Geodesic

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3D GEOMETRIC MODELING & PROCESSING

What is a Geodesic?

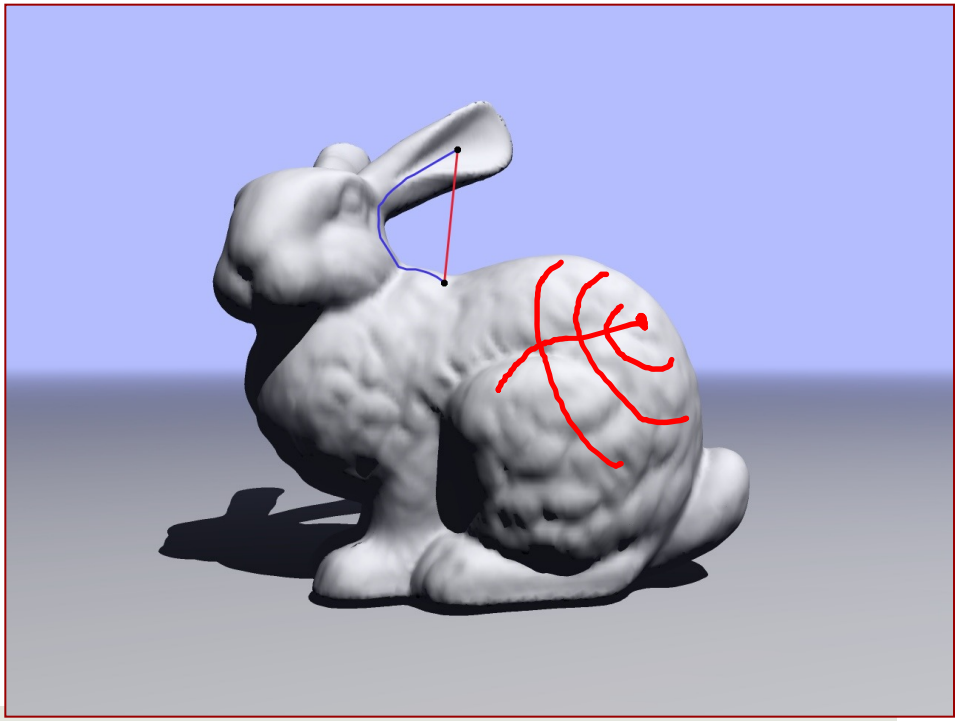
- In the original sense, a geodesic is the shortest route between two points on the Earth's surface.
- Generalized to include measurements in much more general mathematical spaces.
- A curve describing the locally shortest path between two points of a particular space.

Walking on the surface

- We are not interest in the "extrinsic" measurement, but in defining an "intrinsic" metric.
- A metric is a (distance) function:

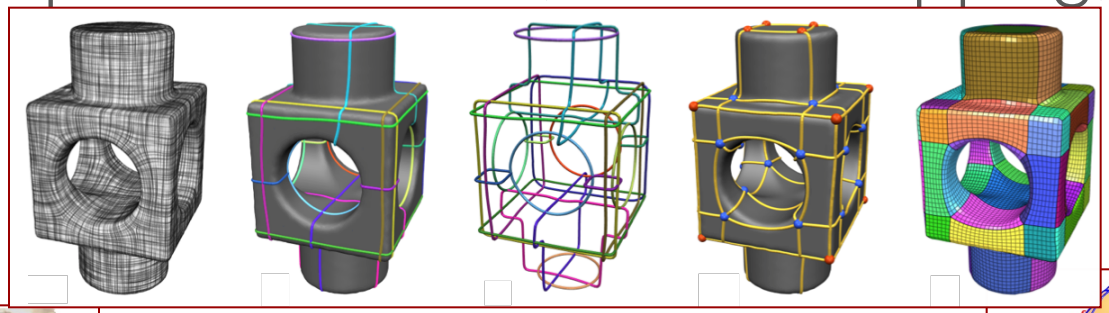
  $d: M \times M \rightarrow R$

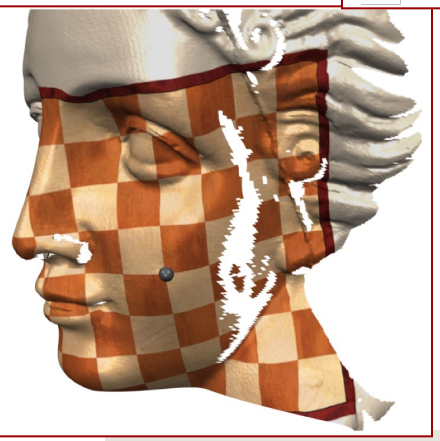
1. $d(x,y) \geq 0$
2. $d(x,y) = 0 \quad iff \quad x=y$
3. $d(x,y) = d(y,x)$
4. $d(x,y) \leq d(x,z) + d(z,y)$

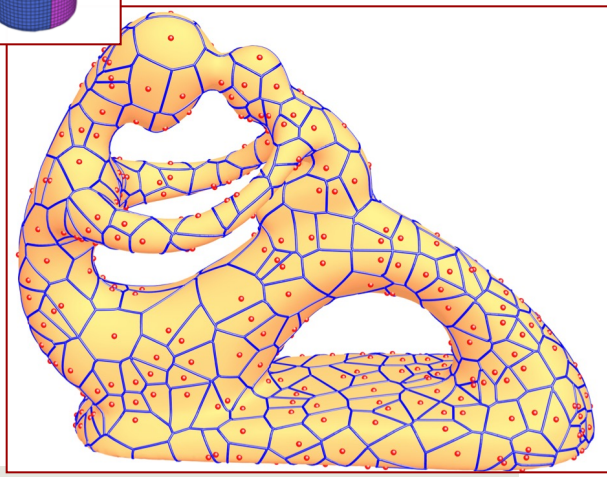


Applications

- Intrinsic geodesic distance computation is a building block for many CG and GP applications, such as:
  - Mesh parameterization, mesh segmentation, shape classification, texture mapping, ...

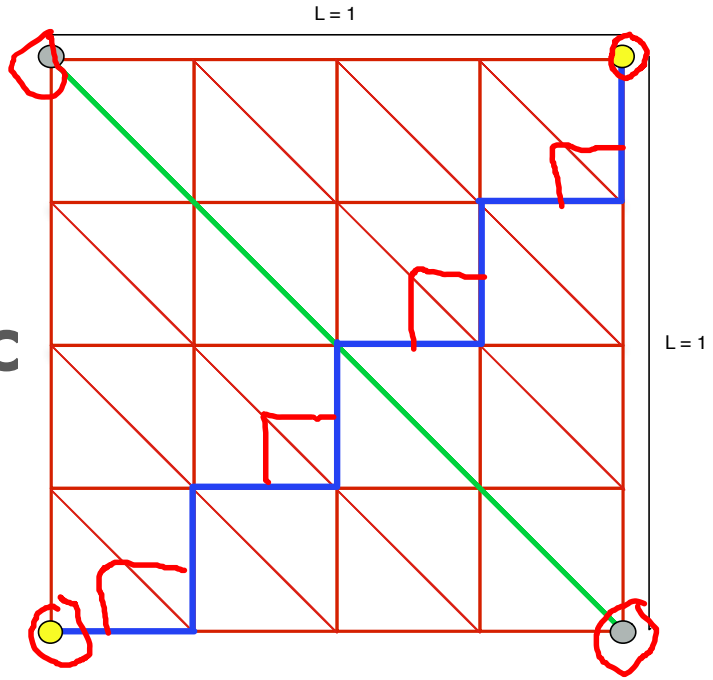






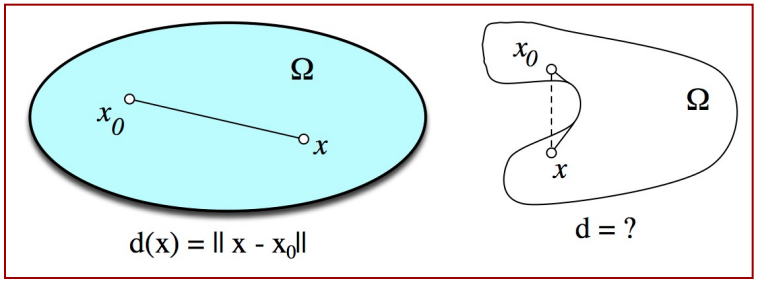
Trivial Approach

- Consider a **unit grid**:
- Apply basic Dijkstra algorithm
- **Gray** pair: $d = \sqrt{2}$
- **Yellow** pair: $d = 2$
- **Dijkstra's computed distanc dependent**



State of the art

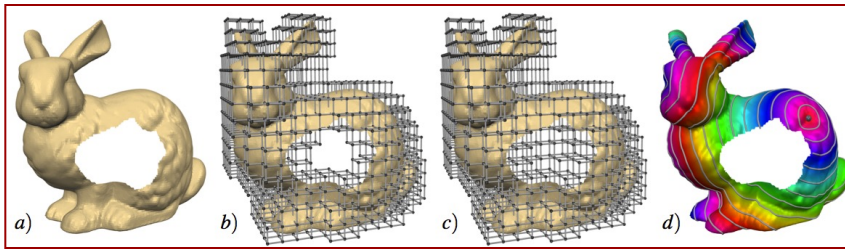
- Two main approaches:
  - "Computational geometry" approach:
    - Aims at computing exact polyhedral distance.

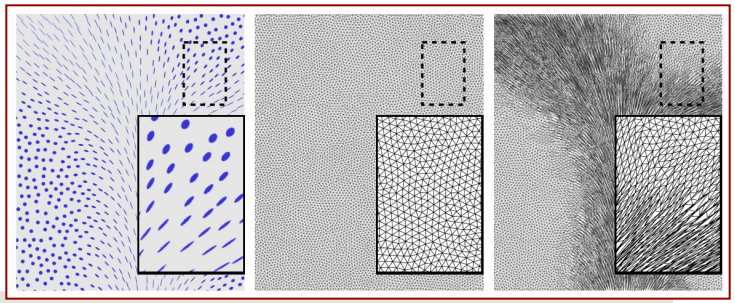


  - "PDE" approach:
    - Aims at solving the Eikonal equation:
      - $||\nabla d|| = 1$

State of the Art

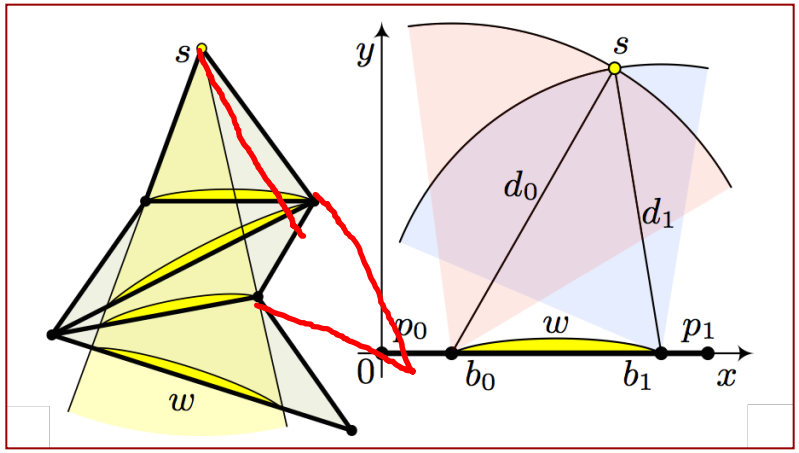
- Computational geometry
  - MMP algorithm
    - Approximations and enhancements
- PDE
  - Fast Marching Method
    - Extension to defected models
  - Heat Method
- Short-Term Vector Dijkstra (anisotropic case)
- Saddle Vertex Graph





Exact Distance

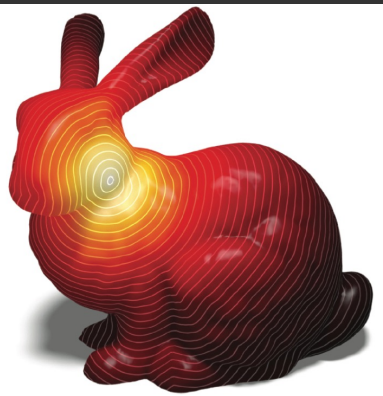
- MMP algorithm: proposed by Mount et. al
- Based on the concept of “windows”, propagated over the mesh edges.
  - Windows encode all shortest paths passing through them.



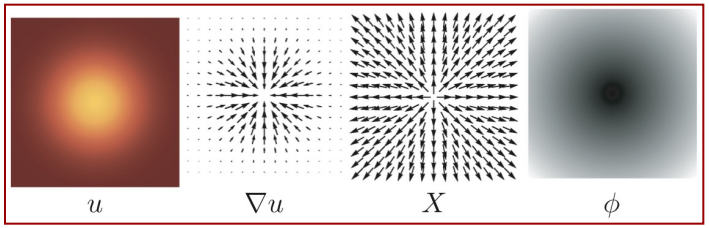
Exact polyhedral distance

- Each window may have O(n) widows
  - Complexity: $O(n^2 \log n)$
  - Subquadratic in practice

Heat method

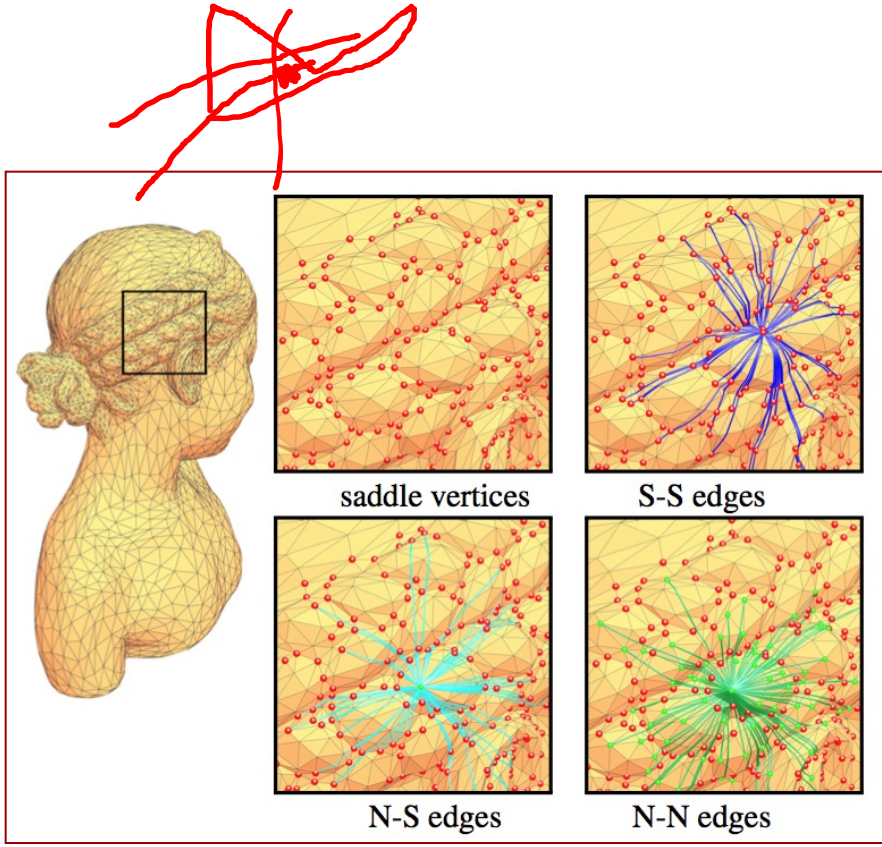


- **Idea**: touch a point on the surface with a scorching hot needle
- Heat kernel measures the heat transferred after some time *t*
- Varadahn's formula relates heat to distance
- Distances are computed solving two linear systems of equations



Saddle Vertex Graph

- **Idea**: subdivide problem into smaller sub-problems
- A saddle vertex is a vertex with total angle greater than 2π (remember Gaussian Curvature)
- Some geodesic information is precomputed and encoded into the SVG.



Saddle Vertex Graph

- Precomputation step builds an approximate version of the SVG
- A geodesic path on the surface is related to a shortest path on the graph
- Issue: On convex polyhedron the SVG becomes very dense