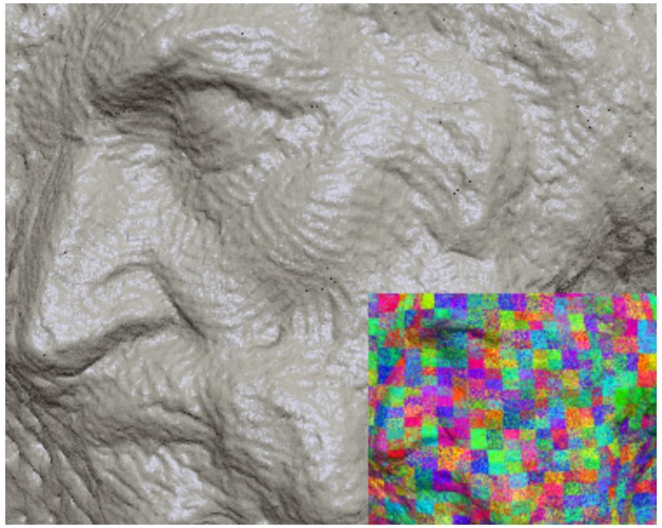
3D GEOMETRIC MODELING & PROCESSING
Sampling

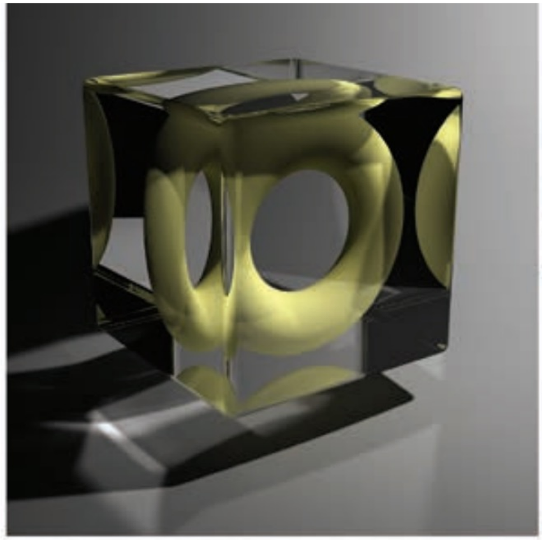
(thanks to Massimiliano Corsini for slides)

Motivations: Rendering

[Layered Point clouds, Gobbetti 2004]

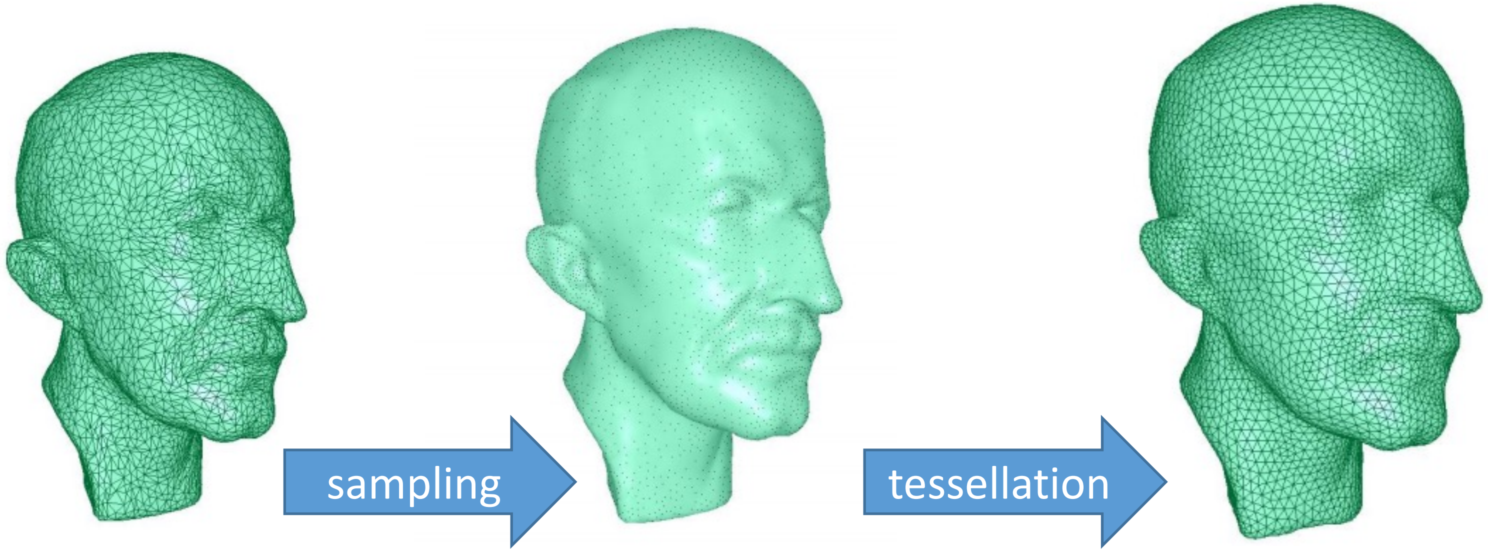


point based rendering



Photorealistic rendering

Motivations: Remeshing



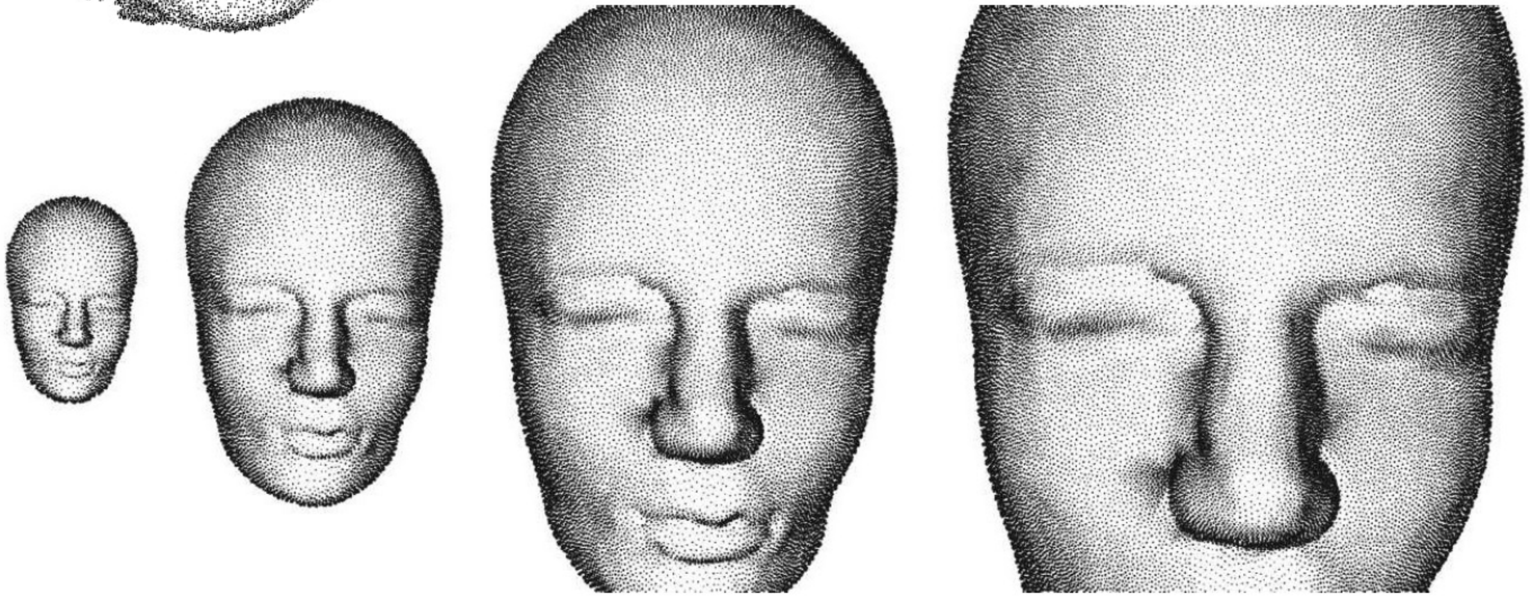
Input mesh Point sampling Output mesh

Motivations: Image/Video stippling

[PixelPie: Maximal Poisson-disk Sampling with Rasterization]

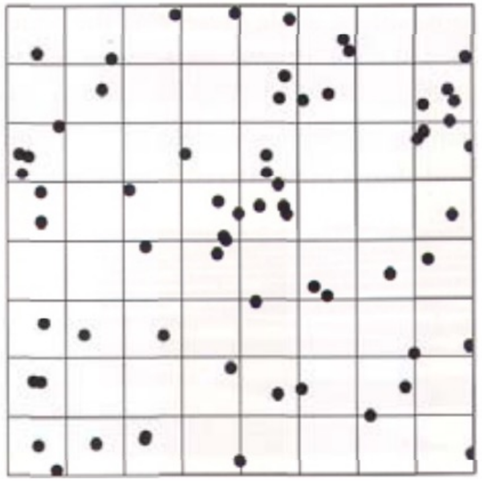
[Bilateral blue noise sampling: Additional algorithms and applications]

[Instant Stippling]



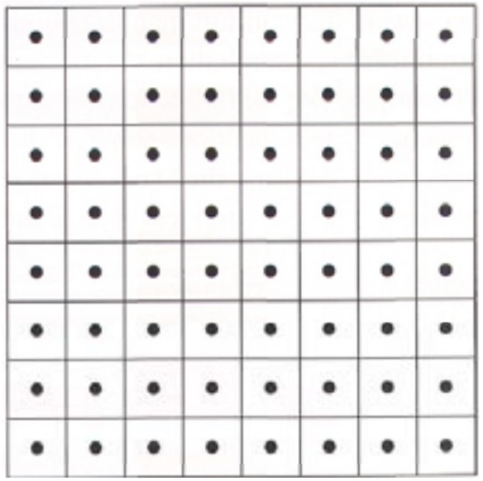
Jittering

Random



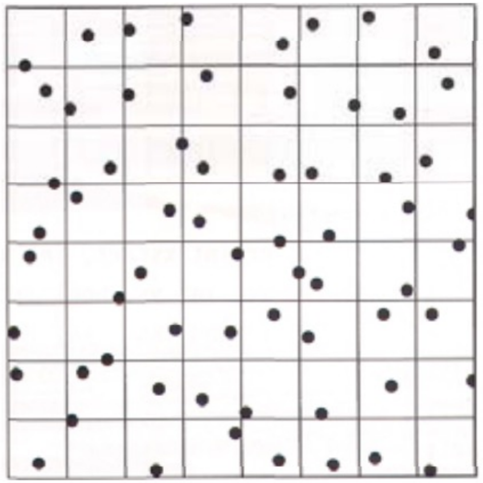
Domain not uniformly sampled

Uniform



Domain uniformly but not in a random way

Jittered



Uniform & random. Trading aliasing for noise

```
S JitteredSampling(){
for each Cell in GRID
   S = S + RandomPointInThe Cell()
return S
}
```

Jittering: from aliasing to noise

256 samples per pixel as reference

1 sample per pixel (jittered)

1 sample per pixel (no jitter)

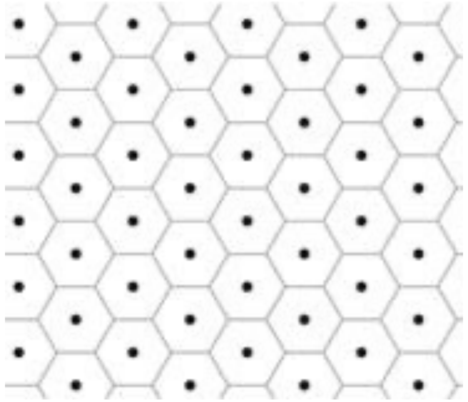
4 samples per pixel (jittered)

How to do sampling

- Characterization
  - Domain: 2D,3D, surfaces..
  - Metric: geodesic, euclidean
  - Shape of the primitive: points, lines, balls
  - Type of algorithms: jittering, dart Throwing, relaxation, Tiling

Poisson Disk Sampling

- A Poisson Disk Sampling is a sampling $X = \{(x_i, r_i)\ i = 1, \ldots, n\}$ such that
  1. Minimal distance: $\forall (x_i, x_j) \in X, ||x_i - x_j|| > \min(r_i, r_j)$
  2. Unbiased sampling: The probability of a region to be covered is proportional to its size
  3. Maximal sampling property: $\Omega \subseteq \bigcup disk(x_i, r_i)$
- Q: is <u>any</u> PDS a «good» sampling?
- A: No, a maximal PDS sampling (for a given radius) is obtained placing samples at the centers of cells of a hesagonal lattice



PDS: Dart Throwing [cook86]

```
DartThrowingPDS
n_miss = 0
do{
          x_cand = RandomPoint()
          if  not x_cand ⊆ covered(X) // no disk in X contains x_cand
           X = X + x_cand // hit
          else
           n_miss = n_miss +1 // miss
} while(n_miss / (n_miss +#X) < threshold )
```

- Works on any domain provided a metric
- Very slow convercence rate, $O(n^2)$ asymptotic complexity
- Maximality not guaranteed in given time (likelability of «hit» tends to 0)
- Many approaches devoted to *efficiency* and *maximality*

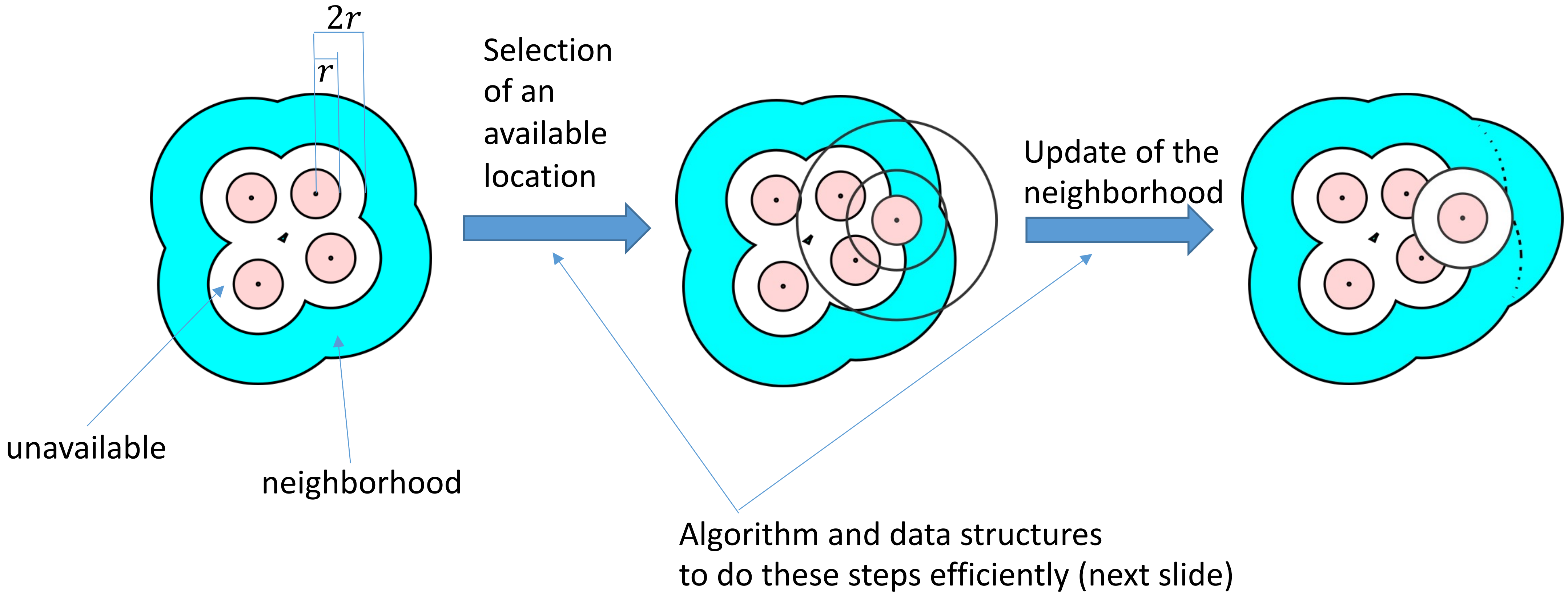
Efficiency in PDS algorithms

- Two basic operations that determine convercence speed of a PDS algorithms

1. choosing a sample location with unbiased probability
2. Testing if a new location is not already covered

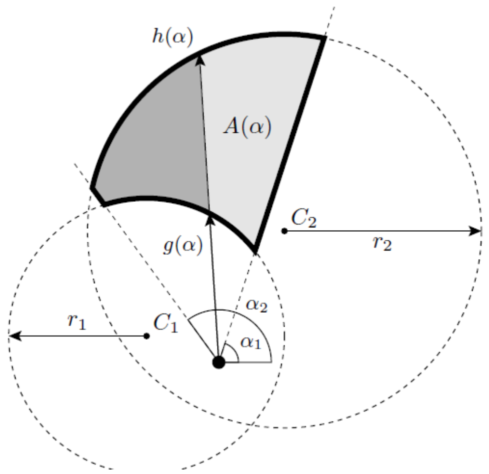
Scalloping [Dunbar06](1/4): Dart Throwing in $O(n \log n)$

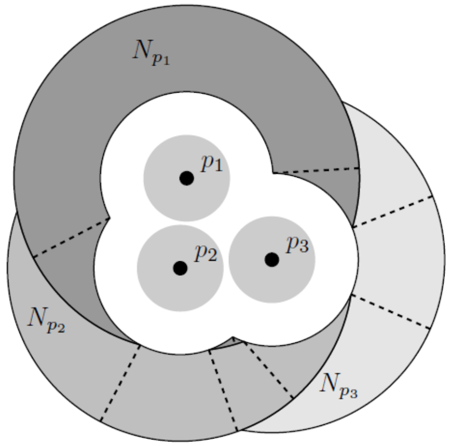
- Core idea: if a sampling is not maximal, there must be an *available* location in the neighborhood of the *unavailable* region



Scalloping (2/4): Dart Throwing in $O(n \log n)$

- *Scalloped regions:*
- *Def. Scalloped sector:* a region of the domain bounded by two circular arcs
- Def: *Scalloped region*: a union of scalloped sectors





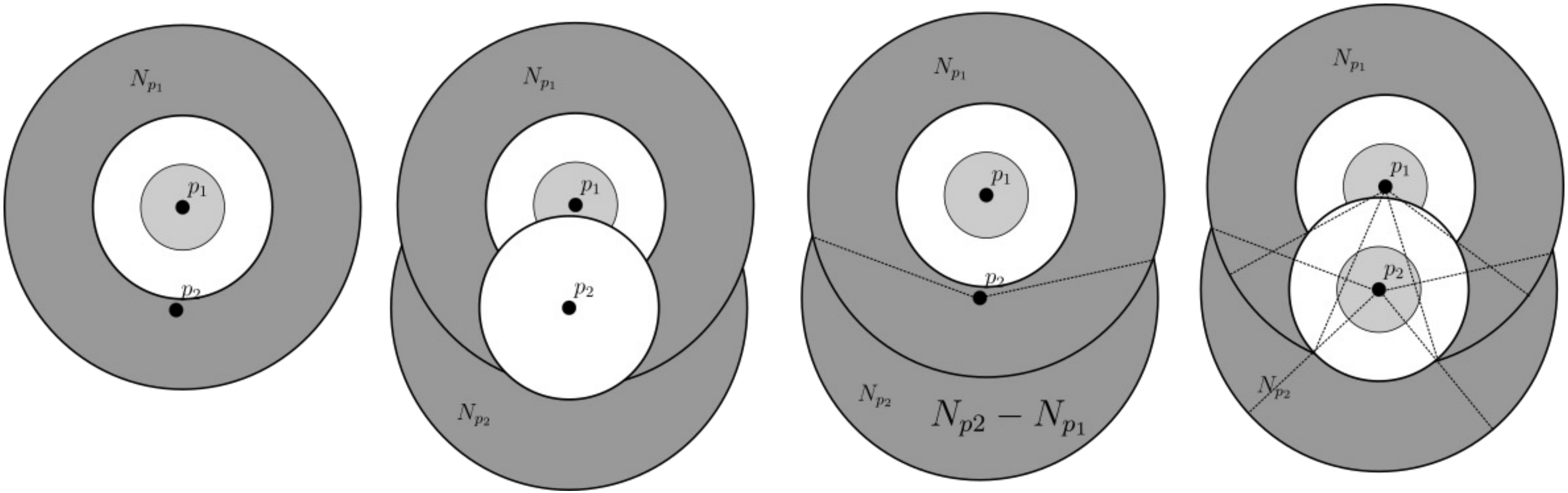
Scalloping (3/4): adding a new disk

Order of insertion

$$N_p = D(p, 4r) - \bigcup_{p' \in P} \begin{cases} D(p', 4r), & p' < p \\ D(p', 2r), & p' \geq p \end{cases}$$

Available neighborhood

disk

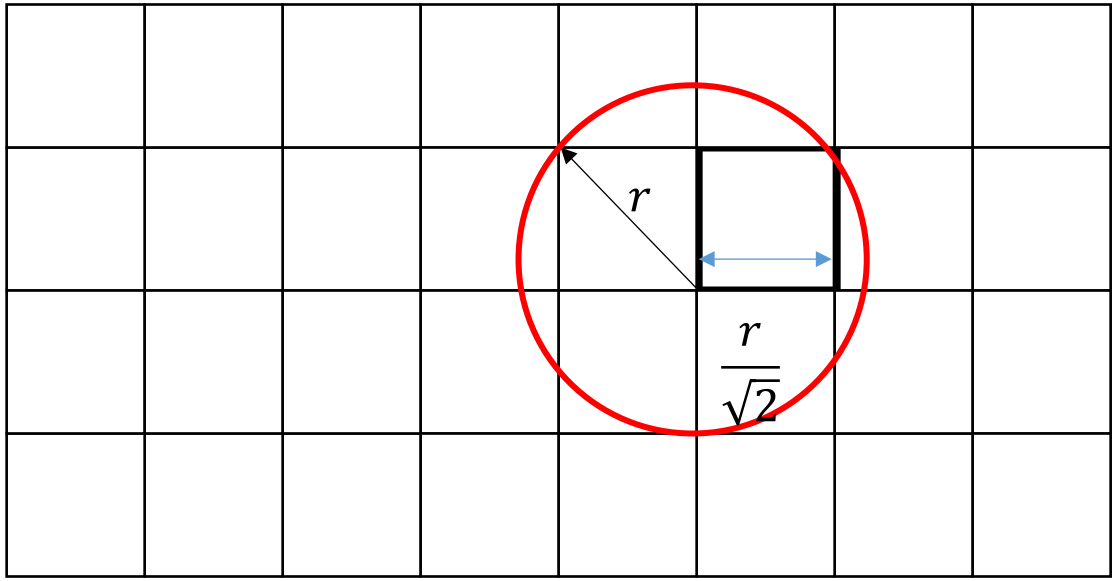


Scalloping (4/4): Speed

- The maximum number of scalloped sectors of r neighboorhod is bounded by a constant
- The update of neighboorhood is limited to the 4r radius from the sample and can be done in O(1)
- Unbiased sampling. All available neighborhoods are stored in a balanced tree O(log N)

Hierarchical Dart Throwing [White07] (1/3)

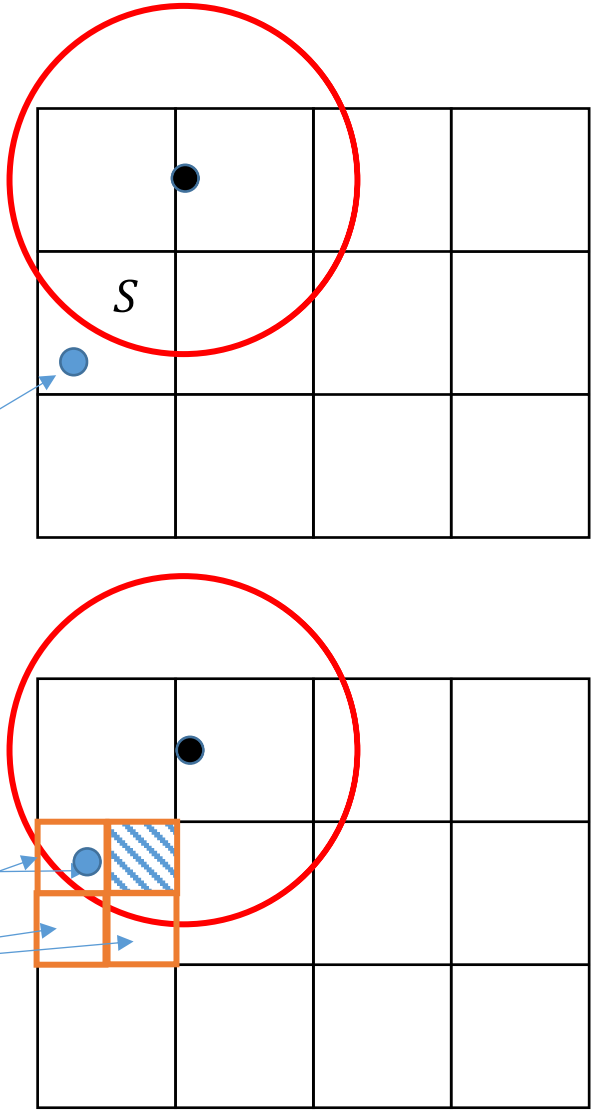
- Regular grid where each cell is the root of a quadtree
- Size of each cell so that it is completely covered by a disk which center is inside the cell
- **def**: *Active cell*
  Cell not yet entirely covered by a disk
- **def:** *Active list with index **i***
  List of active cells at level i
- *Initial condition: Active list L0 contains all the cells of the grid*



Hierarchical Dart Throwing (2/3)

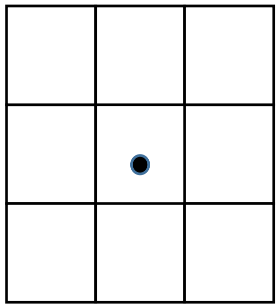
Hierarchical Dart Throwing

```
Put base level squares on active list 0 (the base level).
Initialize the point set to be empty.
While there are active squares
    Choose an active square, S, with prob. proportional to area.
    Let i be the index of the active list containing S.
    Remove S from the active lists.
    If S is not covered by a point currently in the point set
        Choose a random point, P, inside square S.
        If P satisfies the minimum distance requirement
            Add P to the point set.
        Else
            Split S into its four child squares.
            Check each child square to see if it is covered.
            Put each non-covered child of S on active list i + 1.
```



HDT (3/3): Cost

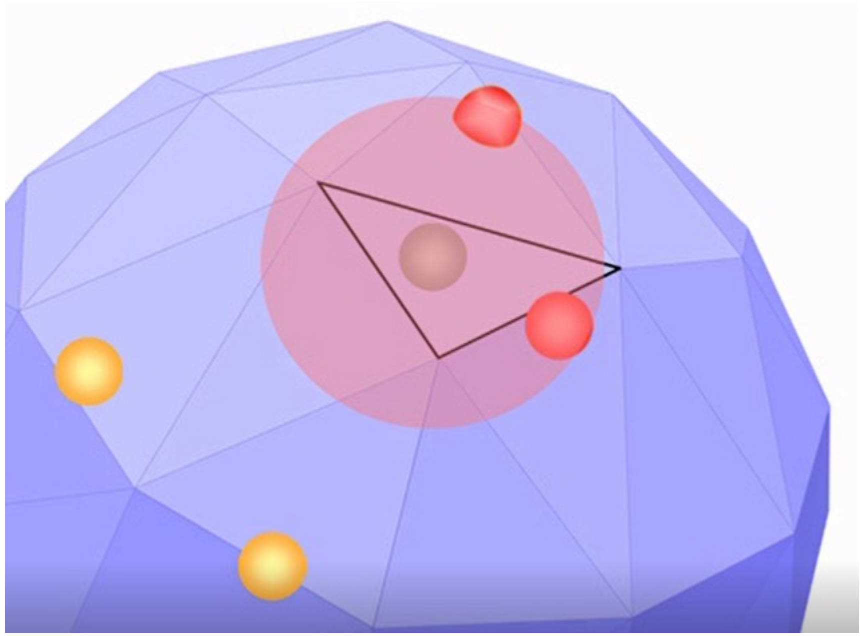
- Coverage test: a secondary uniform grid of cells with size r stores a copy of the point set. By construction, no cell can contain more than 4 points. Therefore, the cost for coverage is constant
- Choice of sample:
  1. keep the sum of areas for each list $a_0, \dots, a_k$
  2. Generate a random number in [0,1]
  3. Find m s.t. $\sum_{i=0}^{m-1} a_i \leq a_{tot} x < \sum_{i=0}^{m} a_i$
  4. Pick a random square in the list



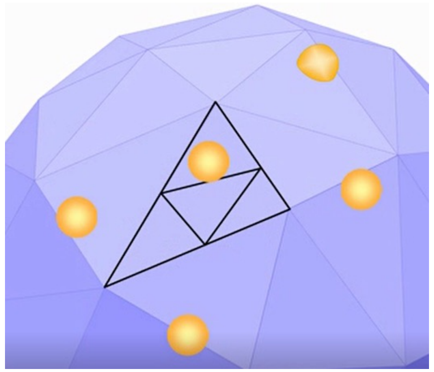
1. Asyntoptic cost: how many cells are created? Authors claim $O(N)$: «*While we cannot provide a rigorous proof, we are convinced that hierarchical dart throwing is O(N) in both space and time on average*»

PDS on surfaces [Cline09] (1/6)

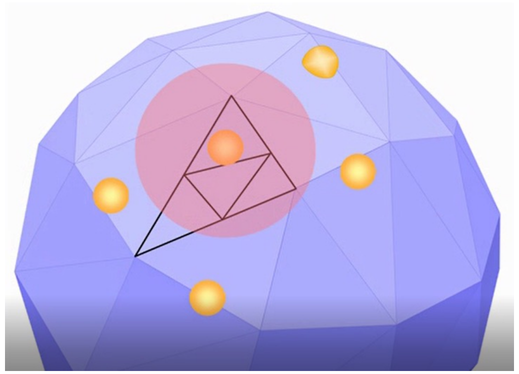
- «Essentially» the same as HDT
  - Replace «uniform grid» with «triangulation»
  - Replace «quadtree» with 1-4 triangle subdivision



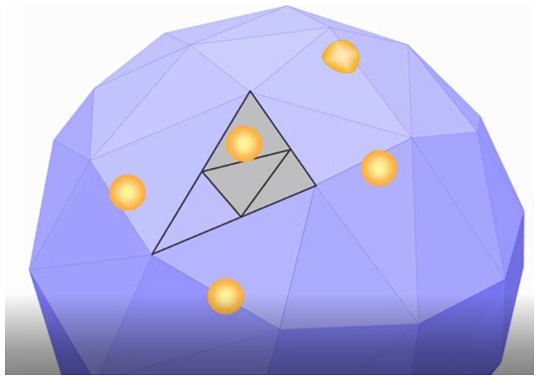
PDS on surfaces [Cline09] (2/6)



Pick a point randomly, check that is not closer than r to any other point



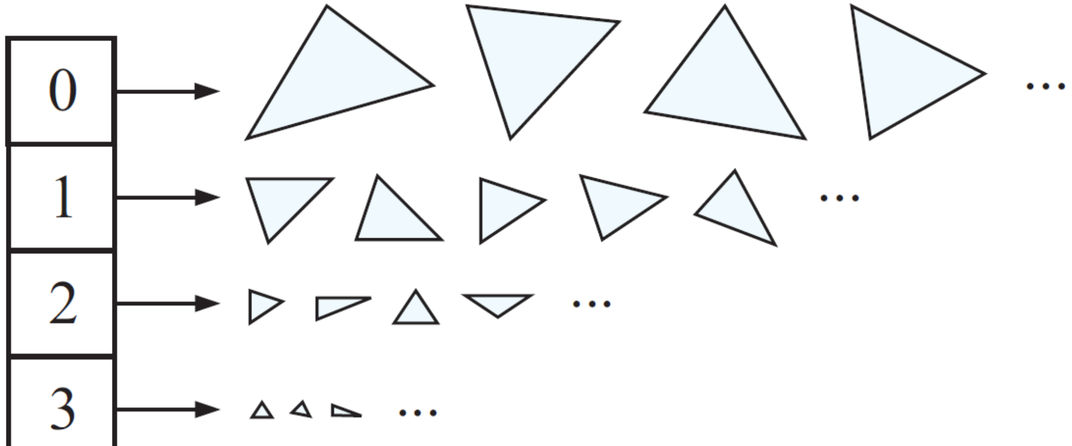
until the triangle containing the point is not entirely within distance r, iterate 1-4 splitting



Remove entirely covered triangles from the activel list.
End when the list is empty.

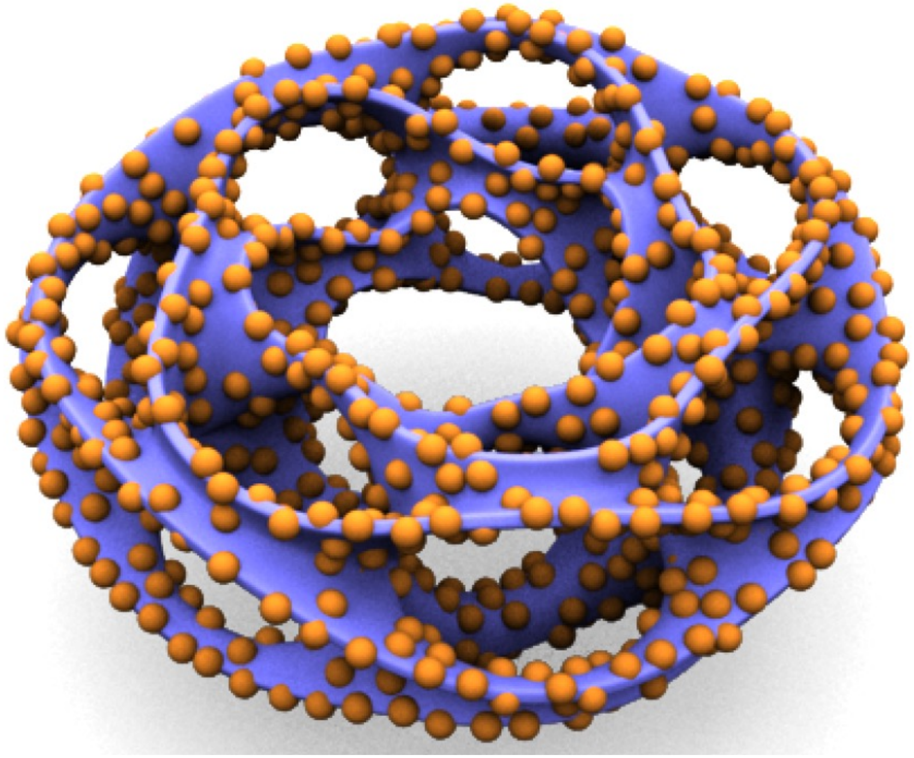
PDS on surfaces [Cline09] (3/6): Cost

- Coverage test. Constant like in [White07]
- Unbiased sampling: Logarithmic binning
  - each bin store a list of triangles within a range area
  1. a = random value in [0,total_area]
  2. Linear search on the bins until sum_b>a
  3. Pick a triangle and accept with prob triangle_area/bin_area. Repeat until accept
  4. Pick a random point in the triangle



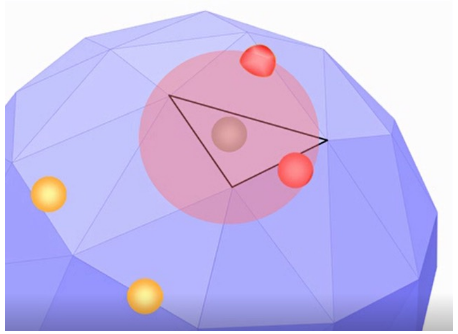
PDS on surfaces [Cline09] (4/6)

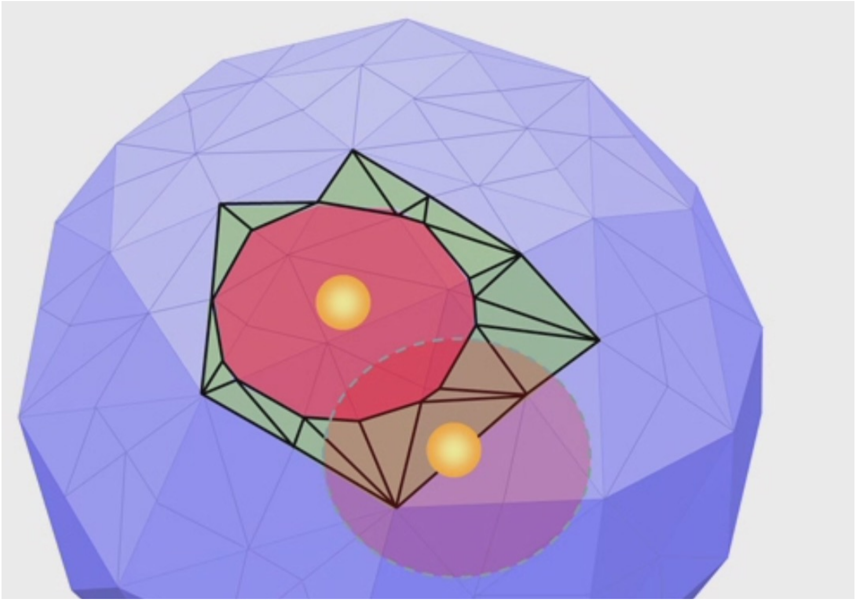
- Does it work well?



PDS on surfaces [Cline09] (5/6)

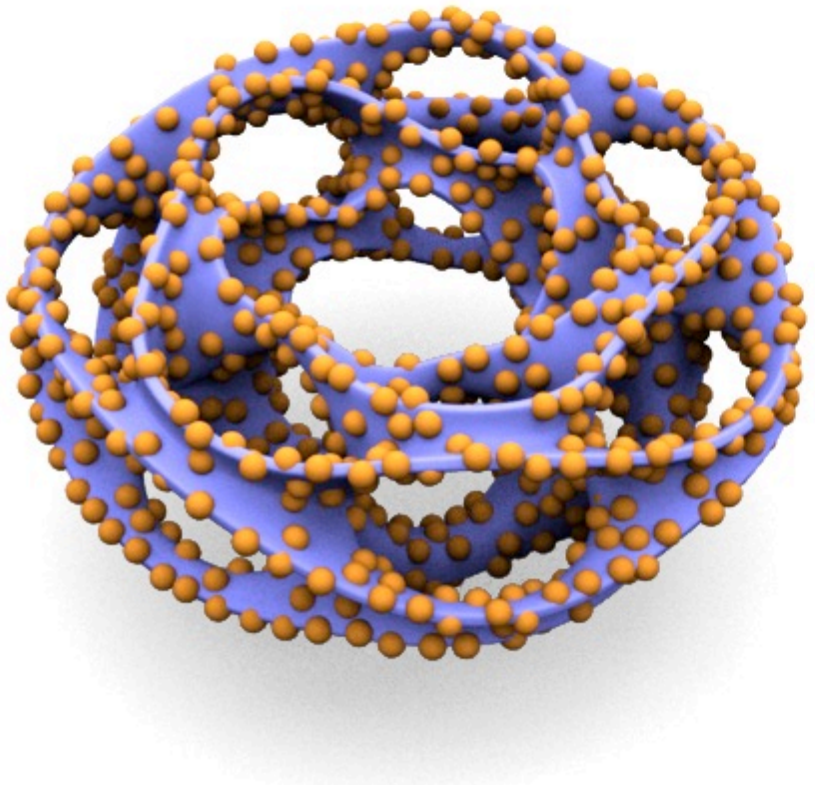
- «Essentially» the same as HDT
  - Replace «uniform grid» with «triangolation»
  - Replace «quadtree» with 1-4 triangle subdivision
  - Replace «Euclidean distance» with «geodesic distance»



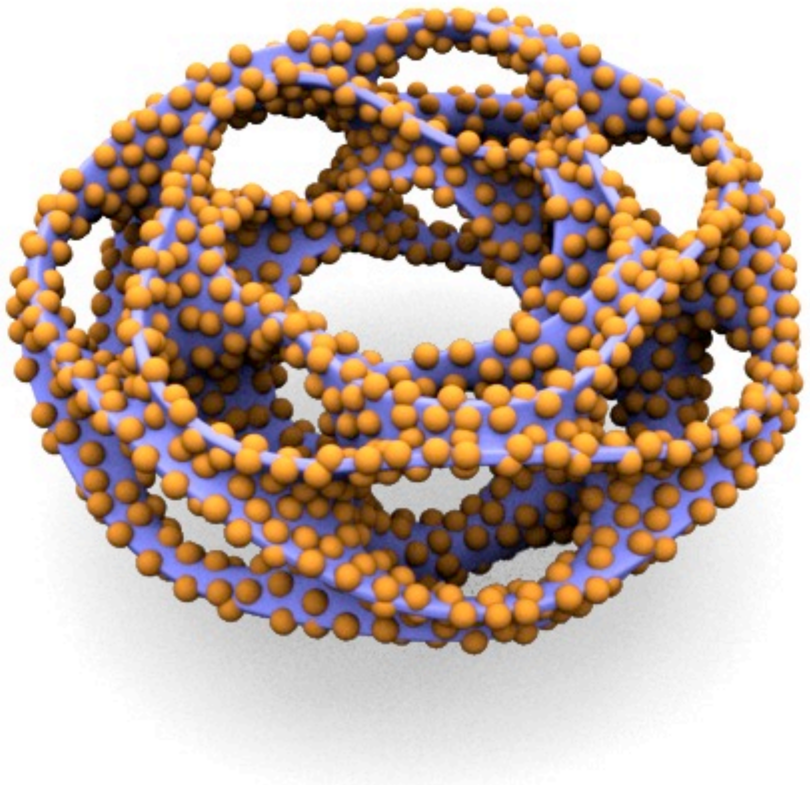


PDS on surfaces (6/6)

Euclidean distance

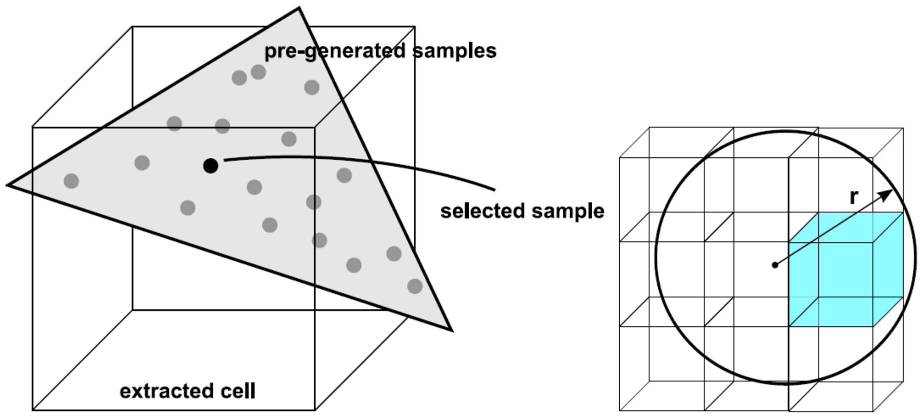


geodesic distance



HDT in 3D space [Corsini12]

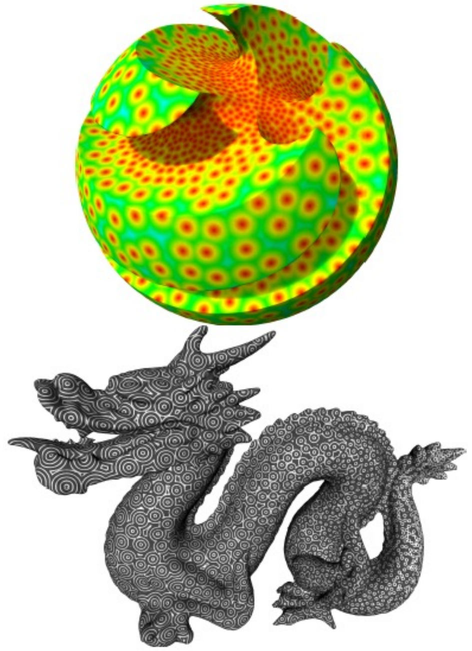
- Surface immersed in a 3D uniform grid where each cell is the root of a oct-tree
- The data structure is a direct extension of HDT except that the cell **is not the** domain but it **contains** the domain (the triangles)
- Basic operation: once an active cell is chosen, pick a point on the contained surface
- Use of *pregenerated samples*



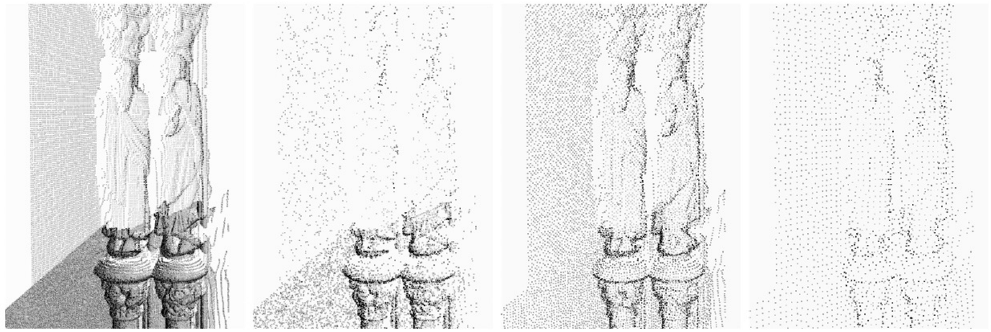
Samples generation

- Variant: just shuffle the initial overasampling and iterate point removal. No hierarchy needed anymore!
  1. Generate the pool: a dense sampling of the surface
  2. Until there are available samples
     1. Pick a sample randomly
     2. Remove the sample closer than he disk radius from the pool
- Q: Both versions work well: where is the trick?
- A: the creation of the pool essentially converts the initial «continuous» domain (faces) in a point sampled one. The algorithm cannot guarantee maximality of the sampling up to the intersample distance of the initial pool

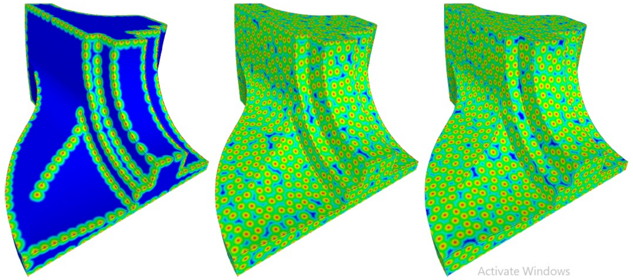
Example uses



Importance sampling / variable radius



Subsampling Point clouds



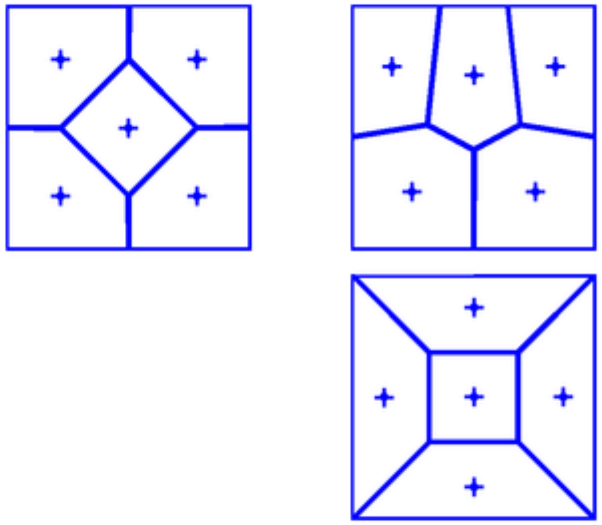
Edge preserving sampling

Relaxation methods

- Methods that iteratively optimize the position of the samples w.r.t. some energy function
- **def**: Centroidal Voronoi Tessellation (Diagram).
  A CVT is a VT where each site (point) lies in the centroid of its region:

$$p_i = \frac{\int_{V_i} x\rho(x)dx}{\int_{V_i} \rho(x)dx}$$

$\rho(x)$ Some density function

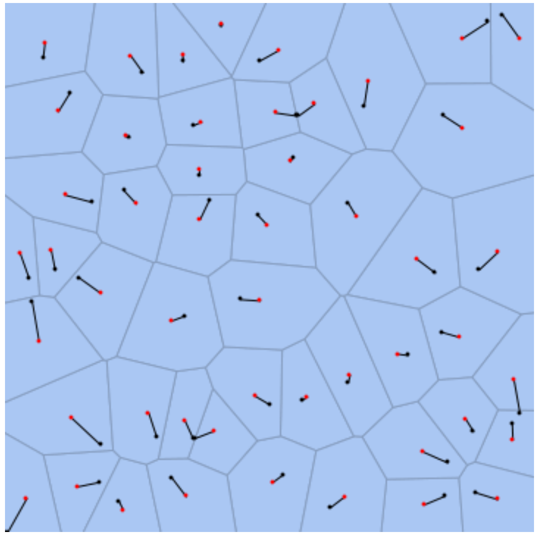


- The minimum of the enery function below is on a CVT

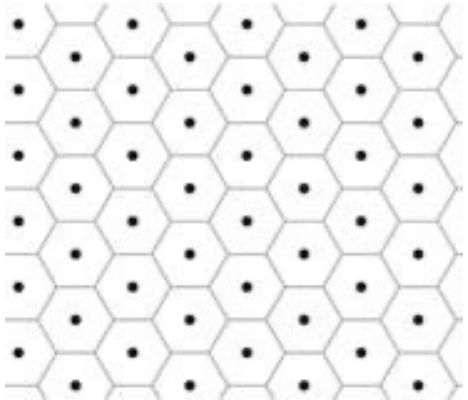
$$\mathcal{F}(S, \mathcal{V}) = \sum_{i=1}^{n} \int_{V_i} \rho(x) \left| x - s_i \right|^2 dx,$$

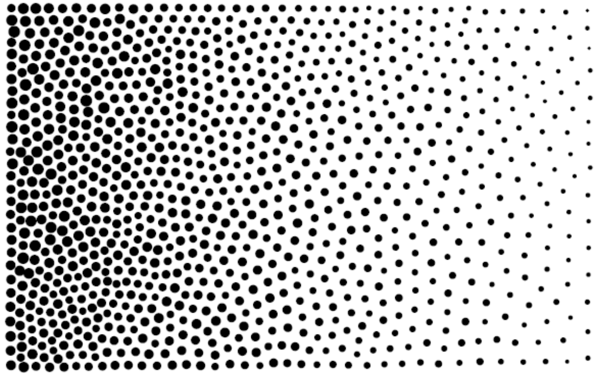
Lloyd's method

1. generate the Voronoi tessellation $V(S)$ in Ω
2. move each site $s_i \in S$ to the centroid p_i of the corresponding Voronoi region $V_i \in V$;
3. if the new sites in S meet some convergence criterion, then terminate; otherwise return to step 1.



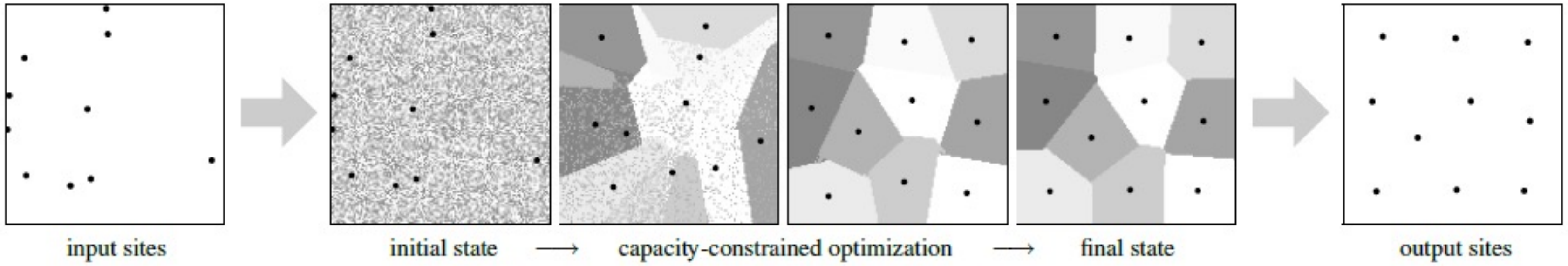
Do CVT necessary lead to a good sampling?



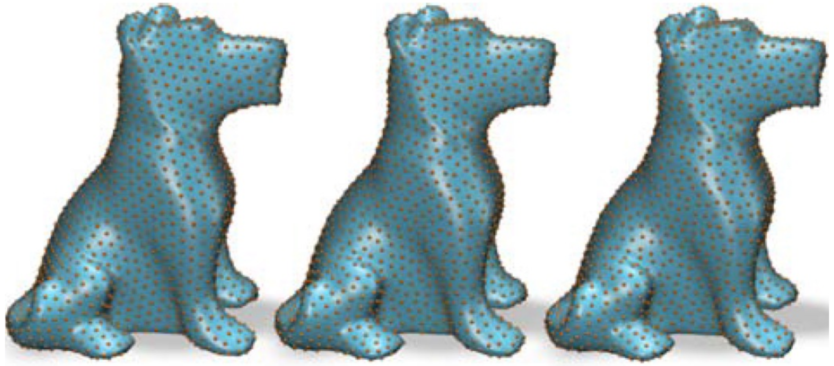


CVT methods

- Methods that modify CVT to obtain better sampling properties
- Capacity Constrained CVT [balzer09] : CVT plus the constraint that each region of the VT has the same area



Extension to surfaces. Capacity contrained Delaunay Triangulation [chen12]



References

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- A Survey of Blue-Noise Sampling and Its Applications Dong-Ming Yan, Jian-Wei Guo, Bin Wang, Xiao-Peng Zhang & Peter Wonka Journal of Computer Science and Technology volume 30, pages439–452(2015)