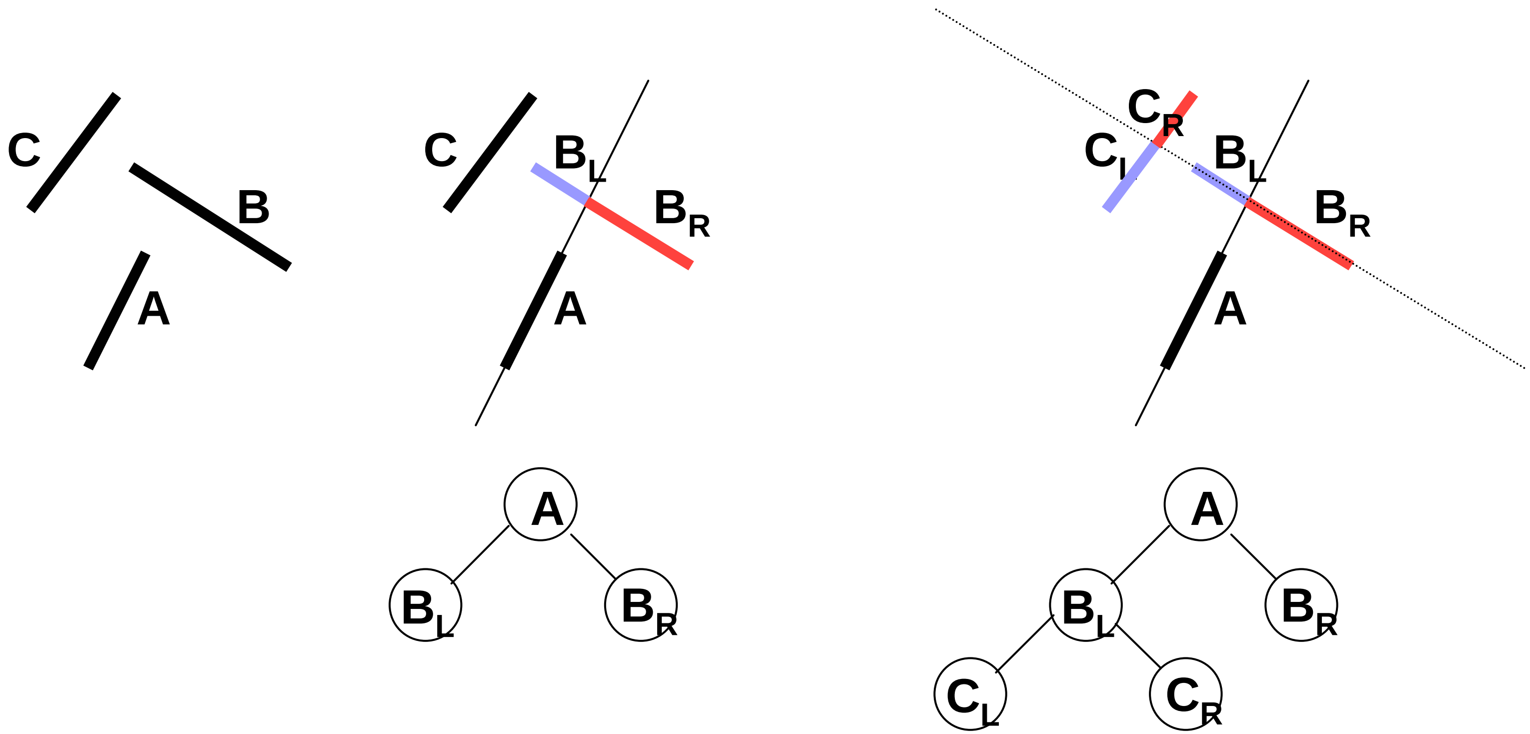
# BSP-Tree For Rendering

- **Not so fast:** set of polygons not always separable by a plane



# Binary Space Partition-Tree (BSP) (3/3 )

- **Auto-partition :**
  - use the extension of primitives as partition planes
  - Store the primitive used for PP in the node



# Bulding a BSP-Tree

- Building a BSP-tree requires to choose the partition plane
- Choose the partition plane that:
  - Gives the  best balance ?
  - Minimize the number of splits ?
  - …..it depends on the application
- Cost of a BSP-Tree

**Cost of visiting $T_{L[r]}$**

$$C(T) = 1 + P(T_L)\ C(T_L) + P(T_R)\ C(T_R)$$

**Probability that $T_{L[R]}$ is visited if T has been visited**

# Bulding a BSP-Tree: example

$$C(T) = 1 + P(T_L)\ C(T_L) + P(T_R)\ C(T_R)$$

$$C(T) = 1 + \qquad |S_L|^{\alpha} + \qquad |S_R|^{\alpha} + \qquad \beta s$$

$$S_{L[R]} = number\ of\ primitives\ in\ the\ left\ [right]\ subtree$$

$$s \quad = number\ of\ primitives\ split\ by\ the\ chosen\ plane$$

- $\alpha, \beta$ *used for tuning*
  - Big alpha, small beta yield a balanced tree (good for in/out test)
  - Big beta, small alpha yield a smaller tree (good for visibility order)

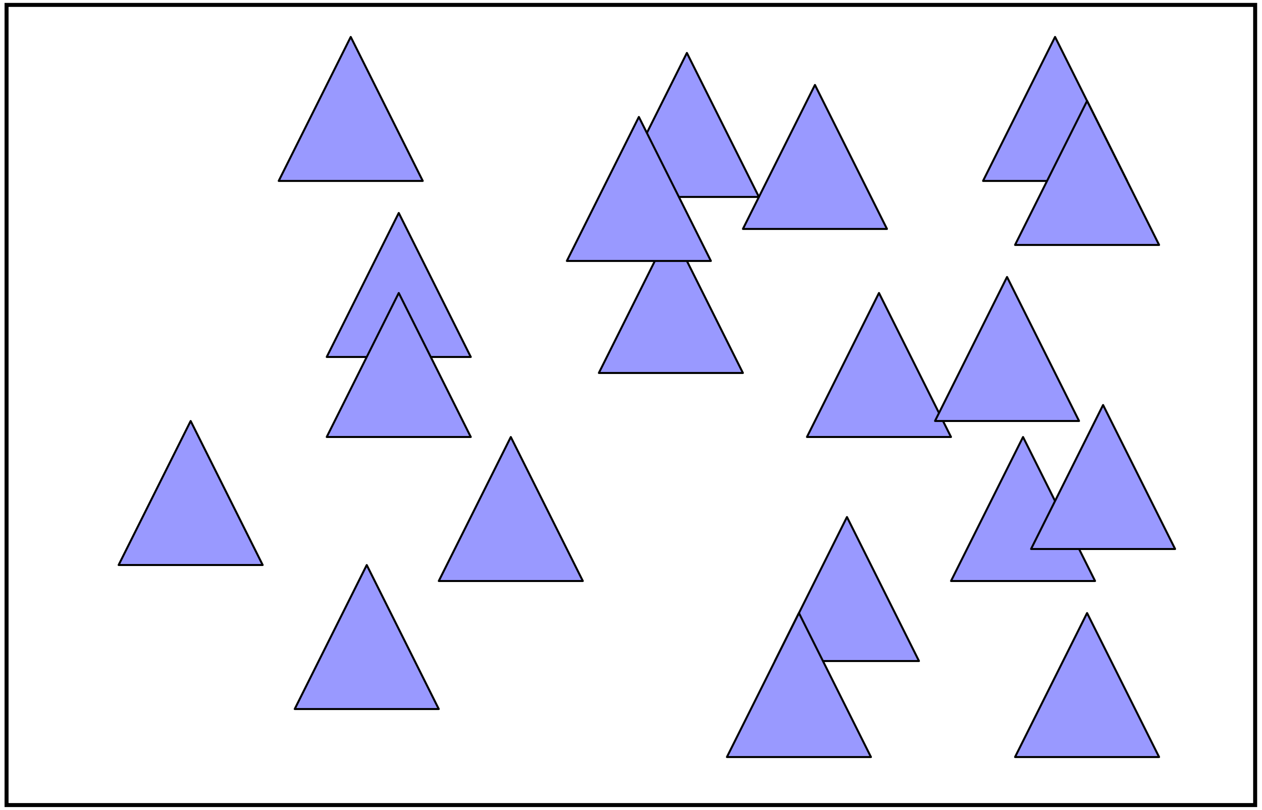
# Binary Space Partition-Tree (BSP)

- **Memory occupation:** $O(n)$
  - For each node:
    - (d+1) floatig point  numbers (in d dimensions)
    - 2 pointers to child node
- Cost of descending the three:
  - d products, d summations (dot product d+1 dim.)
  - 1 random memory access (follow the pointer)
- Less general data structures can be faster/ less memory consuming

# kd-tree

- Kd-tree : k dimensions tree
- È una specializzazione dei BSP in cui i  piani di partizione sono ortogonali a uno degli assi principali
- Scelte:
  - L'asse su cui piazzare il piano
  - Il punto sull'asse in cui piazzare il piano
- Vantaggi sui BSP:
  - determinare in quale semispazio risiede un punto costa un confronto
  - La memorizzazione del piano richiede un floating point + qualche bit

# kD-Trees: esempio



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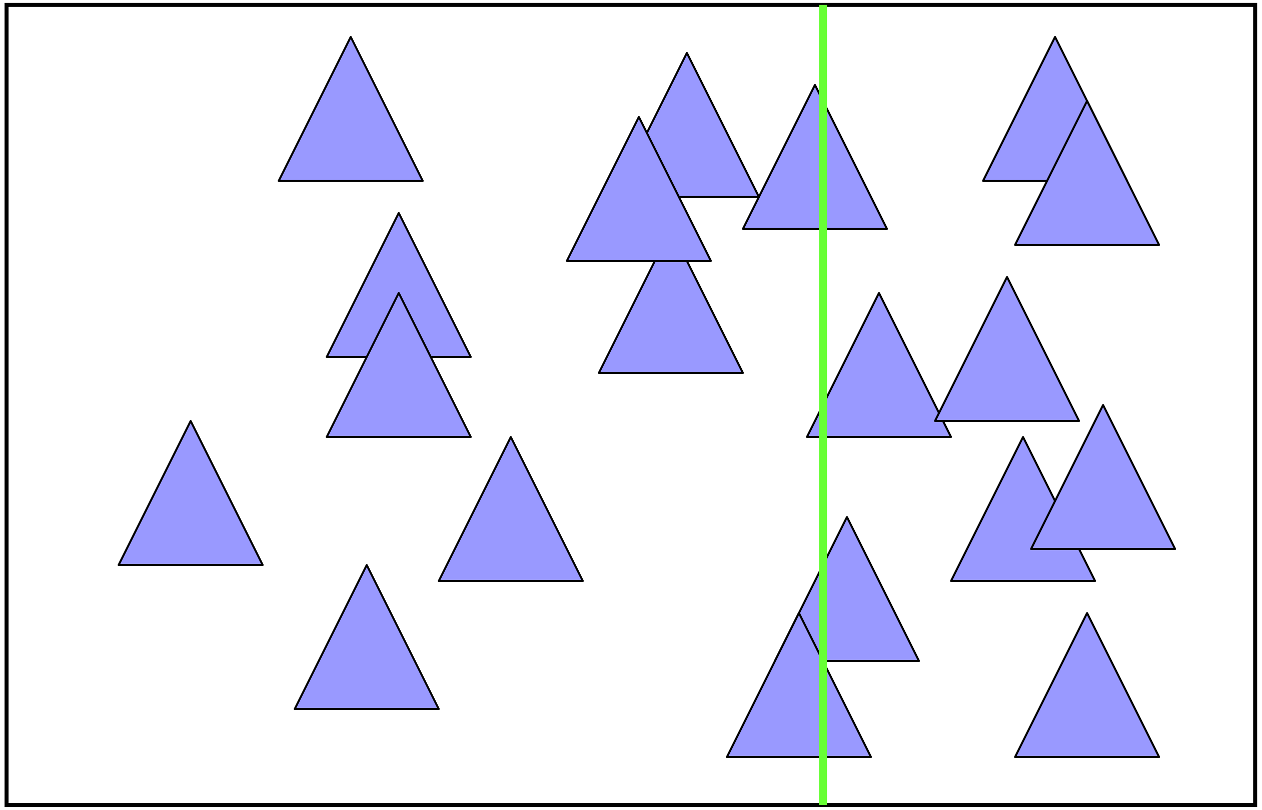

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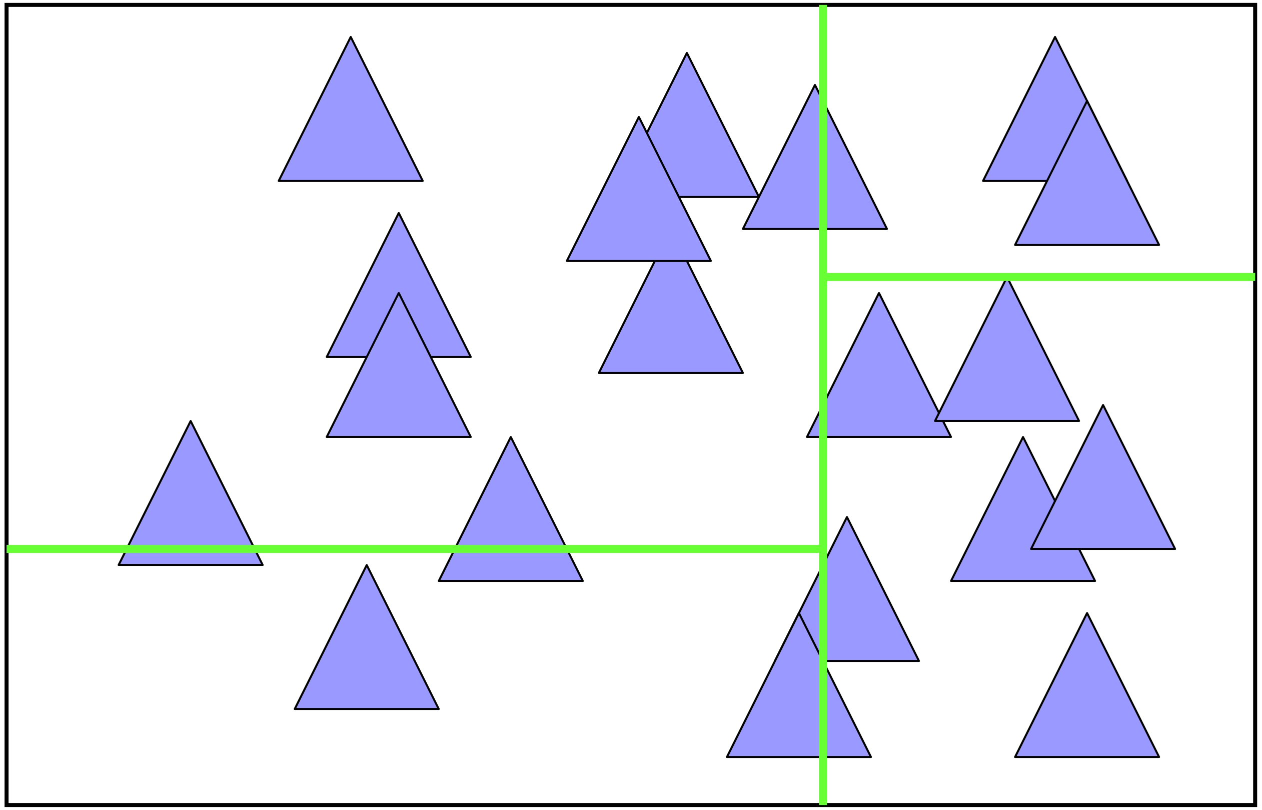

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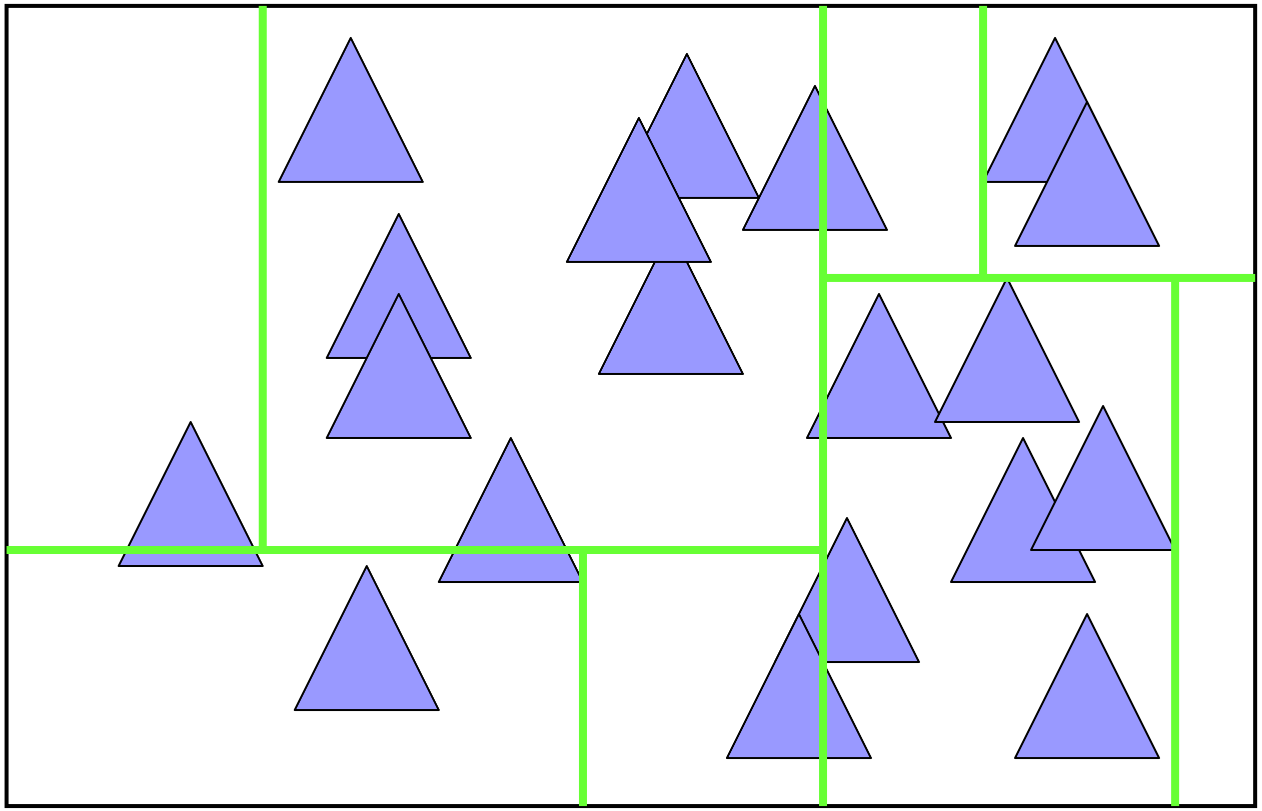


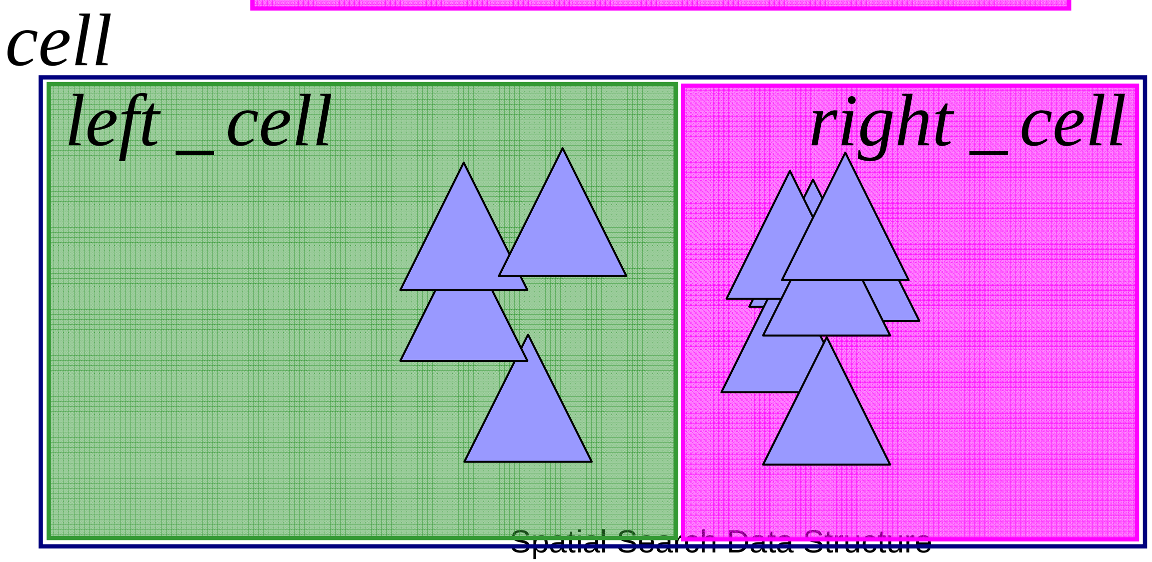
# Costruire un kD-tree

- Dati:
  - axis-aligned bounding box (“cell”)
  - lista di primitive geometriche (triangoli)
- Operazioni base
  - Prendi un piano ortogonale a un asse e dividi la cella in due parti **(in che punto?)**
  - Distribuire le primitive nei due insiemi risultanti
  - Ricorsione
  - Criterio di terminazione **(che criterio?)**
- Esempio: se viene usato per il ray-tracing, si vuole ottimizzare per il costo dell’intersezione raggio primitiva

# Costruire un kD-tree efficiente per RayCast

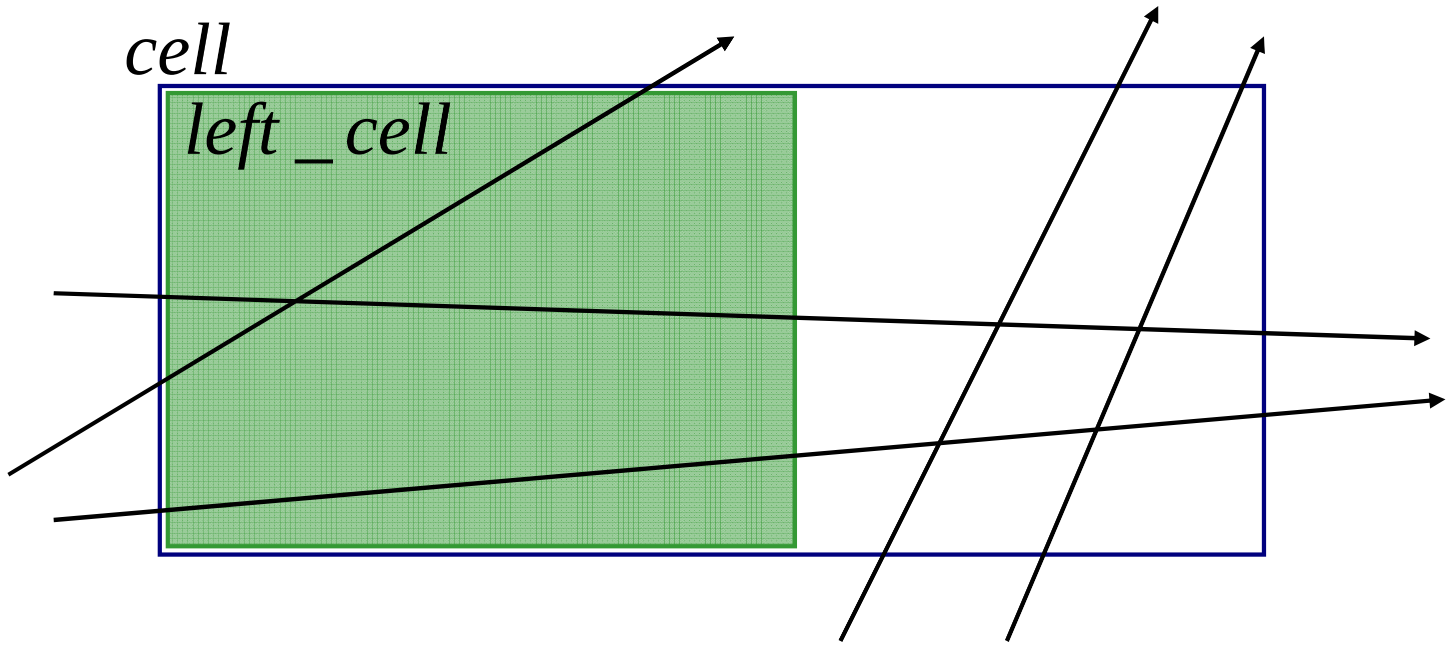
- In che punto dividere la cella?
  - Nel punto che minimizza il **costo**
- Quanto è il costo? Riprendiamo la formula per I BSP

$$Cost(cell) = 1 + \boxed{\mathrm{Prob}(left\_cell \mid cell)\, Cost(Left) +} \boxed{\mathrm{Prob}(right\_cell \mid cell)\, Cost(Right)}$$



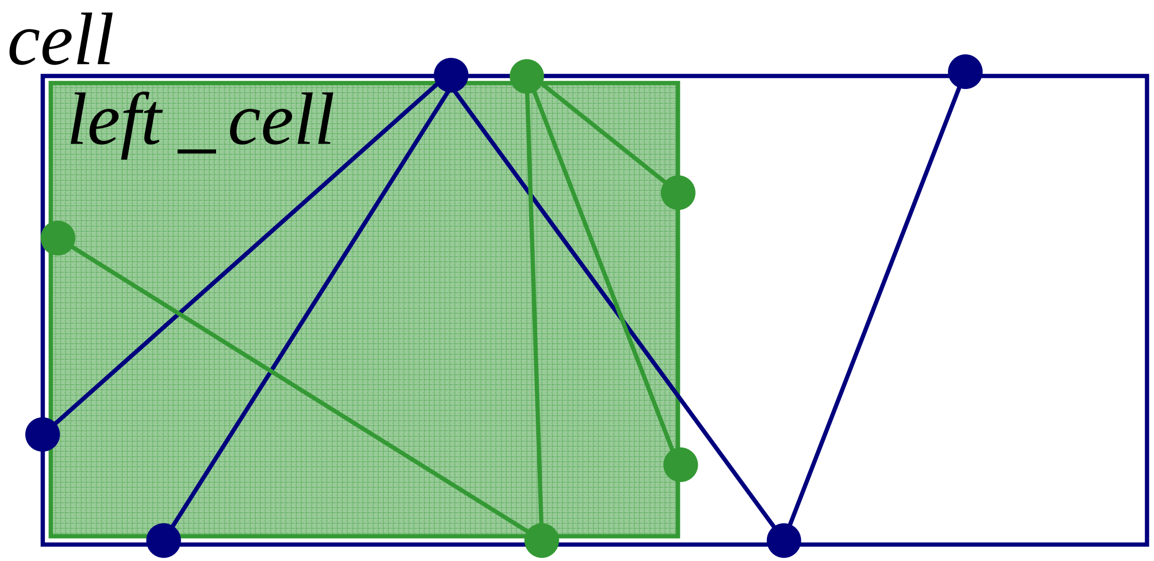
$$\text{Prob}(left\_cell \mid cell)\, Cost(Left)$$

- Sapendo che il raggio interseca la cella *cell*, qual'è la probabilità che intersechi la cella *left_cell* ??



# $\mathrm{Prob}(left\_cell \mid cell)$

$$\mathrm{Prob}[cell \mid left\_cell] = \frac{\#\, raggi\ che\ intersecano\ left\_cell}{\#\, raggi\ che\ intersecano\ \ cell}$$

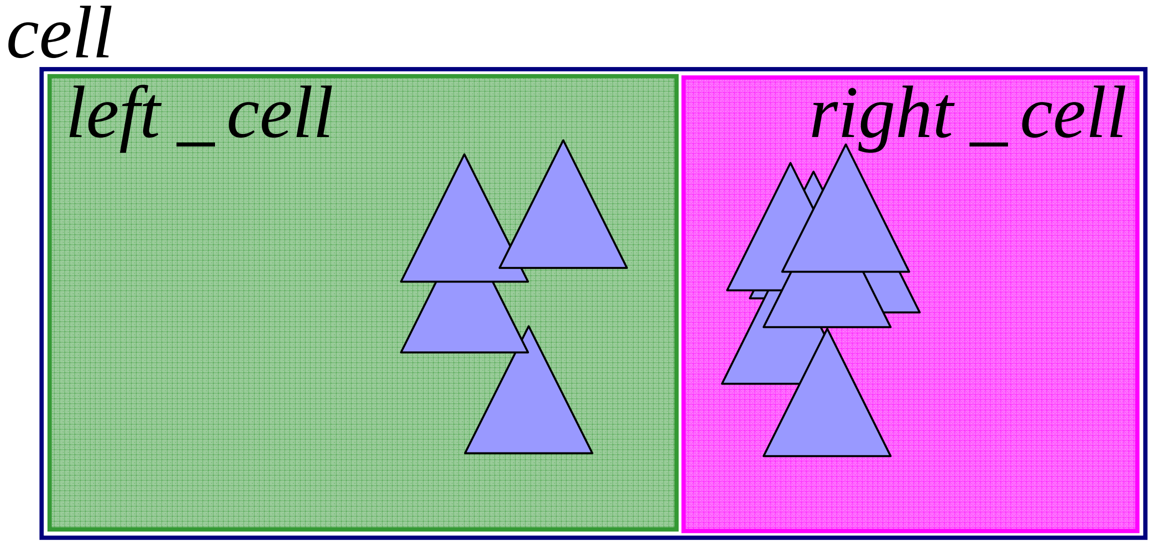



Ogni raggio che interseca una cella corrisponde a una coppia di punti sulla sua superficie. Contiamo le coppie di punti sulla superficie delle celle

$$\mathrm{Prob}[cell \mid left\_cell] = \frac{\int_{\sigma(left\_cell)} \left( \int_{\sigma(left\_cell)} da \right) da}{\int_{\sigma(cell)} \left( \int_{\sigma(cell)} da \right) da} = \frac{Area(left\_cell)^2}{Area(cell)^2} = \frac{Area(left\_cell)}{Area(cell)}$$

$$cost(left\_cell)$$

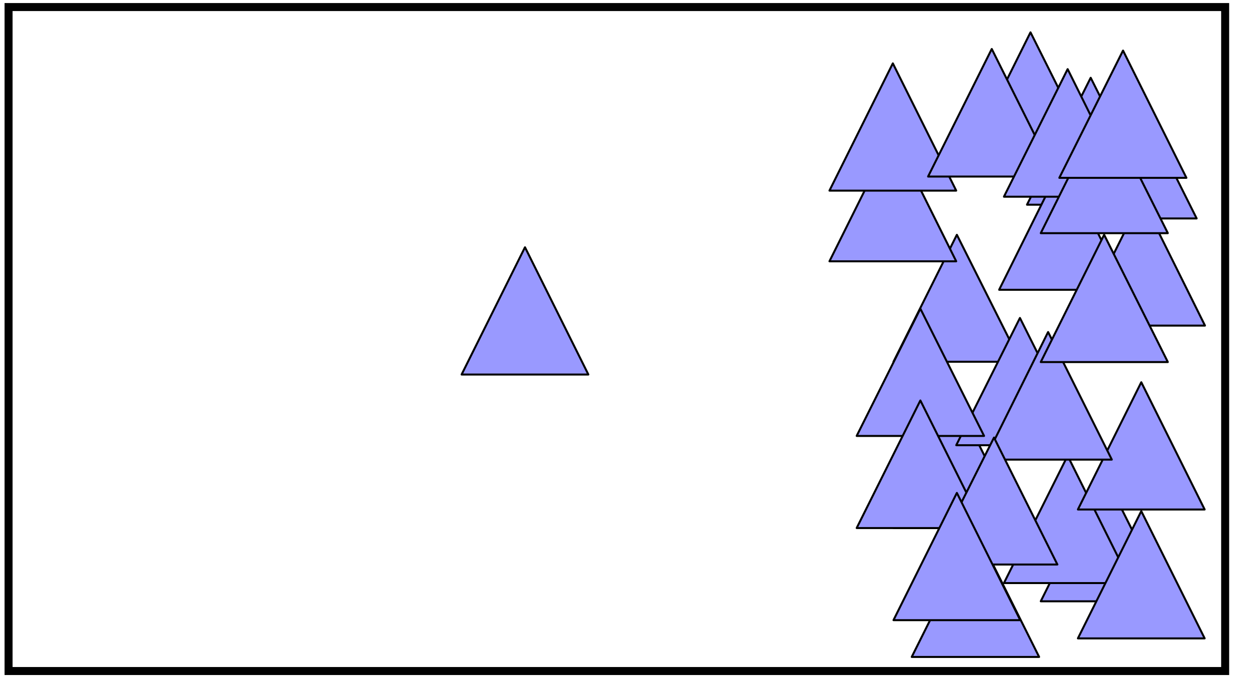
- Sapendo che il raggio interseca la cella *left_cell*, qual'è il costo di testare l'intersezione con i triangoli?
- Si approssima con il numero di triangoli che toccano la cella



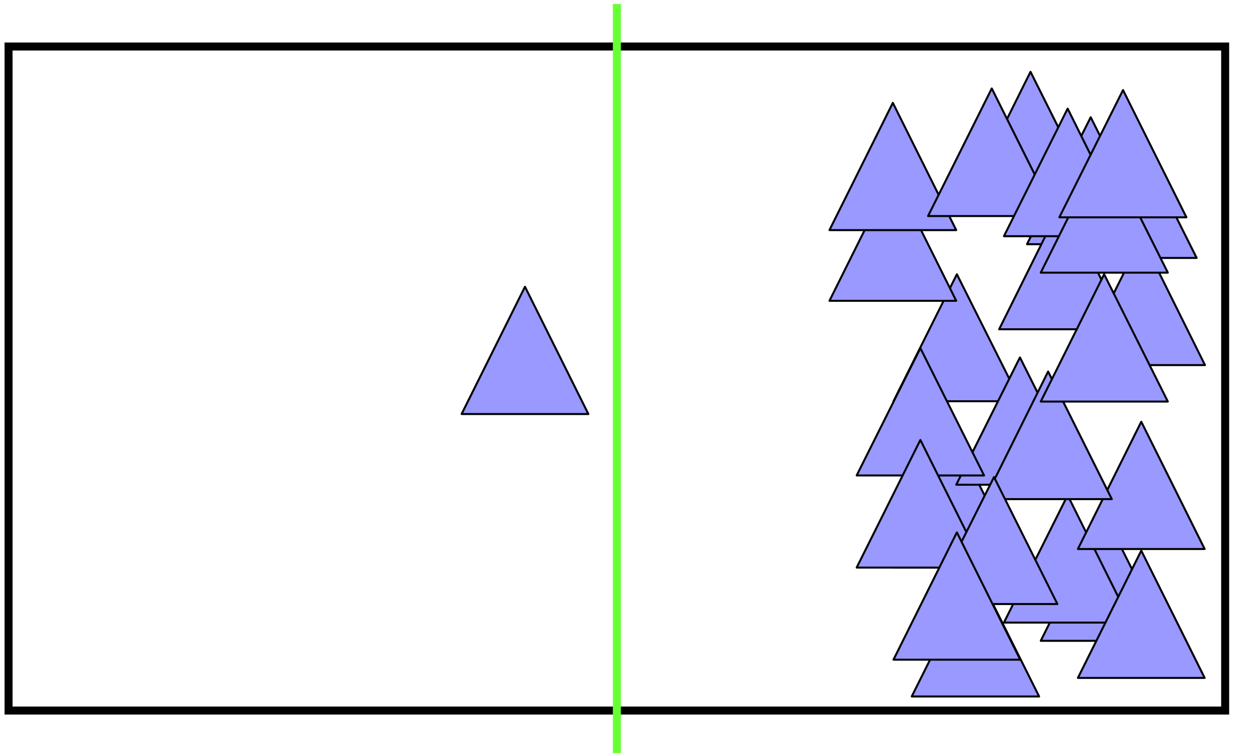

$$Cost(left\_cell) = 4$$

# Esempio

- Come si suddivide la cella qui sotto?

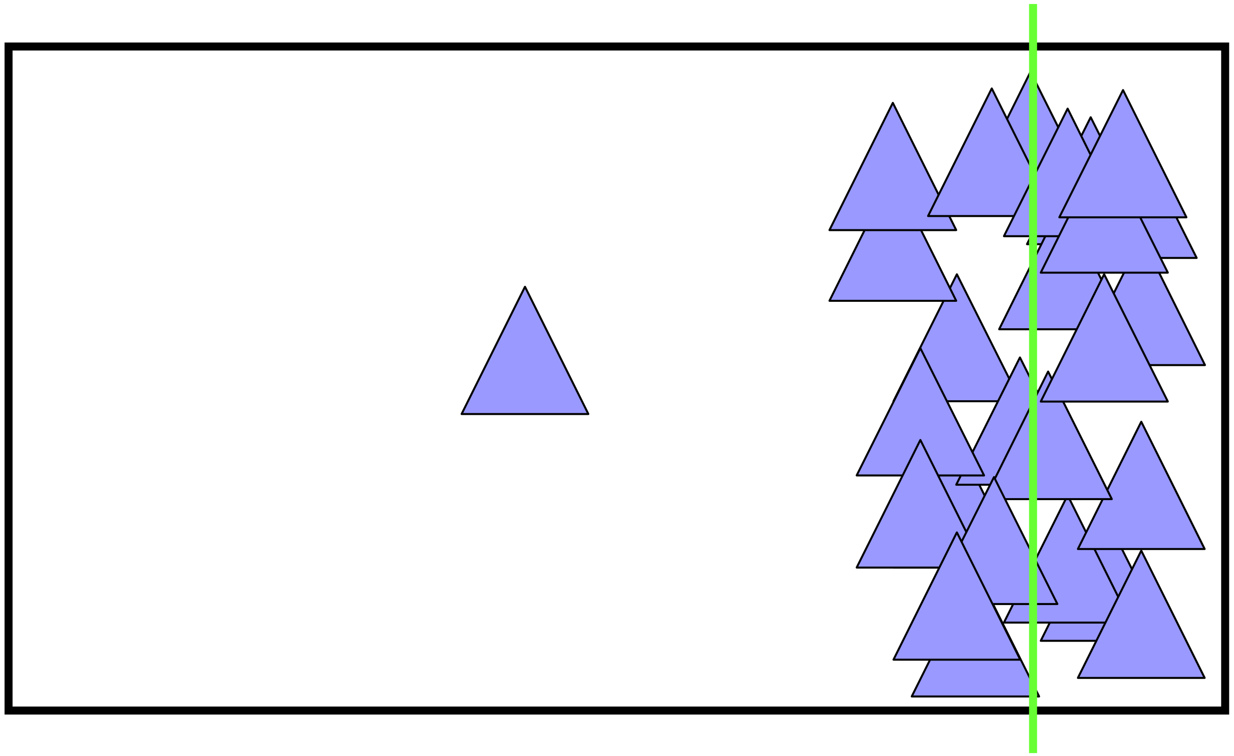


# A metà



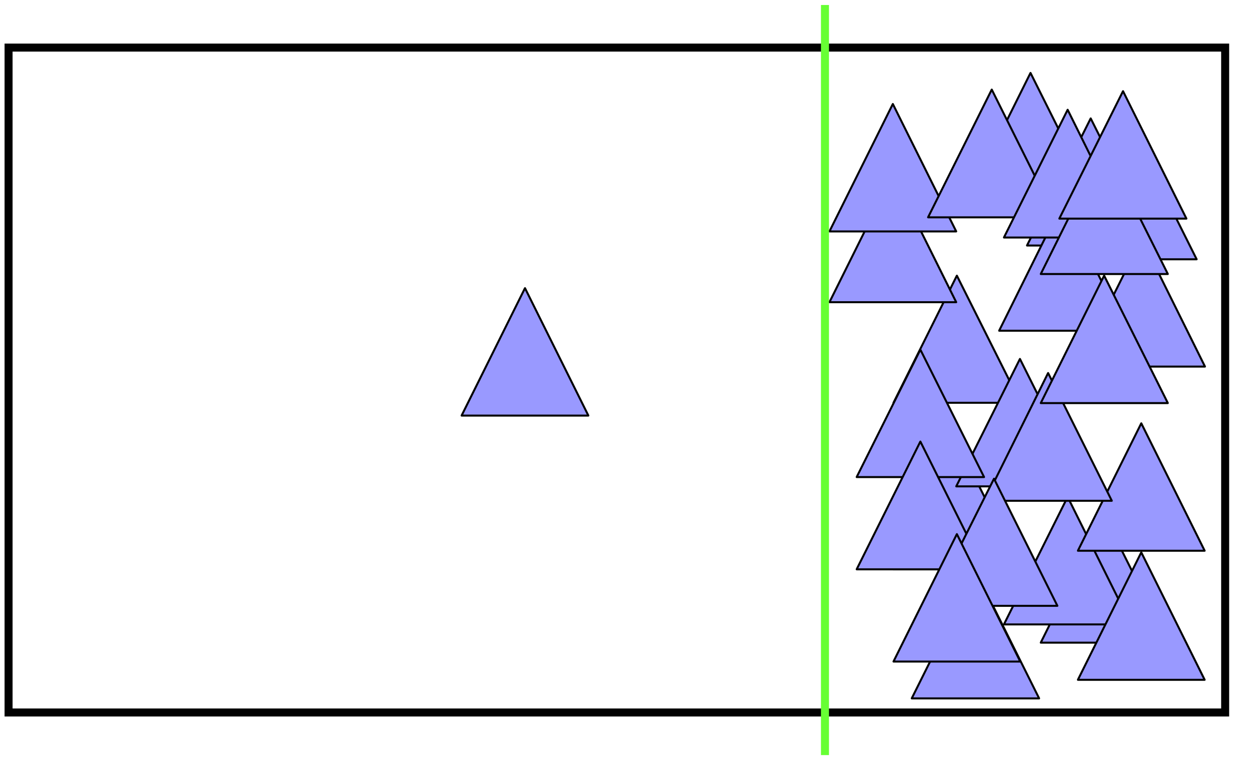
- Non tiene conto delle probabilità
- Non tiene conto dei costi

# Nel punto mediano



- Rende uguali i costi di *left_cell* e *right_cell*
- Non tiene conto delle probabilità

# Ottimizzando il costo



- Separa bene spazio vuoto
- Distribuisce bene la complessità

# Range Query with kd-tree

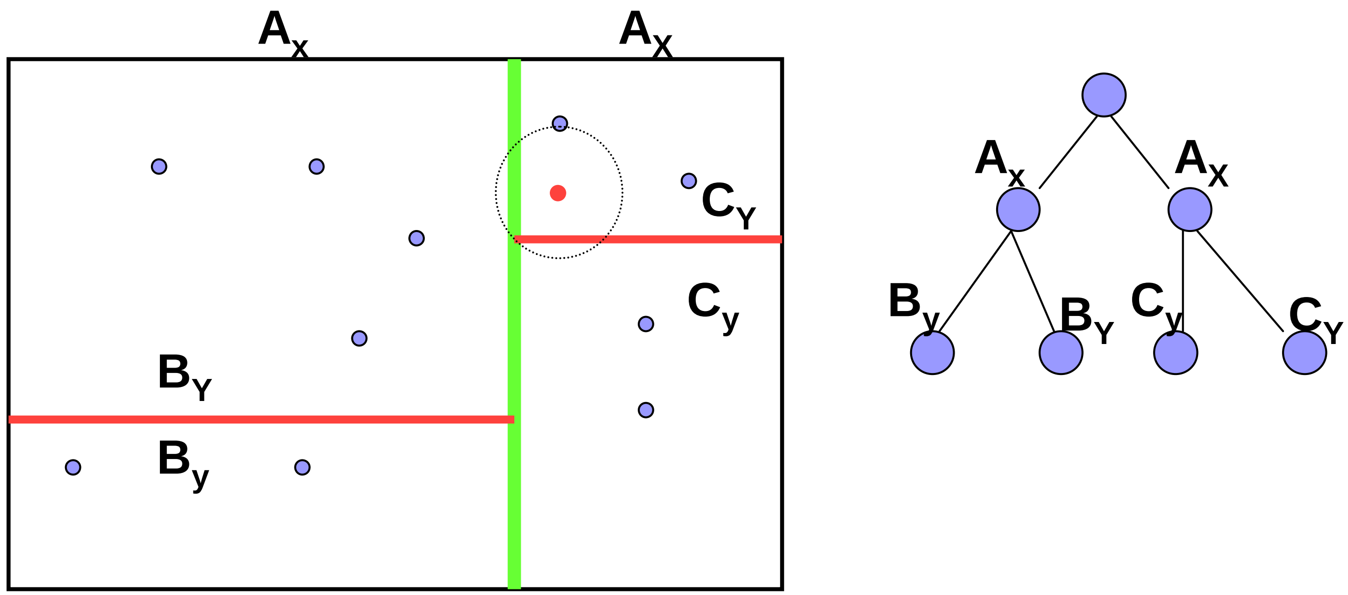
- **Query**: return the primitives inside a given box
- Algorithm:
  - Compute intersection between the node and the box
  - If the node is entirely inside the box add all the primitives contained in the node to the result
  - If the node is entirely outside the box return
  - If the nodes is **partially** inside the box recur to the children
- **Cost:** if the leaf nodes contain one primitive and the tree is balanced:

$$O(n^{1-\frac{1}{d}}+k)$$

  *n* number of primitives, *d* dimension
- $O(n^{2d})$ possible results

# Nearest Neighbor with kd-tree

- **Query**: return the nearest primitive to a given point *c*
- **Algorithm**:
  - Find the nearest neighbor in the leaf containing c
  - If the sphere intersect the region boundary, check the primitives contained in intersected cells

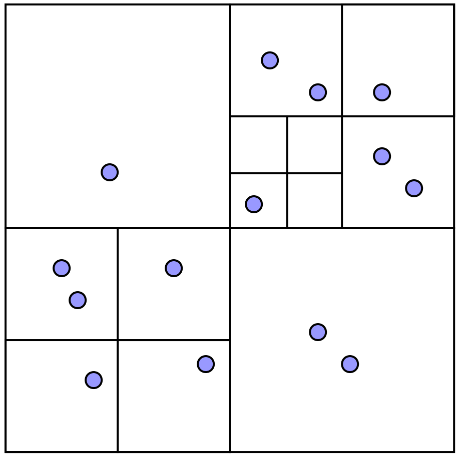



Spatial Search Data Structure

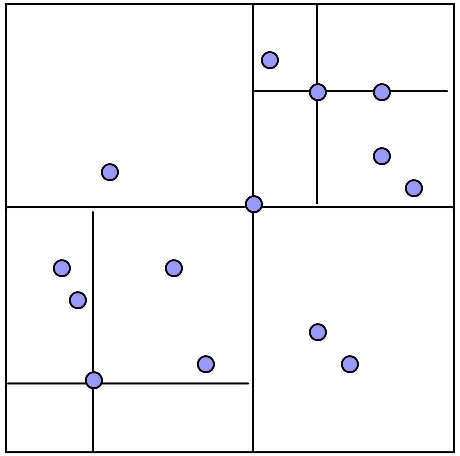
# Quad-Tree (2d)

- The plane is recursively subdivided in 4 subregions by couple of orthogonal planes

**Region Quad-tree**

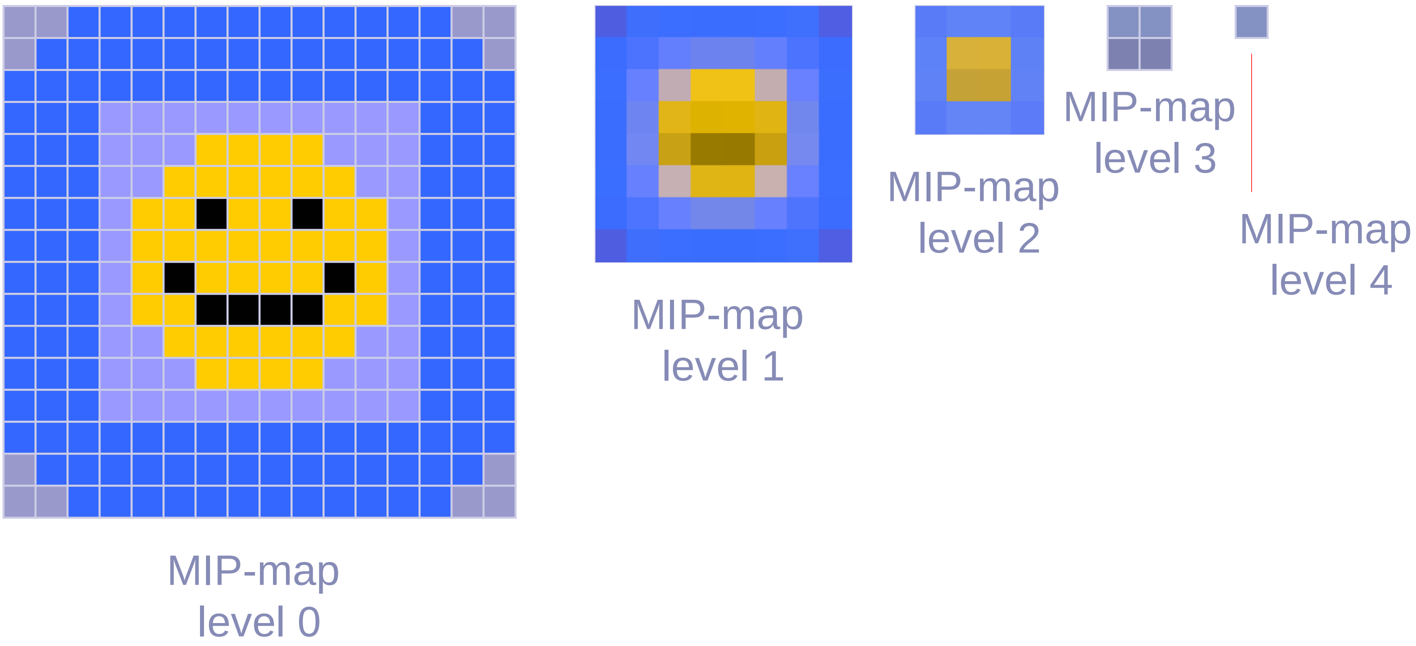


**Point Quad-tree**



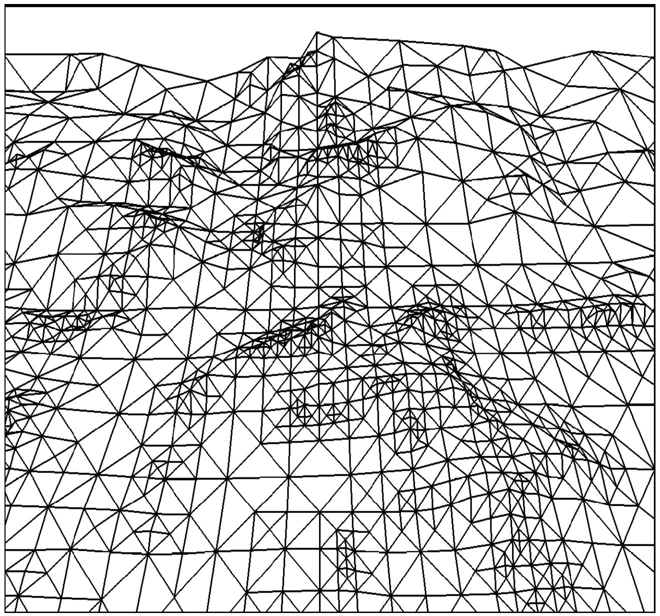
# Quad-Tree (2d): examples

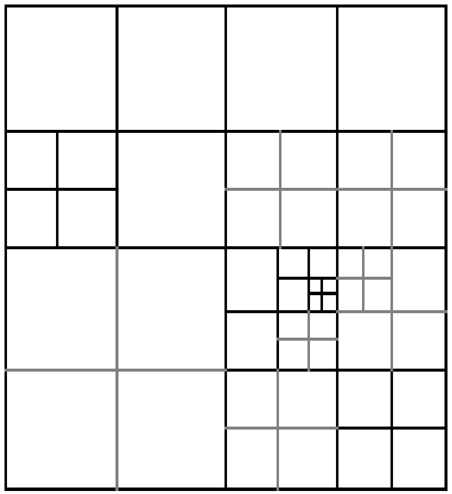
- Widely used:
  - Keeping level of detail of images



# Quad-Tree (2d): examples

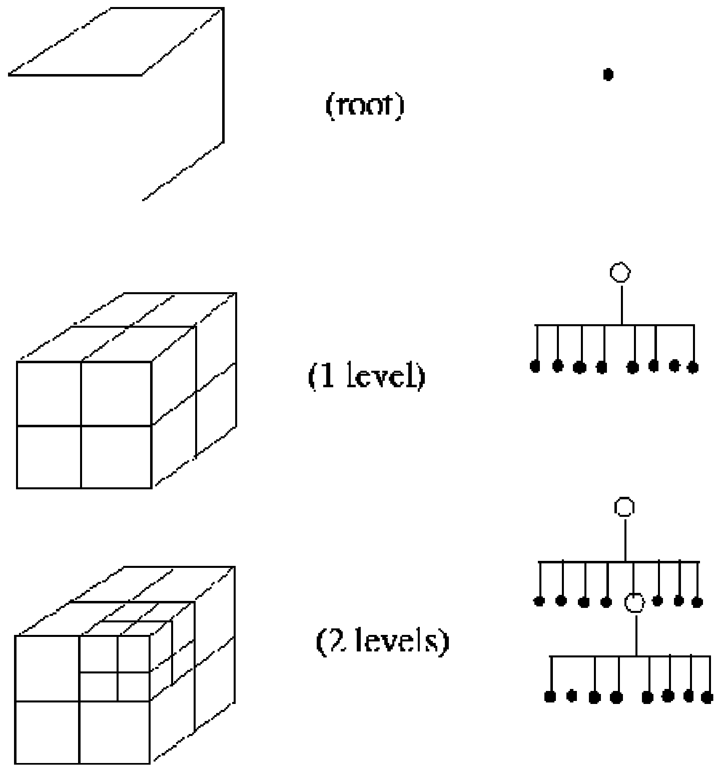
- Widely used:
  - Terrain rendering: each cross in the quatree is associated with a height value





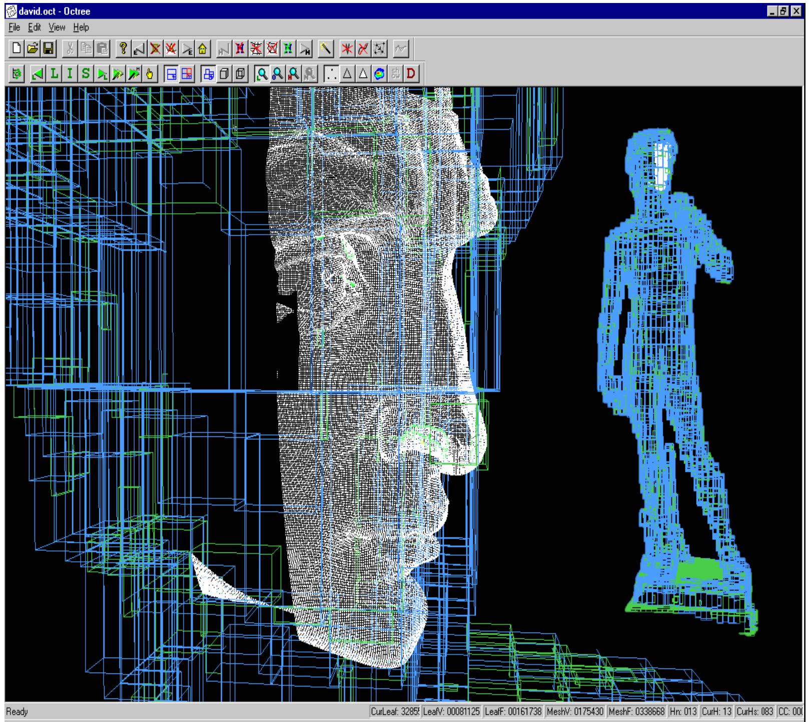
# Oct-Tree (3d)

- The same as quad-tree but in 3 dimensions



# Oct-Tree (3d) : Examples

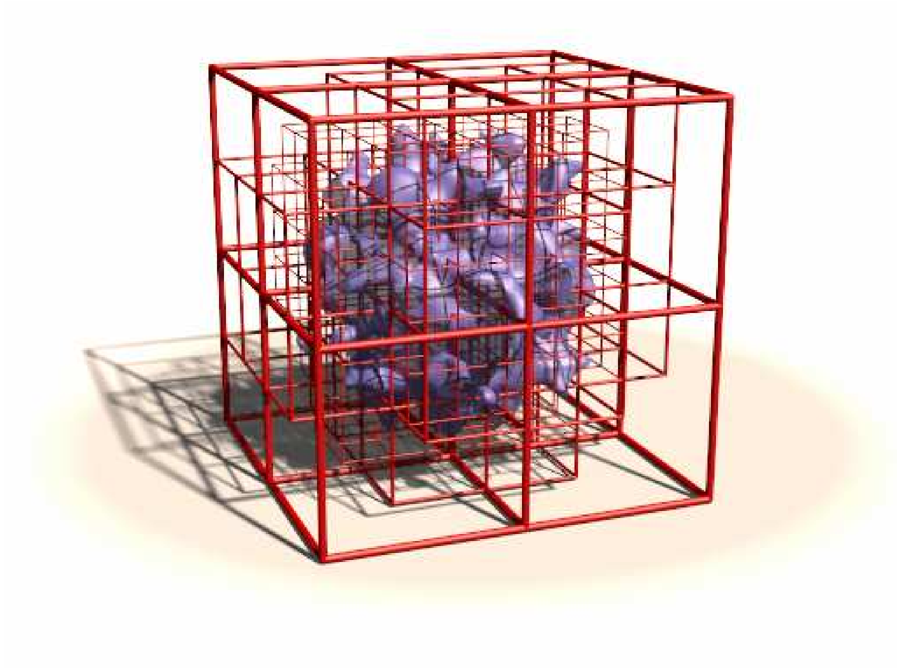
- Processing of Huge Meshes (ex: simplification)
- Problem: mesh do not fit in main memory
- Arrange the triangles in a oct-tree




Spatial Search Data Structure

# Oct-Tree (3d) : Examples

- Extraction of isosurfaces on large dataset
  - Build an octree on the 3D dataset
  - Each node store min and max value of the scalar field
  - When computing the isosurface for alpha, nodes whose interval doesn’t contain alpha are discarded



# Advantages of quad/oct tree

- Position and size of the cells are implicit
  - They can be explored without pointers (convenient if the hierarchies are complete) by using a linear array where:

**quadtree**

$$Children(i) = 4i+1,\ldots,4*(i+1)$$
$$Parent(i) = \lfloor i/4 \rfloor$$

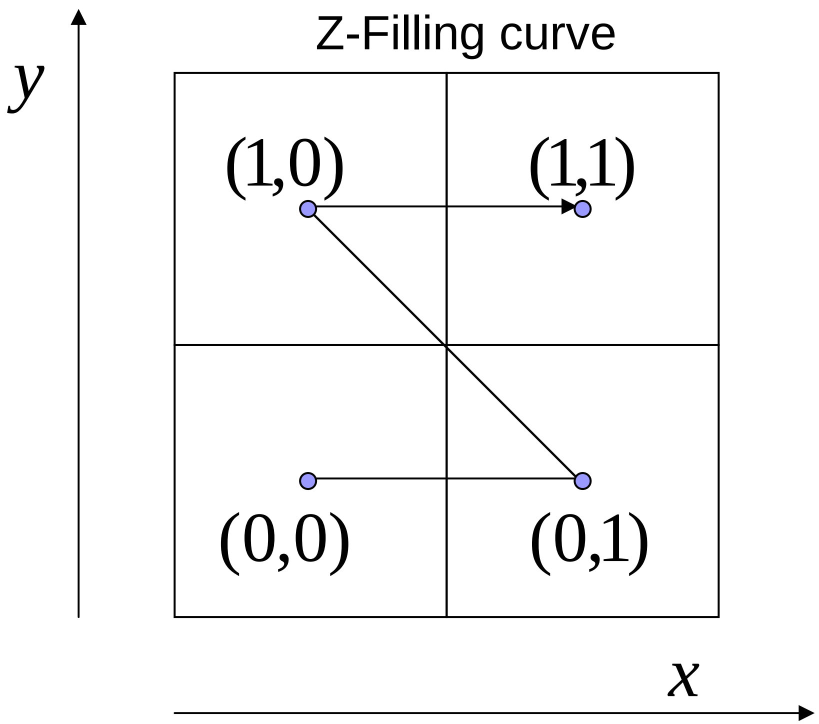
**octree**

$$Children(i) = 8i+1,\ldots,8*(i+1)$$
$$Parent(i) = \lfloor i/8 \rfloor$$

# Z-Filling Curves

- Position and size of the cells are implicit
  - They can be indexed to preserve locality, i.e.

$$Spatially\ close \rightarrow close\ in\ memory$$




Easy conversion between position in space and order in the curve

Just use the 0..1 coordinates as bits

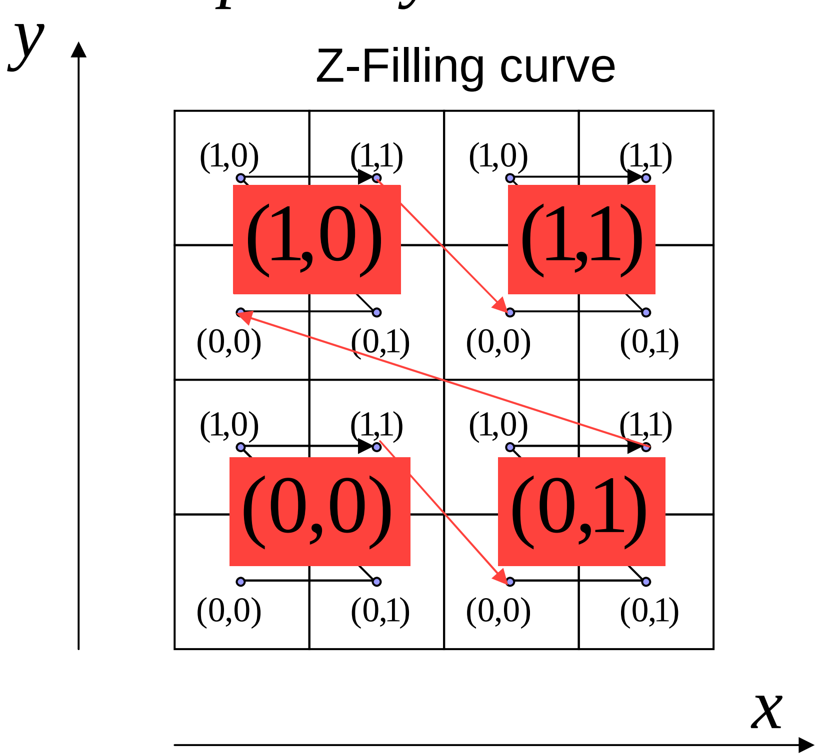
00 01 10 11

# Z-Filling Curves

- Position and size of the cells are implicit
  - They can be indexed to preserve locality, i.e.

$$\textit{Spatially close} \rightarrow \textit{close in memory}$$

Z-Filling curve



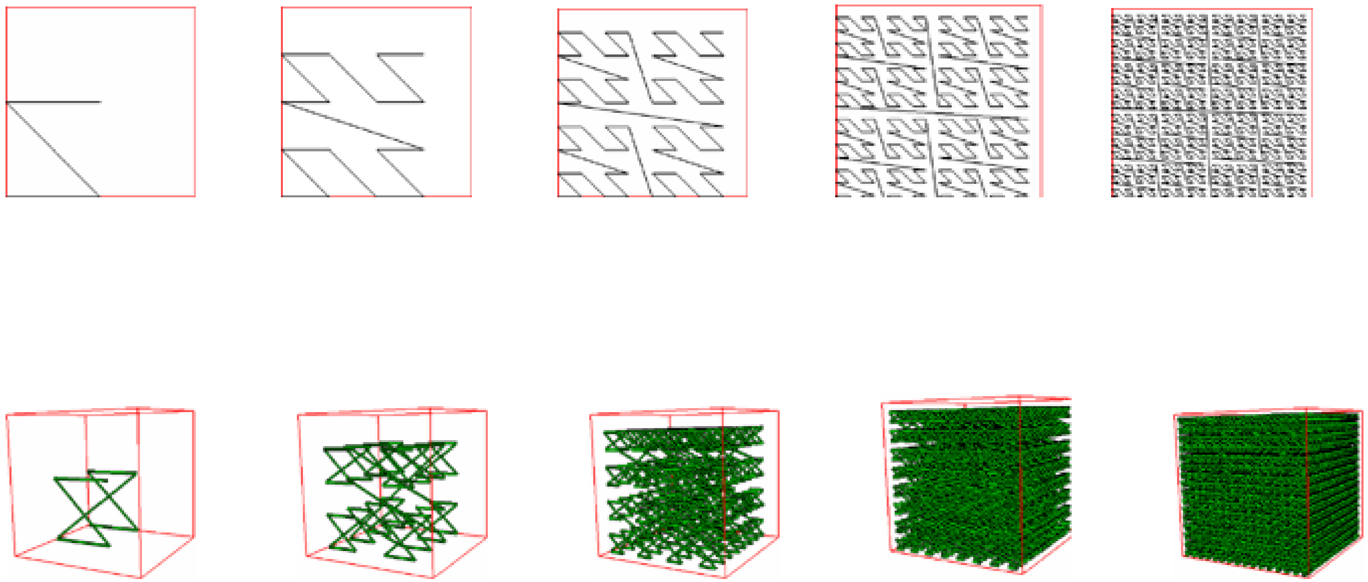

00 00
00 01
00 10
00 11
01 00
01 01
01 10
01 11
10 00
10 01
10 10
10 11
11 00
11 01
11 10
11 11

# Z-Filling Curves

- Conversion from spatial coordinates to index.
  - Write the coord values in binary
  - Interleave the bits

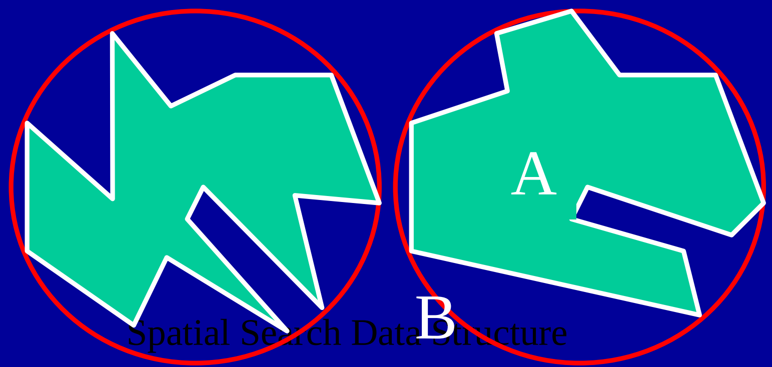
$$
\begin{array}{ccccccccccc}
x & = & & b_0^x & & b_1^x & & b_2^x & \ldots & & b_n^x \\
y & = & b_0^y & & b_1^y & & b_2^y & & \ldots & b_n^y & \\
id & = & b_0^y & b_0^x & b_1^y & b_1^x & b_2^y & b_2^x & \ldots & b_n^y & b_n^x
\end{array}
$$

# Hierarchical Z-Filling Curves



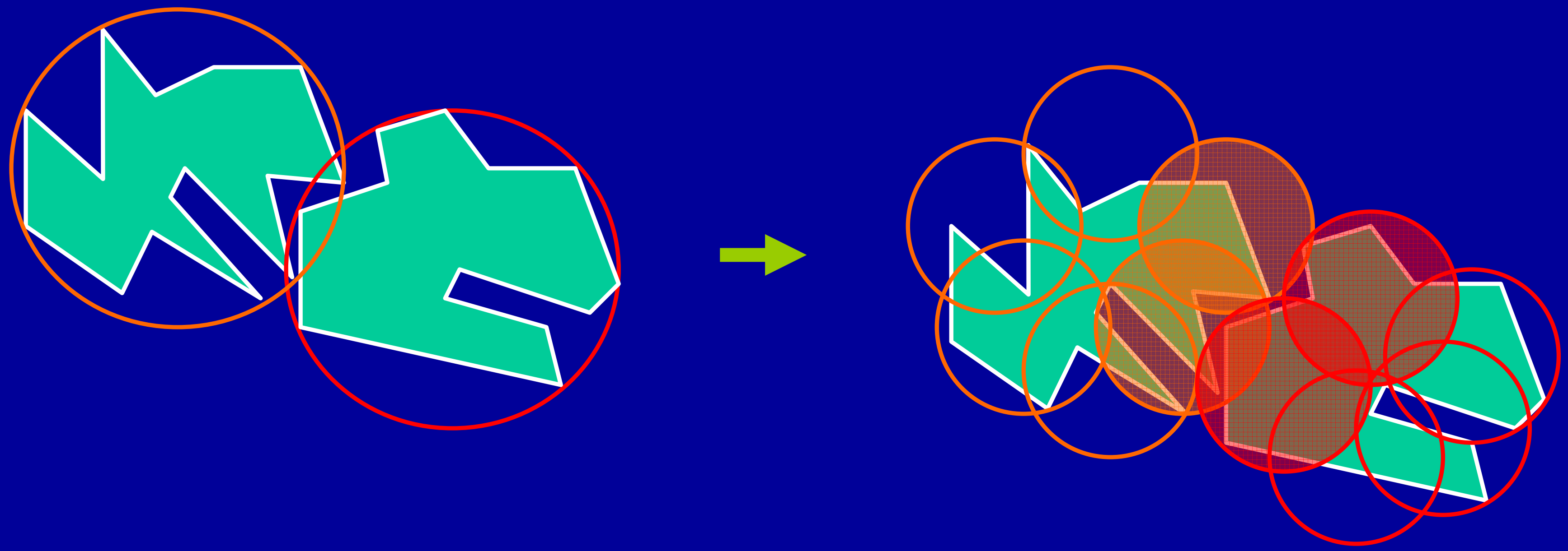
# Bounding Volumes Hierarchies

- If a volume B includes a volume A, it is called *bounding volume* for A
- No object can intersect A without intersecting B
- If two bounding volumes do not overlap, the same hold for the volumes included



# The Principle

- What if they do overlap?
- Refine.



Spatial Search Data Structure

# Questions!

- What kind of Bounding Volumes?
- What kind of hierarchy?
- How to build the hierarchy?
- How to update (if needed) the hierarchy?
- How to transverse the hierarchy?

All the literature on CD for non-convex objects is about answering these questions.

# Cost

$$T_c = N_v*C_v + N_n*C_n + N_s*C_s$$

v : visited nodes

n :  couple of bounding volumes tested for    overlap

s : couple of polygons tested for overlap

N: number of

C: Cost

# BHV - Desirable Properties (2)

- The hierarchy should be able to be constructed in an automatic predictable manner
- The hierarchical representation should be able to approximate the original model to a high degree or accuracy
  - allow quick localisation of areas of contact
  - reduce the appearance of object repulsion

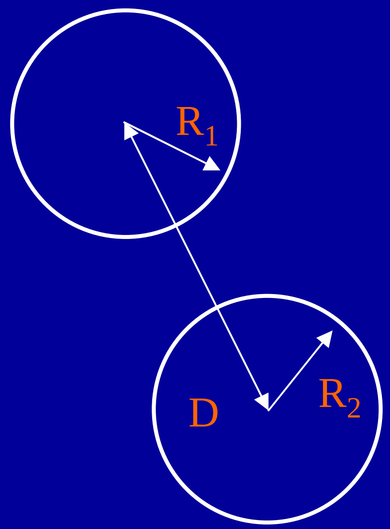
# BHV - Desirable Properties

- The hierarchy approximates the bounding volume of the object, each level representing a tighter fit than its parent
- For any node in the hierarchy, its children should collectively cover the area of the object contained within the parent node
- The nodes of the hierarchy should fit the original model as tightly as possible

# Sphere-Tree

[O'Rourke and Badler 1979 , Hubbard 1995a & 1996, Palmer and Grimsdale 1995, Dingliana and O'Sullivan 2000]

- Nodes of BVH are spheres.
- Low update cost $C_u$
  – translate sphere center
- Cheap overlap test $C_v$

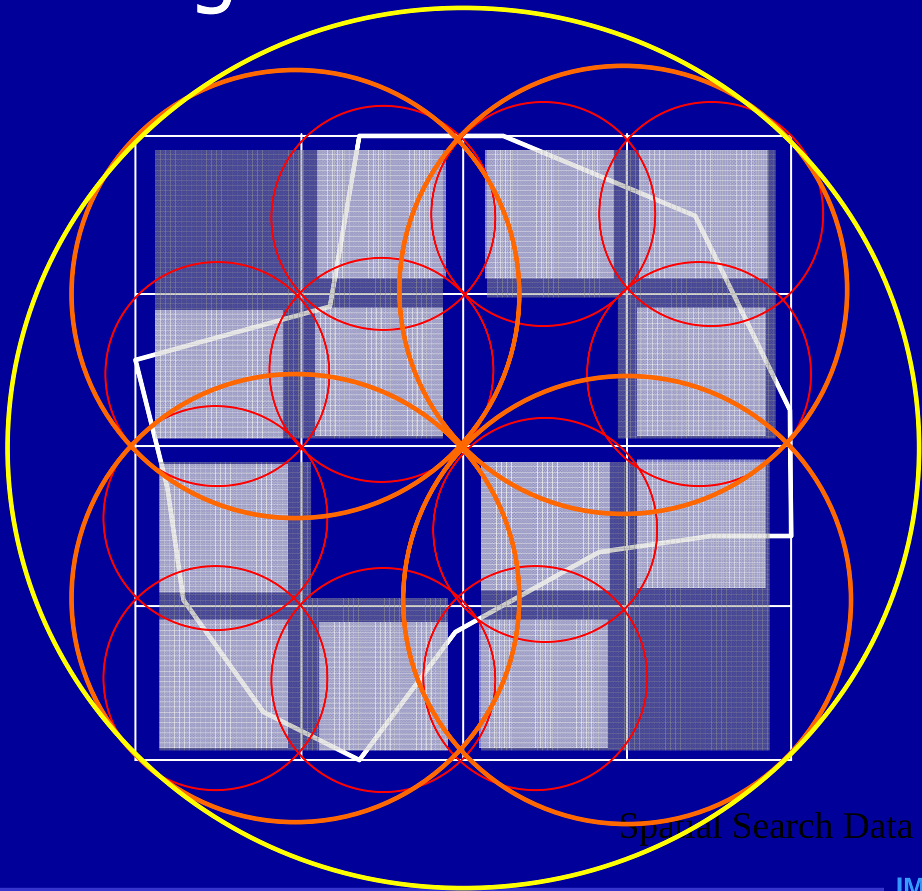


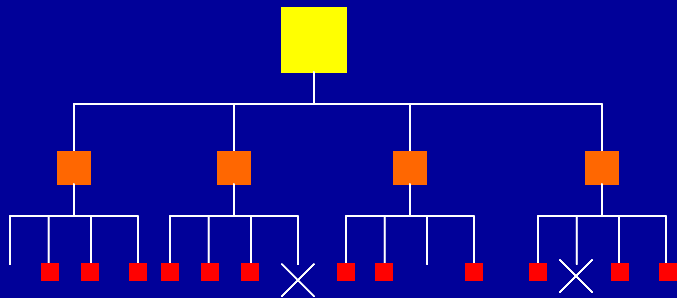

$$D^2 < (R_1 + R_2)^2$$

- Slow convergence to object geometry
  – Relatively high $N_u$ & $N_v$

# Sphere-Tree Construction Dingliana and O’Sullivan 2000

- Spheres placed around the boxes of a regular oct-tree

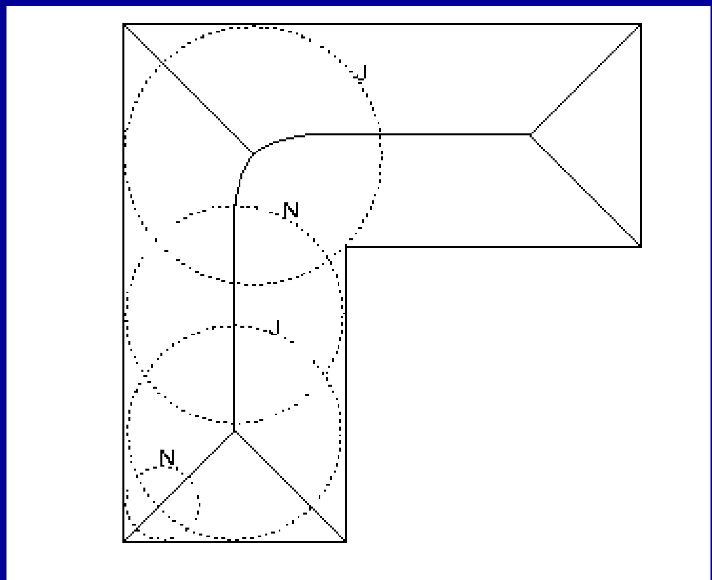


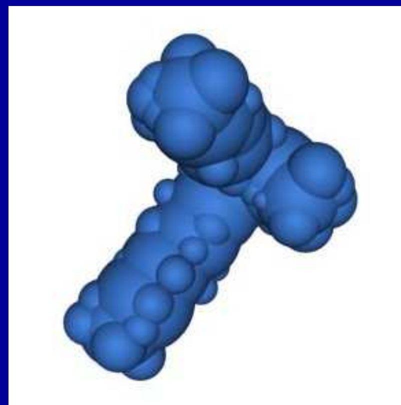


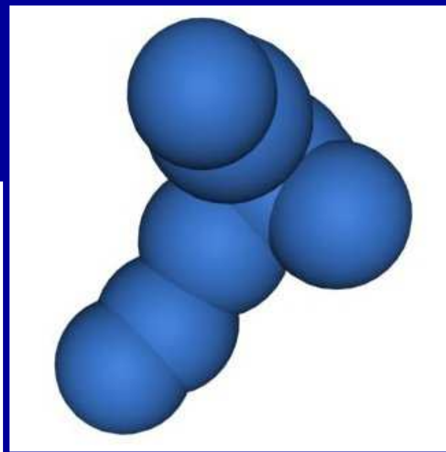
Spatial Search Data Structure

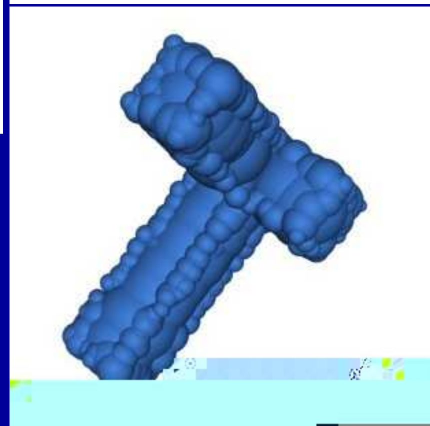
# Sphere-Tree Construction Hubbard 1995a & 1996,

- Spheres placed along the Medial-Axis (transform)







# Axis-Aligned Bounding Box

[van den Bergen 1997]

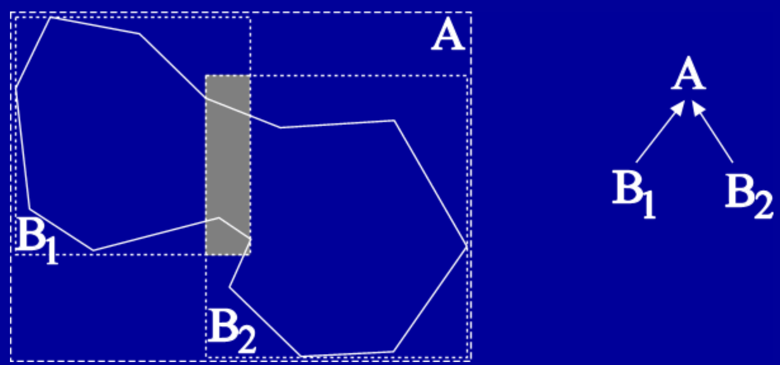
- The bounding volumes are axis aligned boxes (in the *object* coordinate system)
- The hierarchy is a binary tree (built top down)
- Split of the boxes along the longest edge at the median (equal number of polygons in both children)

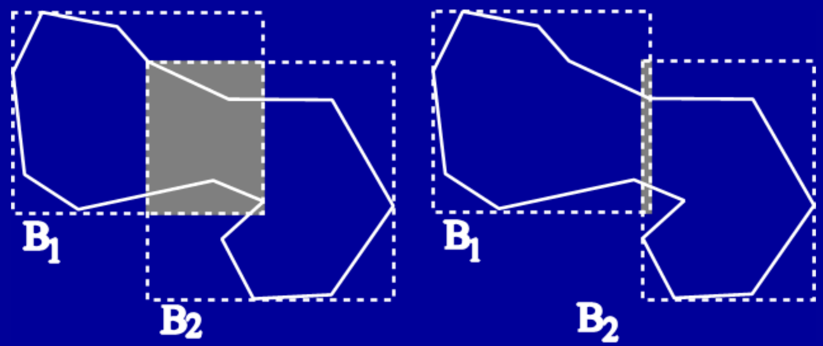
# Axis-Aligned Bounding Box

- The hierarchy of boxes can be quickly updated :
- let $Sm(R)$ be the smallest AABB of a region R and $r_1, r_2$ two regions.

$$Sm(Sm(r_1) \cup Sm(r_2)) = Sm(r_1 \cup r_2)$$

- The hierarchy is updated in *O(n)* time
- Note: this is not the same as rebuilding the hierarchy

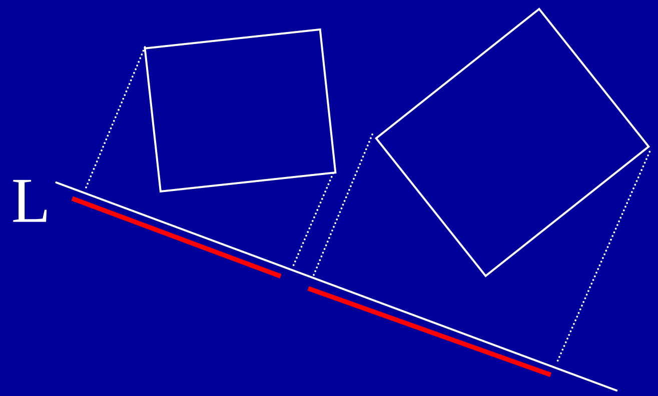



*refitting* *rebuilding*

# AABB - Overlap

If two **convex polyhedra** do not overlap, then there exists a direction L such that their projections on L do not overlap. L is called Separating Axis

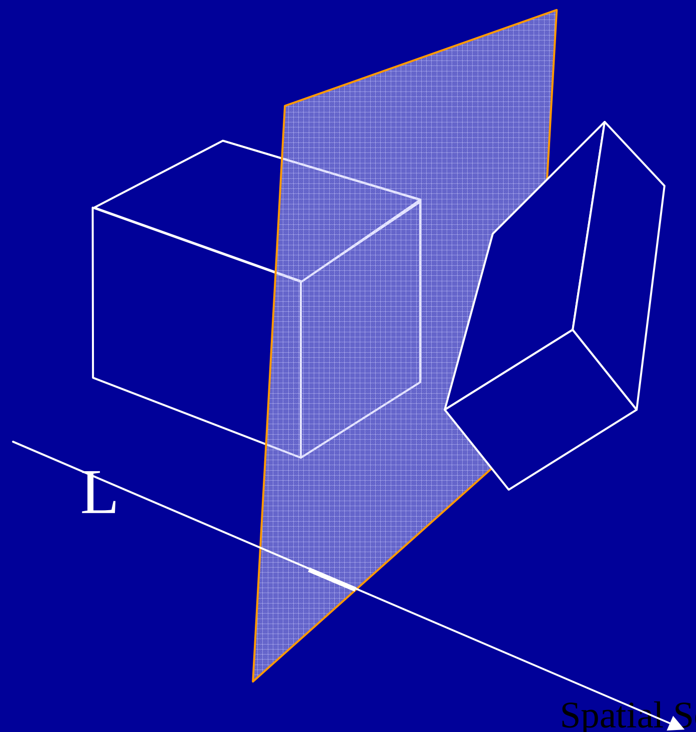



Separating Axis Theorem: L can only be one of the following:

- Normal to a face of one of the polyedra
- Normal to 2 edges, one for each polyedron

# AABB - Overlap

Ex: There are 15 possible axes for two boxes: 3 faces from each box, and 3x3 edge direction combinations



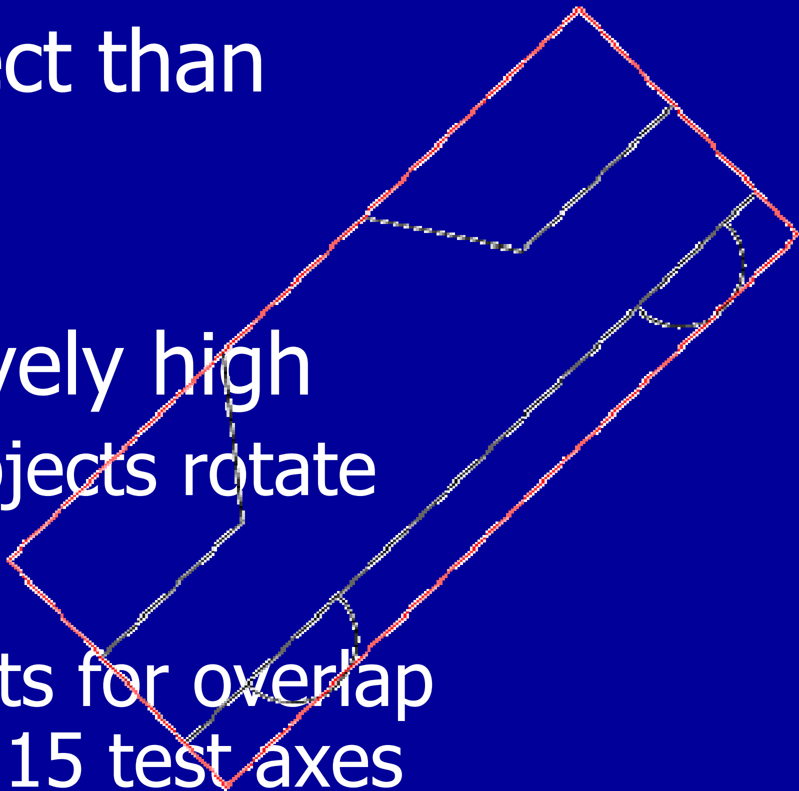

Note: SA is a normal to a face 75% of the times

Trick: Ignore the tests on the edges!

# Object Oriented Bounding Box

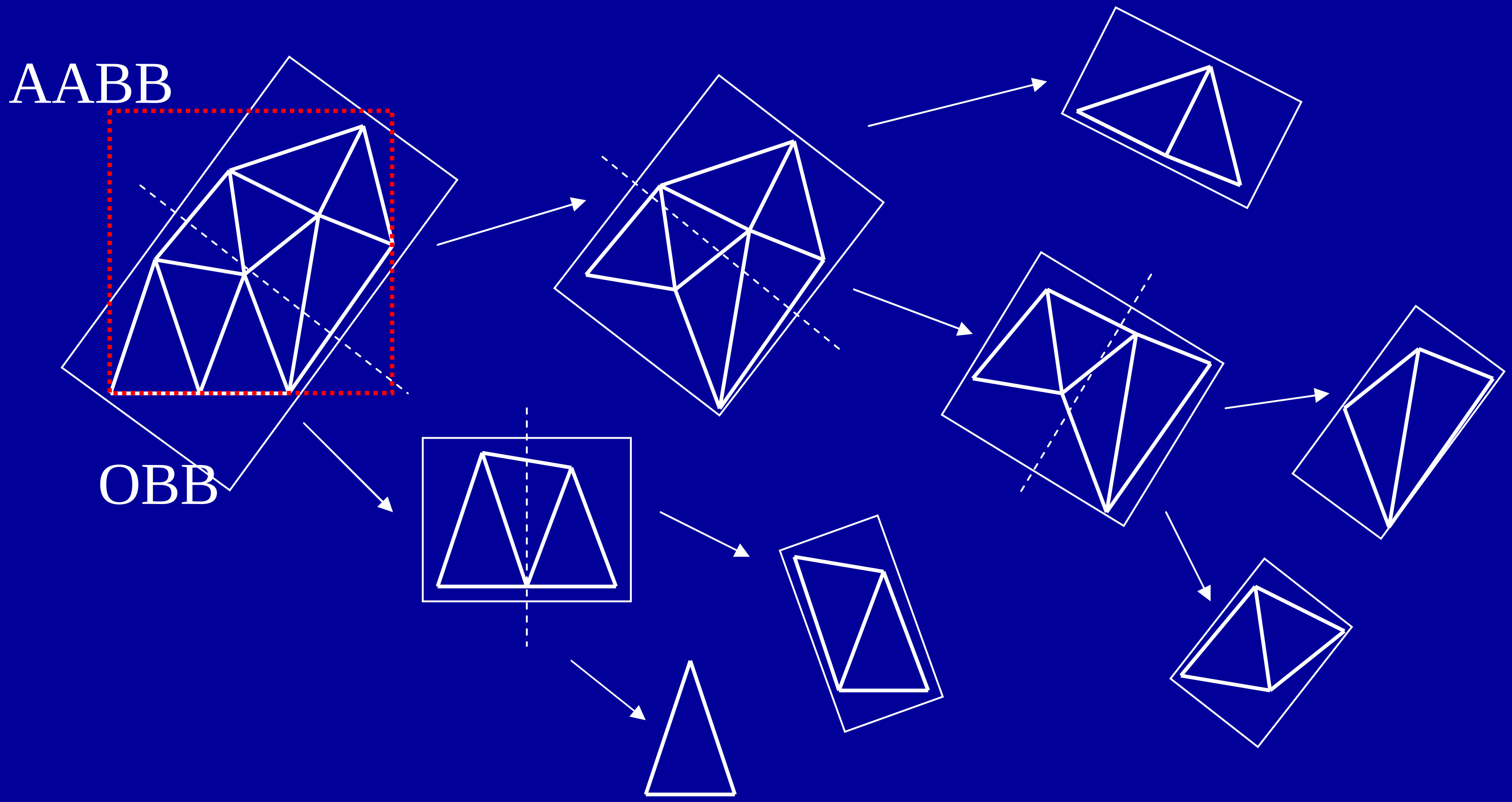
[Gottschalk et al. 1996]

- Better coverage of object than AABB
  - Quadratic convergence
- Update cost $C_u$ is relatively high
  - reorient the boxes as objects rotate
- Overlap cost $C_v$ is high
  - Separating Axis Test tests for overlap of box's projection onto 15 test axes



# Oriented Bounding Box

[Gottschalk et al. 1996]




Spatial Search Data Structure

# Building an OBB

- The OBB fitting problem requires finding the orientation of the box that best fits the data
- Principal Components Analysis:
  - Point sample the convex hull of the geometry to be bound
  - Find the mean and covariance matrix of the samples
  - The mean will be the center of the box
  - The eigenvectors of the covariance matrix are the principal directions – they are used for the axes of the box
  - The principle directions tend to align along the longest axis, then the next longest that is orthogonal, and then the other orthogonal axis

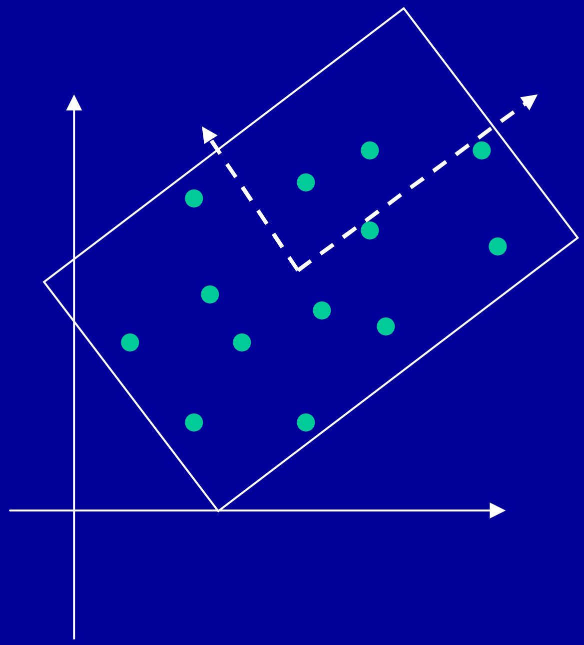
# Principal Component Analysis

$$c = \frac{1}{3n}\sum_{h=1}^{n} p^h$$

$$Cov_{ij} = \frac{1}{3n}\sum_{h=1}^{n}(p_i^h - c_i)(p_j^h - c_j)$$

$$Cov \quad is \quad symmetric \Rightarrow eigen\ vectors$$

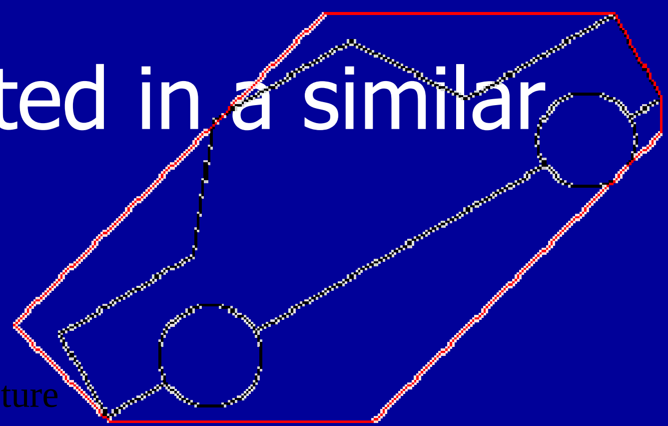
$$form\ an\ orthogonal\ basis$$



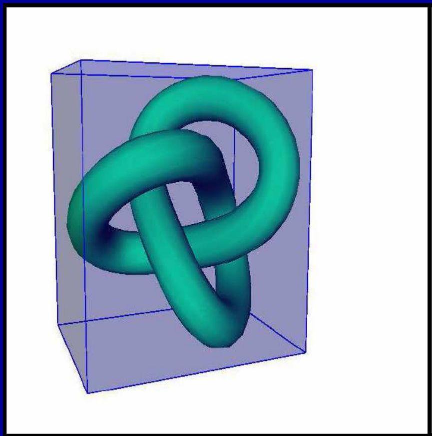
# Discrete Oriented Polytope

[Klosowski et al. 1997]

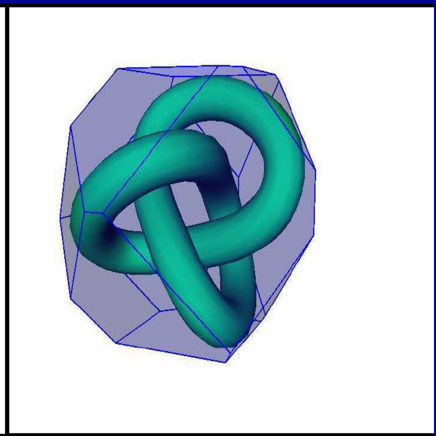
- Convex polytope whose faces are oriented normal to *k* directions: $n \in \{0, \pm 1\}^d$
- Overlap test similar to OBB
  - *k/2* pairs of co-linear vectors
  - k/2 overlap tests
- *k*-DOP needs to be updated in a similar way as the AABB
- AABB is a 6-DOP



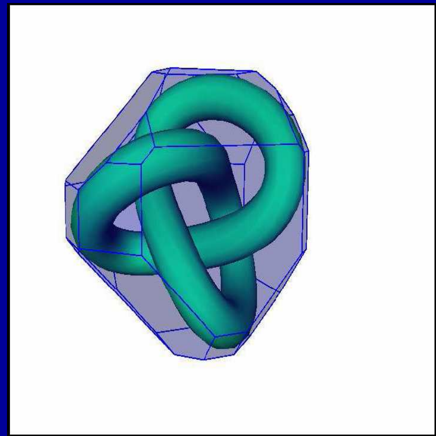
# K-Dops examples



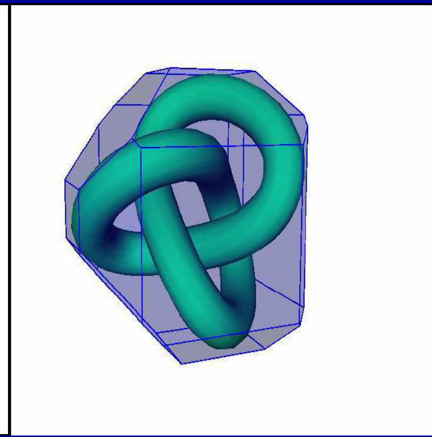
6-dop



14-dop



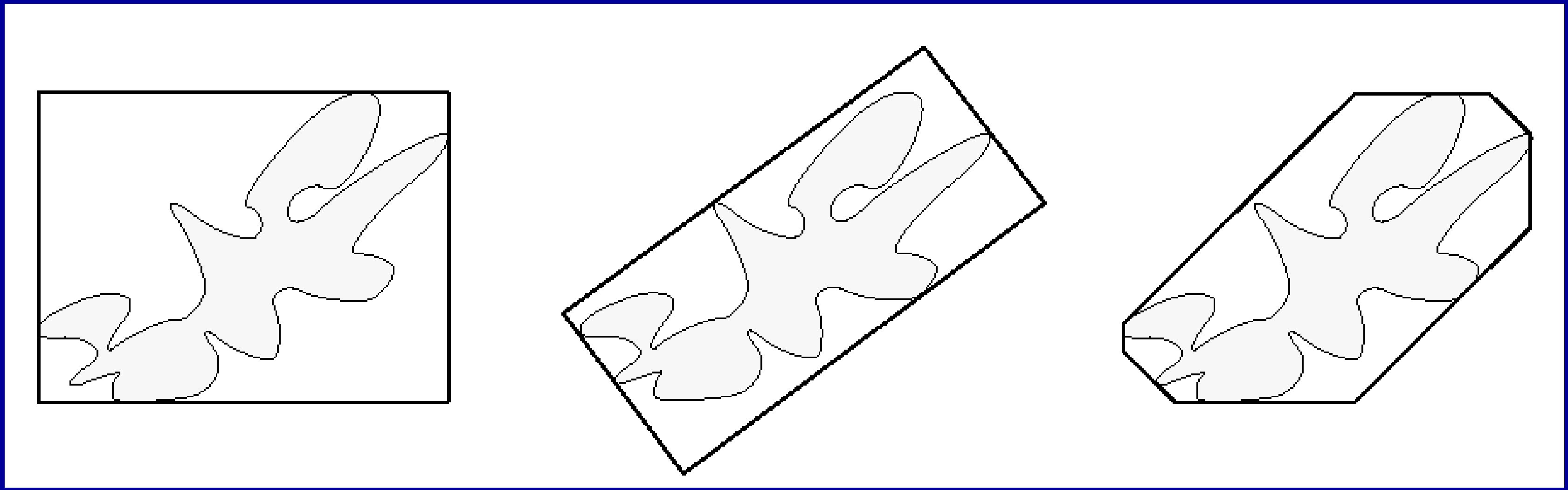
18-dop



26-dop

# Discrete Oriented Polytope

[Klosowski et al. 1997]



AABB OBB 6-DOP