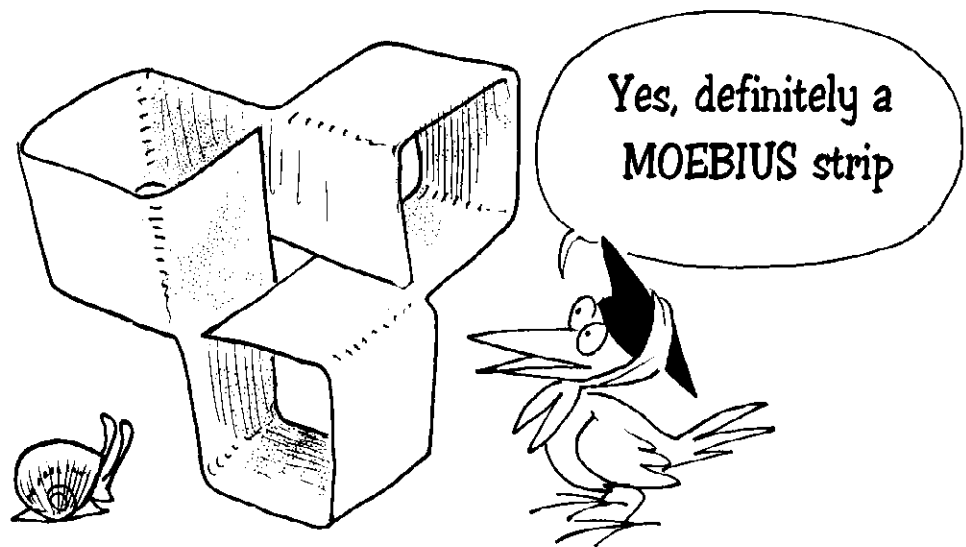


Knowledge without Border (Ssavoir sans Frontières)

The Adventures of Archibald Higgins

TOPO THE WORLD

Jean-Pierre Petit



Translated by John Murphy

The Association Knowledge without Borders, founded and chaired by Professor Jean-Pierre Petit, astrophysicist, aims at spreading scientific and technical knowledge in as many countries as possible and in as many languages as possible. To this end, all his popular scientific works, which cover a period of thirty years, and more particularly the illustrated albums he has created, are now freely accessible. Anyone is now free to duplicate the present file, either in digital form or in the form of printed copies and circulate these copies to libraries, within the context of schools or universities or associations whose aims would be the same as the association, provided that they do not derive any profit from this circulation and that they do not have any political, sectarian or confessional connotations. These pdf files may also be put on line in the computer networks of school and university libraries.



Jean-Pierre Petit intends to create numerous other works which will be accessible to a larger audience. Even illiterate people will be able to read them because the written parts will “speak” when the readers click on them. Thus it will be possible to use these works to support literacy schemes. Other albums will be "bilingual" in so far as it will be possible to switch from one language to another selected language with a mere click. Hence another tool made available to develop language skills.

Jean-Pierre Petit was born in 1937. He made his career in French research. He worked as a plasma physicist, he directed a computer science centre, he has created softwares, he has published hundreds of articles in scientific magazines, dealing with subjects ranging from fluid mechanics to theoretical cosmology. He has published about thirty books which have been translated in numerous languages.

The association can be contacted on the following internet site:

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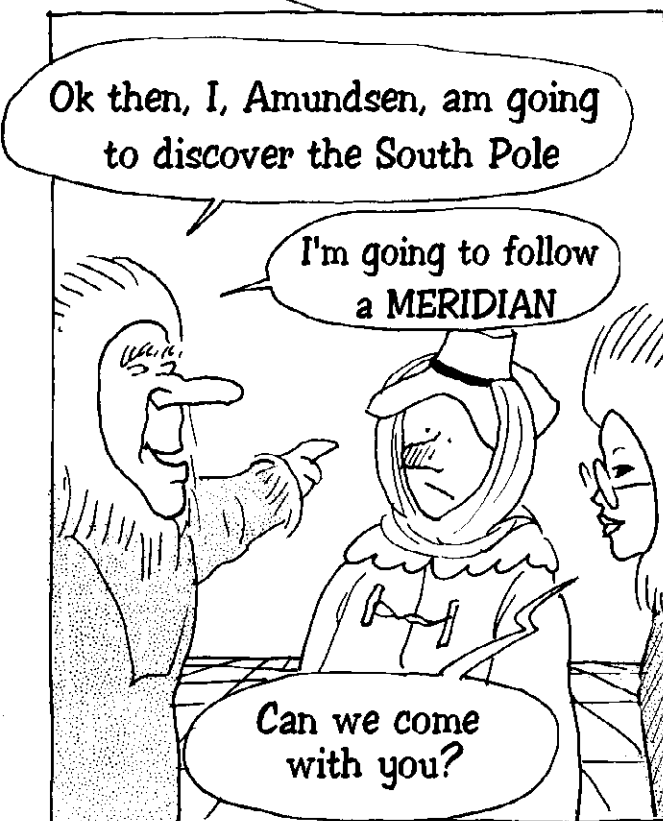
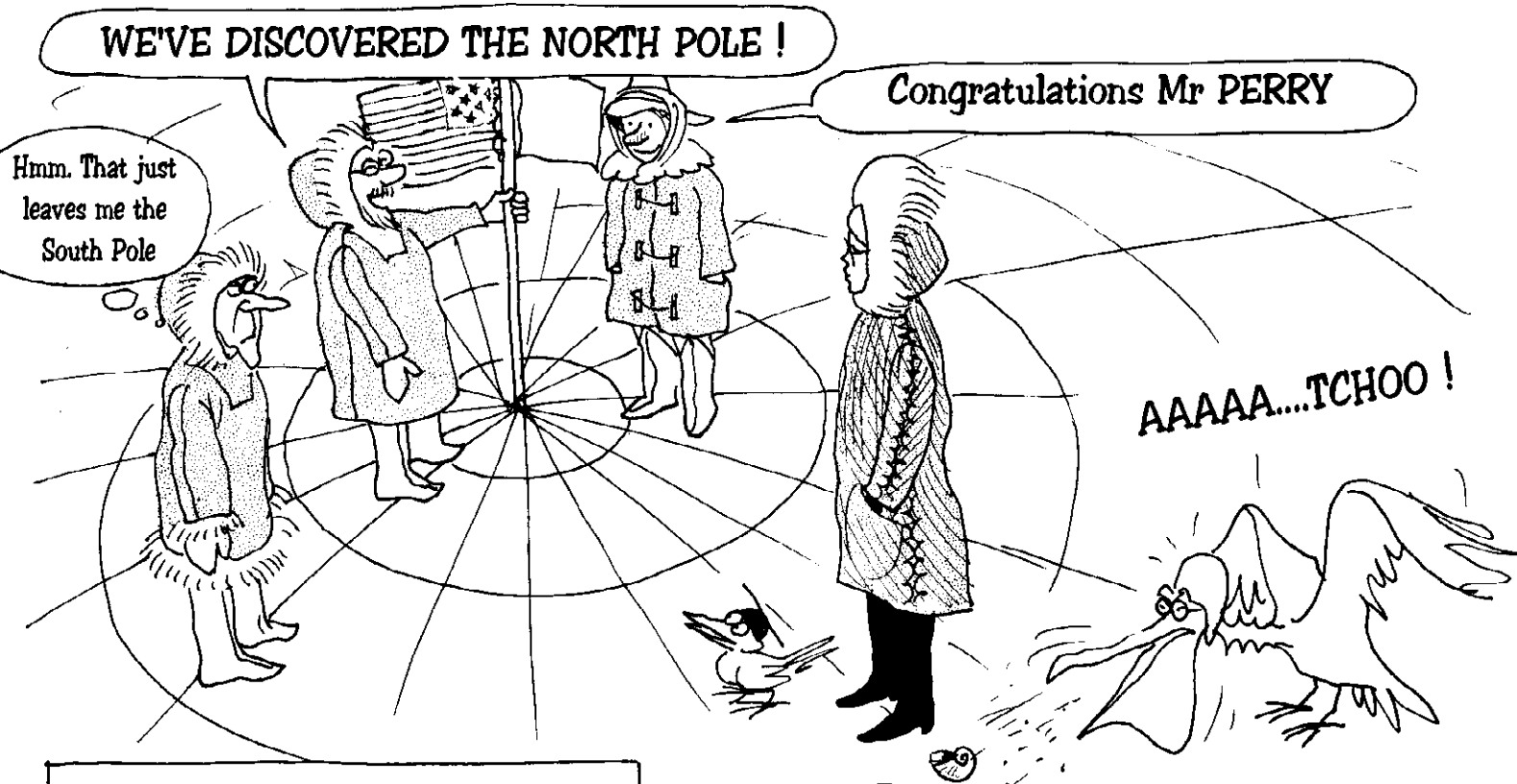
Warning to the reader

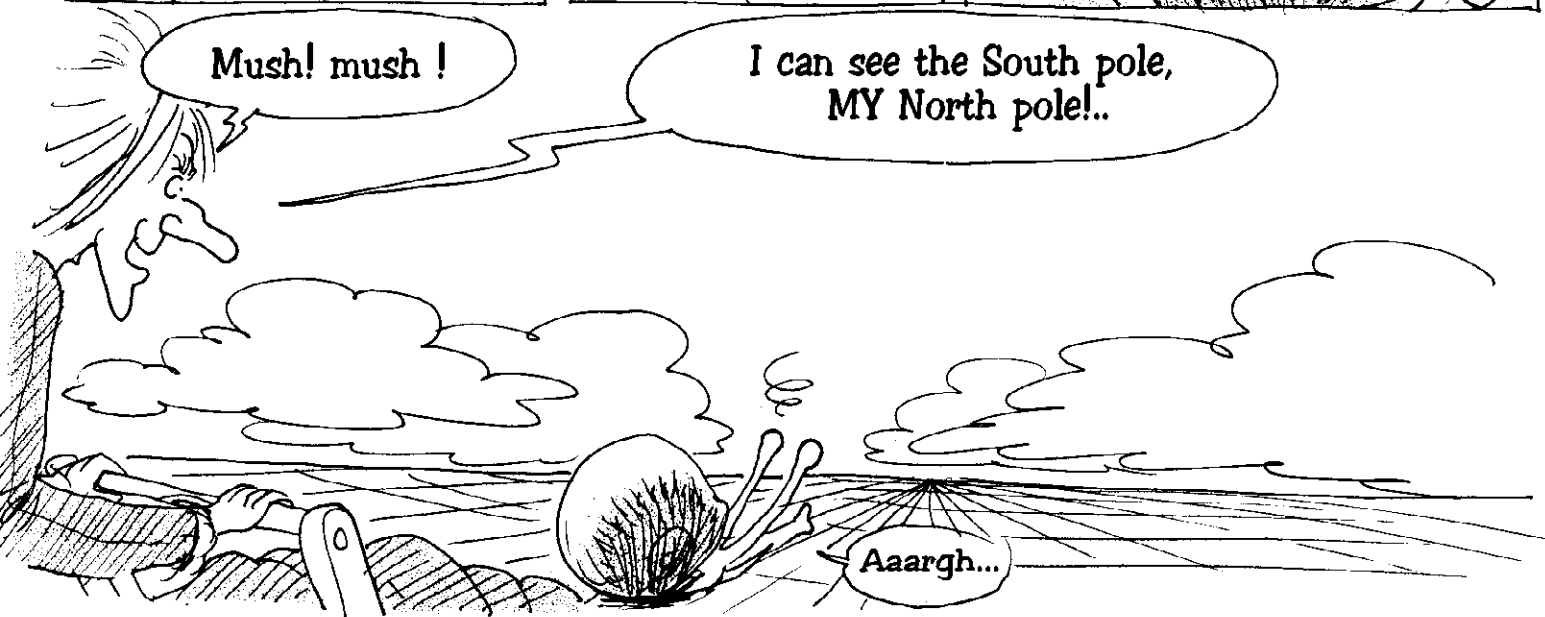
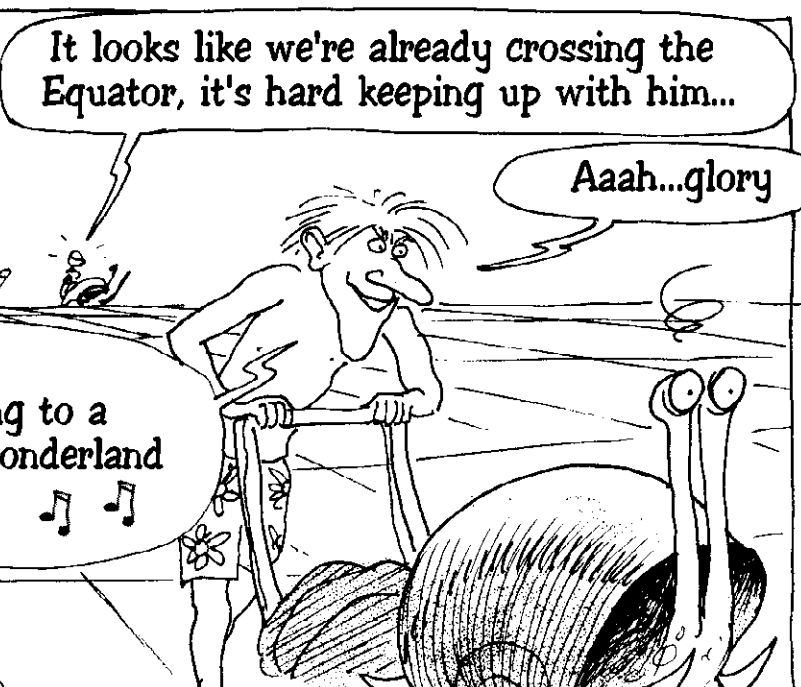
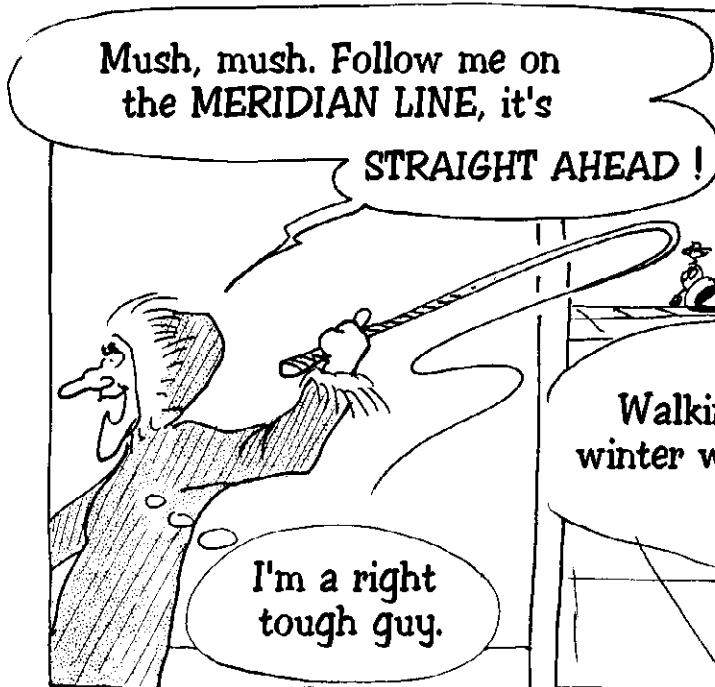
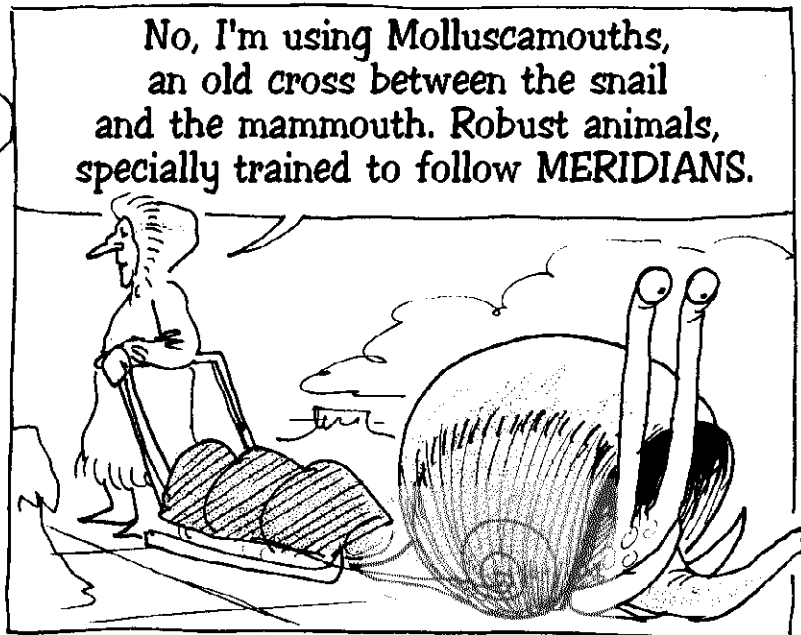
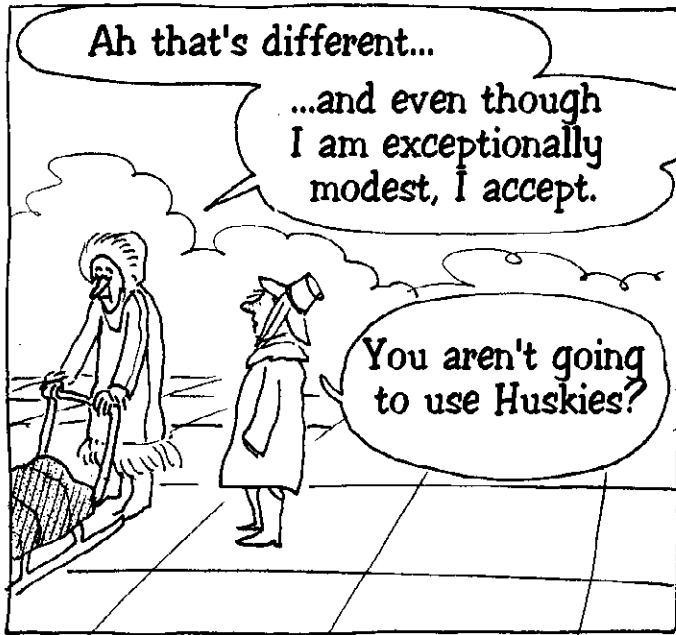
It is best to avoid reading this album :

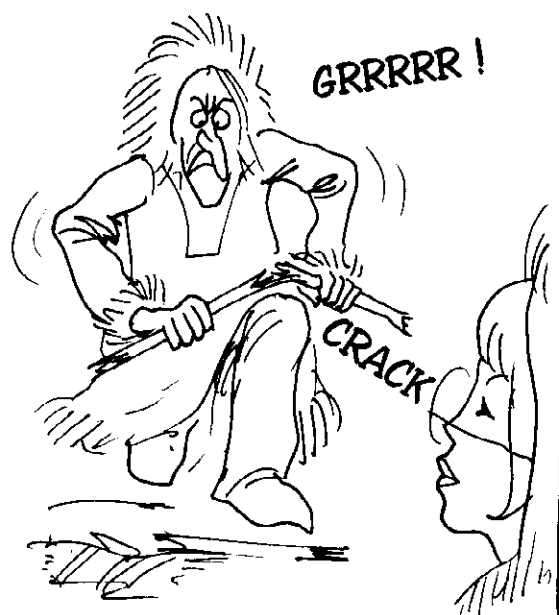
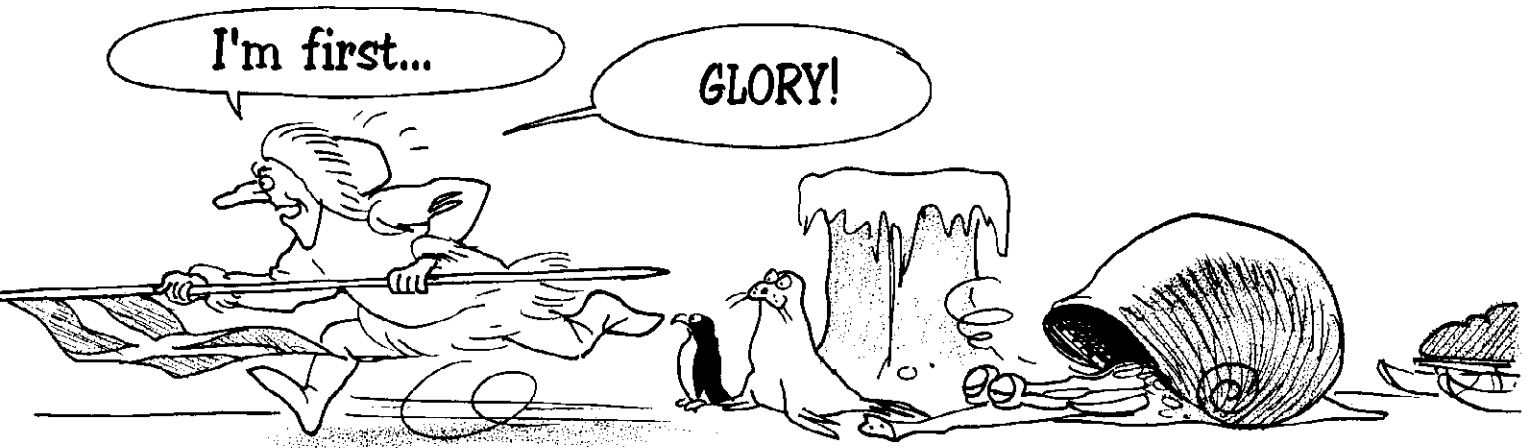
- In the evening before going to bed
- after a heavy meal
- or when you're certain about nothing, because this will only make it worse

The author

THE PLANET WITHOUT A SOUTH POLE







And not a word to anyone about this OK!

Hey, look!

My flag!
It's disappearing!!!

What!!?!

Calm down Mr Amundsen

Hey, have you finished mucking about you lot?

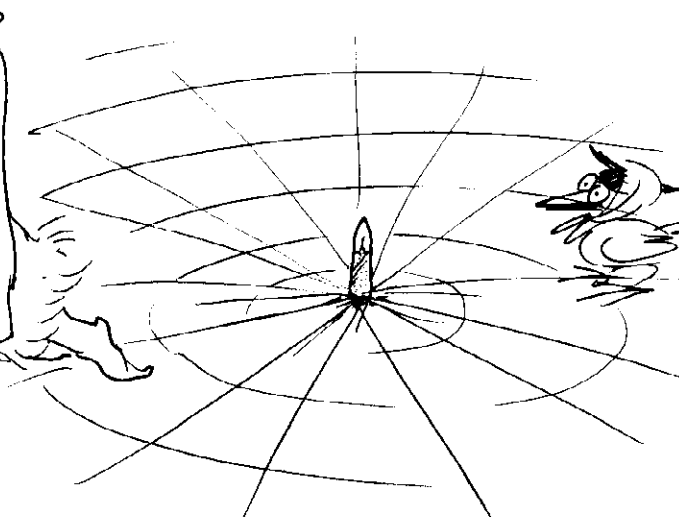
Funny, I sounded like the voice of Mr Perry...

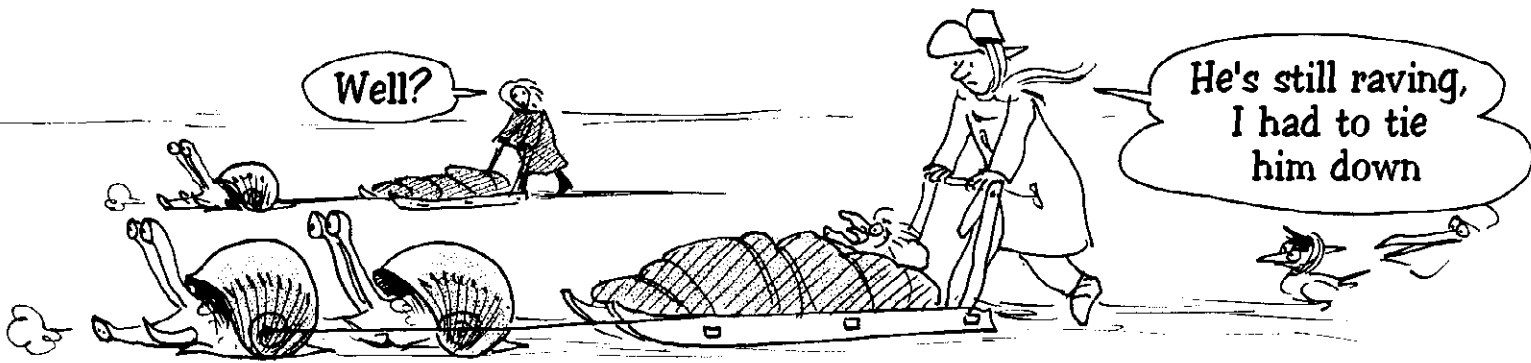
Come on Mr Amundsen,
let's go home

He's in shock

We'll try to find out what this is all about

GLGBL..

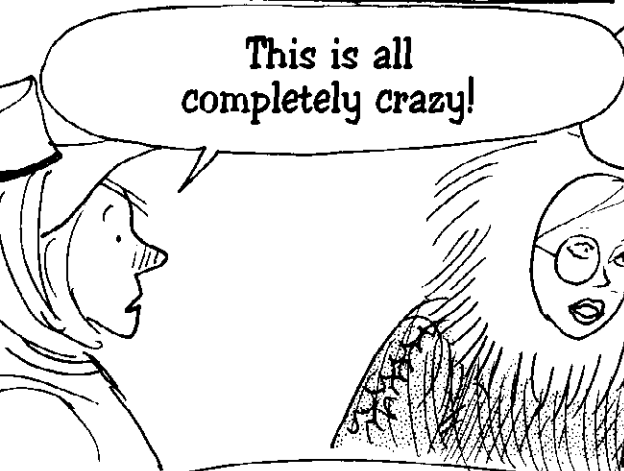
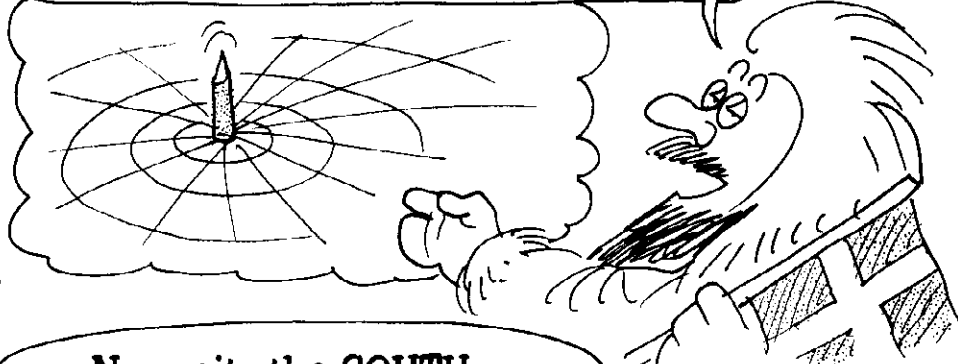




The Molluscamouths slid along without a sound on the frozen meridians



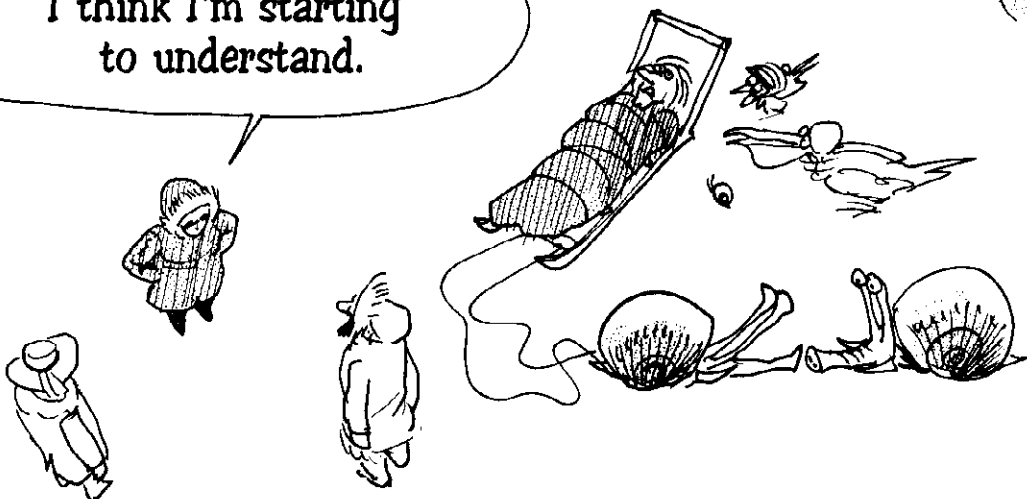
Something amazing happened while you were gone. My flag suddenly disappeared and another one marked "SOUTH POLE" took its place!!



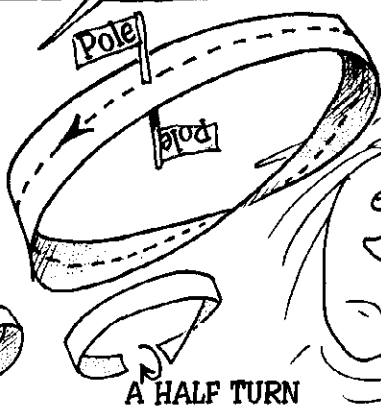
No wait...the SOUTH POLE flag, does it appear point first?

Yes, why do you ask that?

I think I'm starting to understand.



It's obvious if we consider the NEIGHBOURHOOD of the meridian we followed to be a UNILATERAL SURFACE (*), a MOEBIUS STRIP, with a single side (see "Here's Looking at Euclid", p54)



You mean that the south pole where we were earlier was only the north pole upside down?

So where is the REAL south pole?

It's rather strange

So what's happening ?

Apparently we've lost the south pole

Let's think.

Oh, nice one !

What are they saying ?

Well according to Sophie we are on a sort of sphere with only one side !..

That's NUTS !

Hi, how are things where you live ?

(*) A strip that is twisted a half turn before the two ends are stuck, it then has only one side.

Oh, much like here really

Well if we want to get Mr Amundsen out of his difficult situation, first of all we have to understand the **SHAPE** of this strange planet. Let's use some basic principles of **TOPOLOGY**. For that, we'll decompose all objects into:

CONTRACTILE CELLS



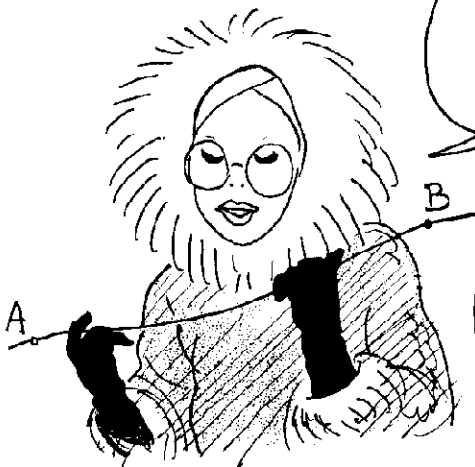
This indecomposable object seems to be a **POINT**...

What can you do with a point ?

An object, considered as an ensemble of points, occupies a certain place in space. It would be contractile if it could shrink and become a single point, but by **RUNNING THROUGH ITSELF**

Take this element of a curve for example. It's an **OBJECT WITH ONE SPATIAL DIMENSION**

Ah yes, the position of a point on this curve can be pinpointed using just one quantity, the curvilinear abscissus, or the length of the line separating one point from another taken as the origin.



I can put a piece of the curve inside a bit of hollow pasta, and inside it can shrink, shrink...

Just like mercury in a thermometer.

Is every curve **CONTRACTILE** then ?

No, **CLOSED** curves aren't

Yes but you just need to cut it !

OK, but then the **CURVE** becomes a **SEGMENT**. It is no longer **CLOSED**.

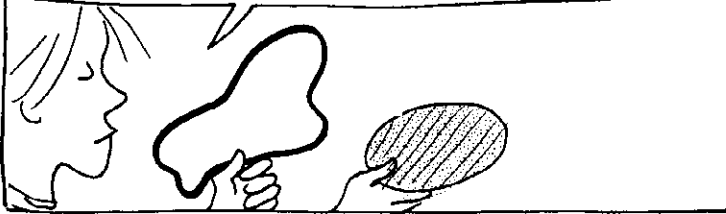
If I take a circle for instance, I can shrink it according to a point like this no?

A **CIRCLE** is therefore not **CONTRACTILE** and the same goes for any a closed curve, whether it's planar or not.

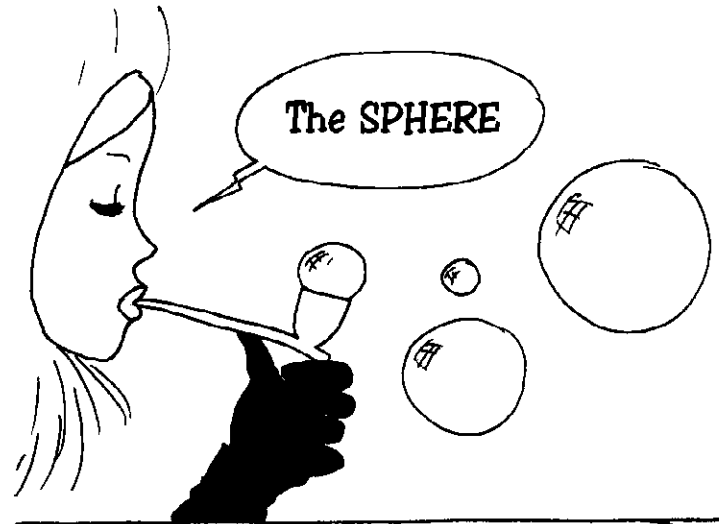
No, that doesn't work because it no longer runs through itself, it develops outside the space that it occupied in the beginning.

However a **DISC**, a **SURFACE** element, IS contractile.

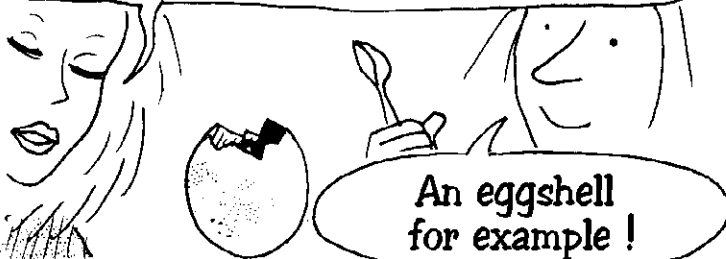
This disc is a SURFACE element, so is a TWO DIMENSIONAL object. OK. So what TWO DIMENSIONAL object is to a disc as a circle is to a segment ?



The SPHERE



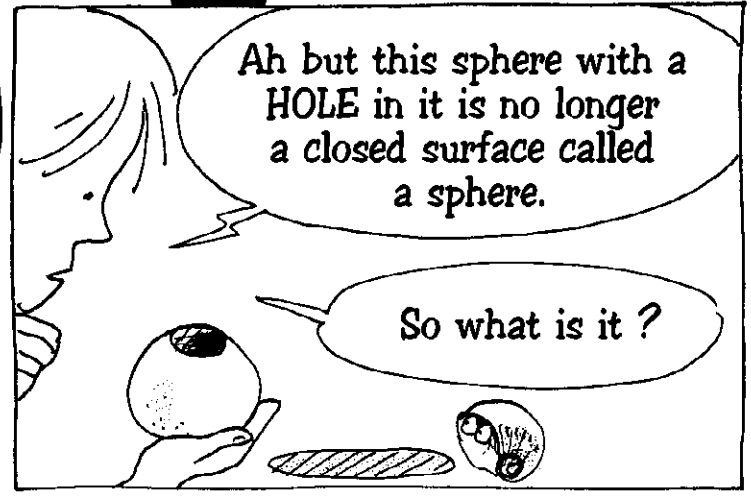
To contract a closed curve you have to break it. Same thing for a sphere or an object of the TYPE sphere.



An eggshell for example !

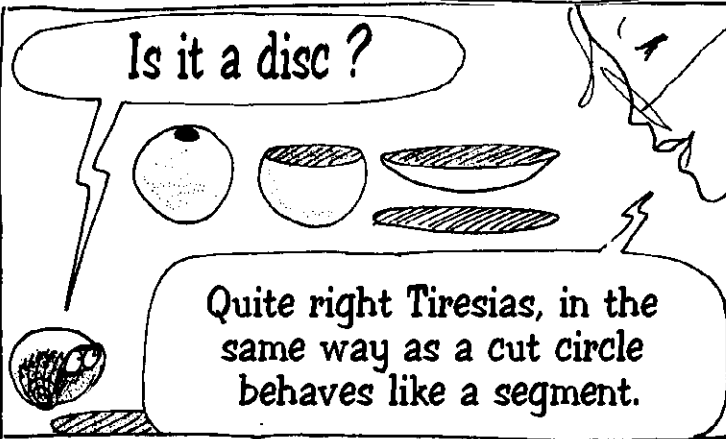
Ah but this sphere with a HOLE in it is no longer a closed surface called a sphere.

So what is it ?



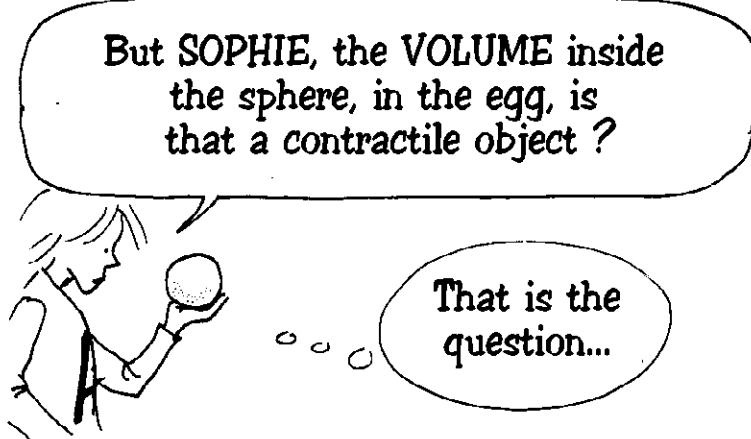
Is it a disc ?

Quite right Tiresias, in the same way as a cut circle behaves like a segment.



But SOPHIE, the VOLUME inside the sphere, in the egg, is that a contractile object ?

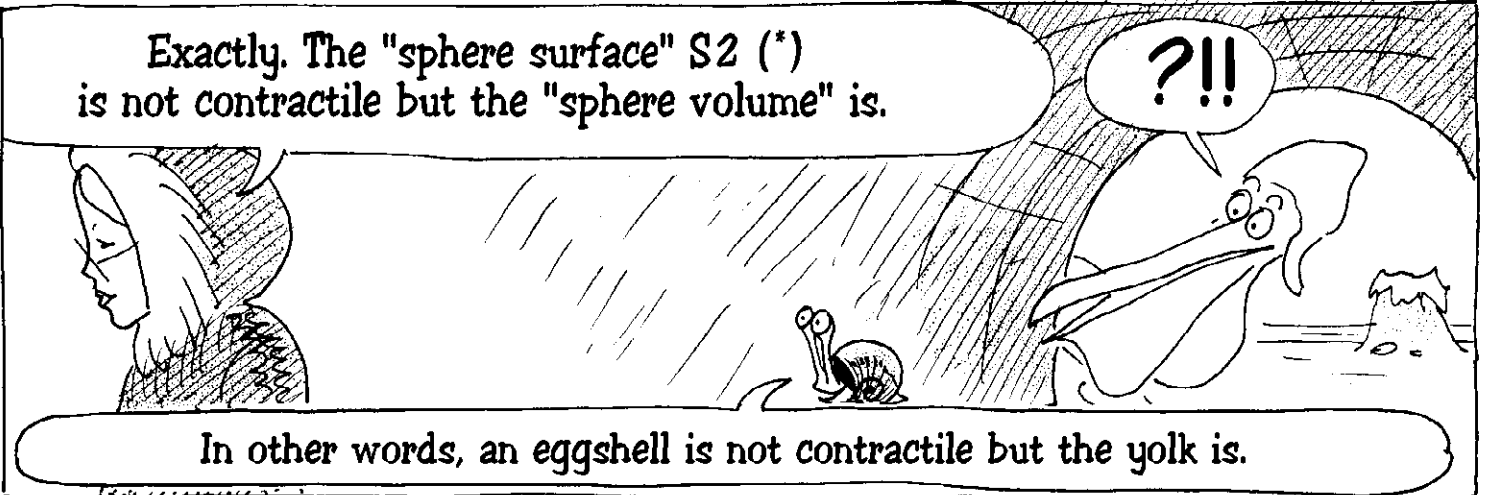
That is the question...



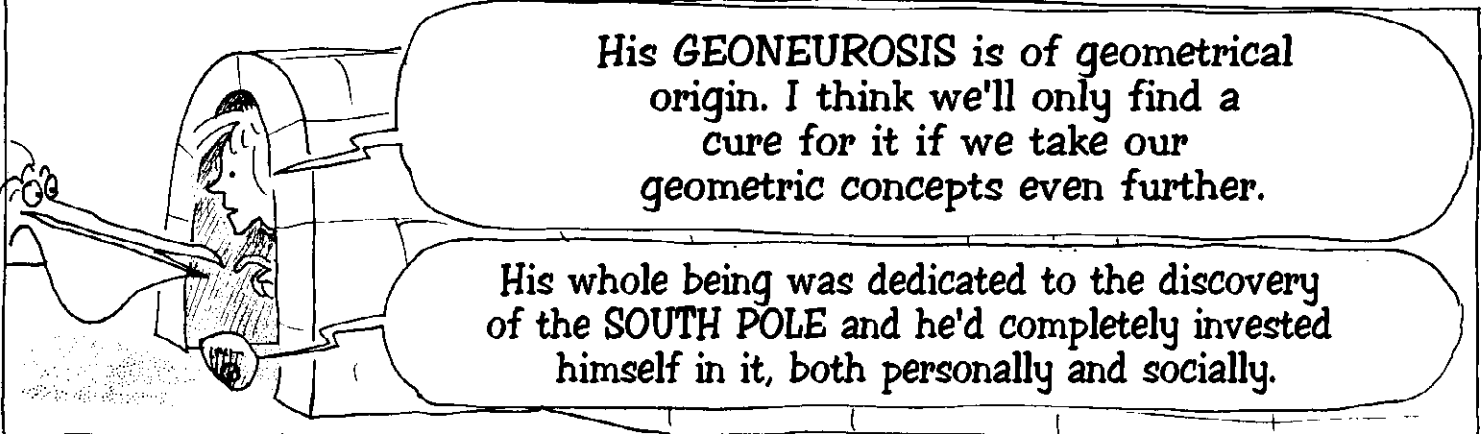
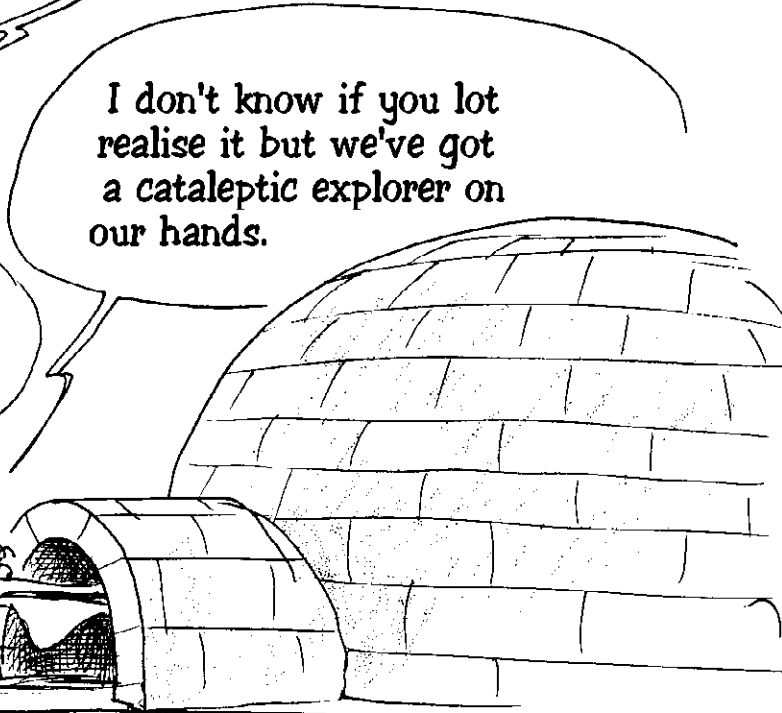
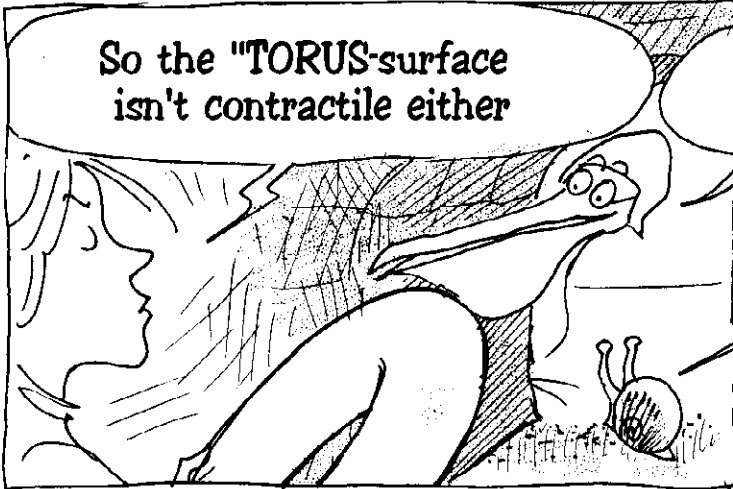
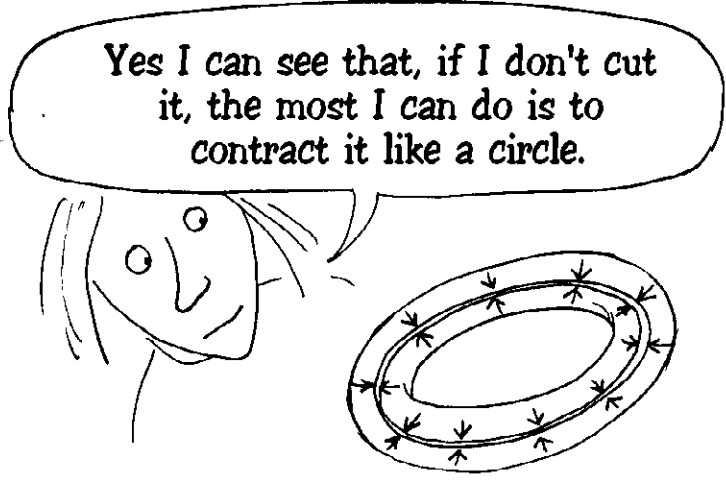
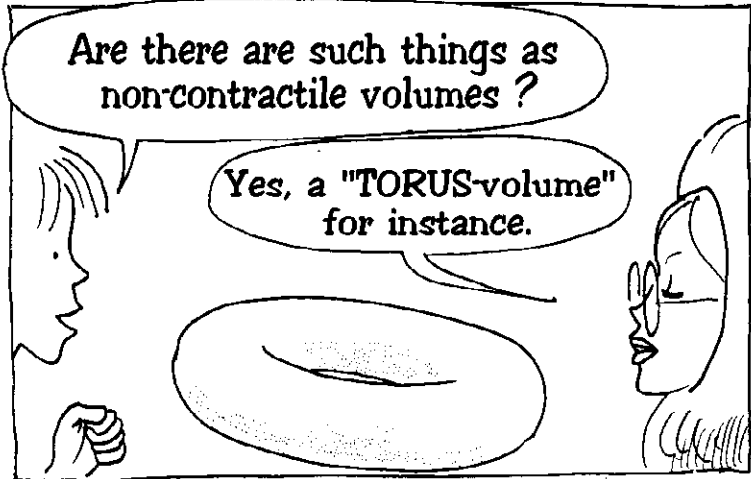
Exactly. The "sphere surface" S^2 (*) is not contractile but the "sphere volume" is.

?!!

In other words, an eggshell is not contractile but the yolk is.



(*) See: HERE'S LOOKING AT EUCLID.



Alas yes, his misadventure has brought him face to face with a situation he can't handle

Very nice, but the only real solution is to find out where the blinkin' South Pole has gone.

An sudden, brutal calling into question of his Self !

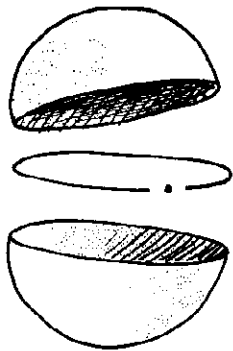
CELLULAR DECOMPOSITION

Every geometrical object will be decomposed into elements, **CONTRACTILE** cells of all dimensions: **POINTS, SEGMENTS, SURFACES, VOLUMES** etc.

So what dimension does a **POINT** have?

By extension we can say that a **POINT** has **ZERO** dimension.

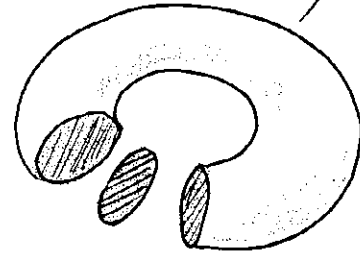
And to decompose a circle you just have to consider it to be a segment closed on itself by a **POINT**. If I remove the point, the segment remains.



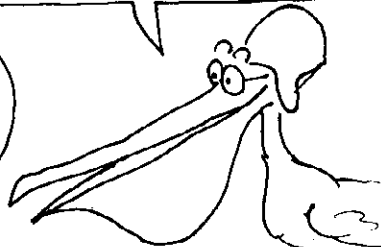
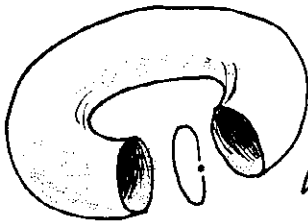
A "SPHERE SURFACE" S^2 can be decomposed into two caps and a segment closed by a point.



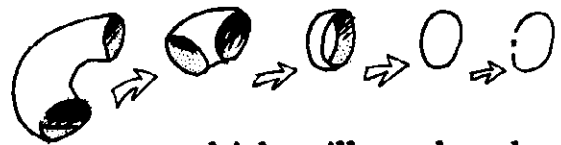
A "TORUS VOLUME"? Well I just need to cut it with a disc



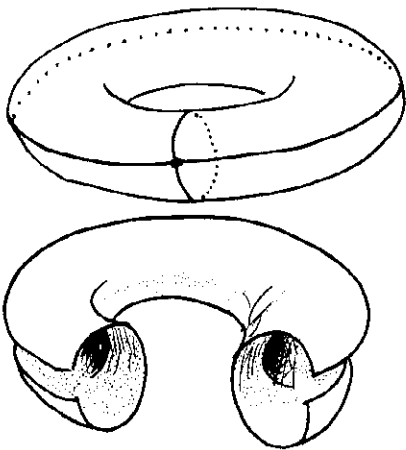
And for a "TORUS SURFACE"... I cut it with a circle which itself is cut at a point



The torus cut in this way will contract as a circle:



which will need to be decomposed into a segment and a point in its turn



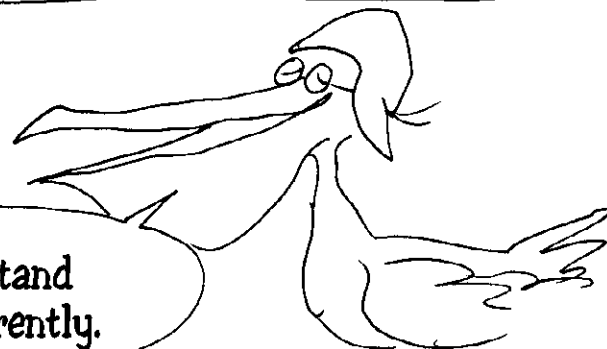
Here is another solution with one point, two segments and one face, where all the elements are contractible.



Ok, but what good is all that to us?



To help understand the world apparently.



THE EULER-POINCARÉ CHARACTERISTIC



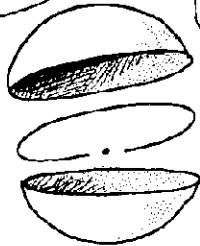
With the object decomposed in this way, we will create a number X , equal to the number of points, less the number of segments, plus the number of contractile surface elements, less the number of contractile volumes (*), and we'll call this number X , the EULER-POINCARÉ CHARACTERISTIC

So for the circle $X = 1 - 1 = 0$

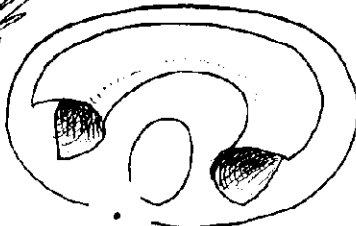
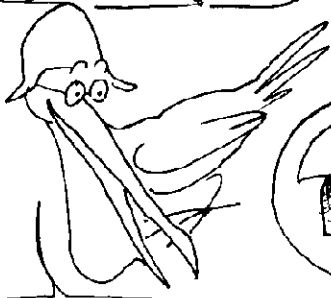


1 point, 1 segment

For the SPHERE SURFACE
 $X = 1 - 1 + 2 = 2$



One point, one segment, two caps

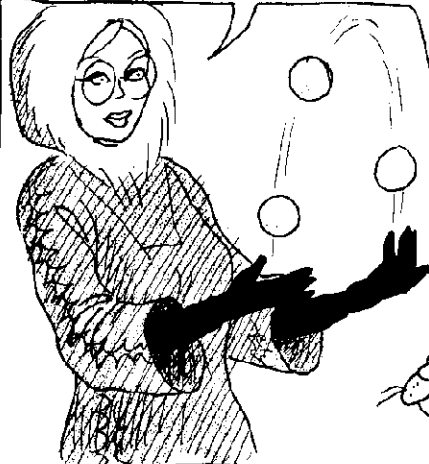


Let's see, for the torus-surface, one point, two segments, one surface element
 $X = 1 - 2 + 1 = 0$

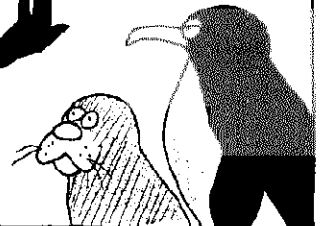
That is to say 1 point, 2 segments and 1 contractile surface element.



The characteristic of the SPHERE-VOLUME is obviously -1 , whereas that of the TORUS-VOLUME is $1 - 1 = 0$ (see the drawing on the top right of page 14)



top right of page 14)



(*Which immediately extends to a number of dimensions superior to three (it's an alternate sum)

Now listen carefully: the characteristic X is **INDEPENDENT OF THE DECOMPOSITION MODE** (in contractile cells)!!

For example, this closed curve has been cut into eight segments linked by eight points but its characteristic is still nil.

It certainly is.

Let's look at this decomposition of a sphere: 4 summits, 6 segments, 4 faces, so I've got $X = 4 - 6 + 4 = 2$

And here, 8 summits, 12 segments, 6 faces so $X = 8 - 12 + 6 = 2$

You can try in any way you want, you'll still end up with 2

Well doggone!

Astonishing!

Here's a useful theorem: if an object is the union of two objects, its characteristic is the sum of the two objects that compose it.

The Management

The Torus-Volume has a characteristic nil

If a handle is added, a unit is being added to the characteristic.

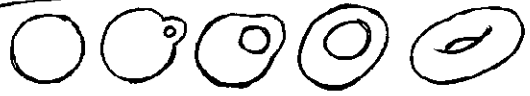
By extension the FOUASSE-VOLUME (*) will have a characteristic equal to the number of holes less one unit.

I suppose that it's the same for a FOUASSE-SURFACE ?

* Fougasse: An olive oil based bread made in southern France

Not at all ! A FOUGASSE-SURFACE can't contract like a disc with N holes, be serious !

Oops. I blew it.

 We can go from a SPHERE-SURFACE (characteristic 2) to a TORUS-SURFACE (characteristic zero) by adding a handle, meaning that the handle reduces the characteristic of a surface by 2 units.

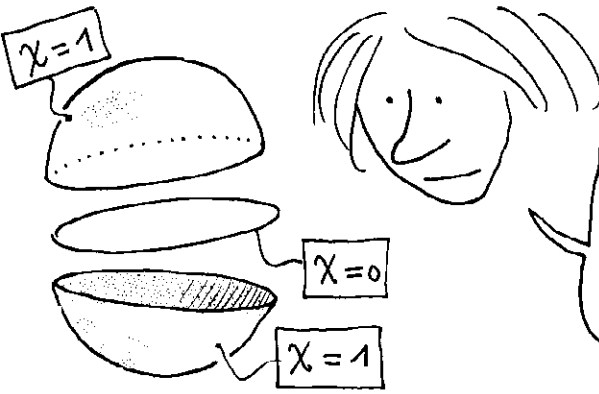
So the characteristic of the FOUGASSE-SURFACE is equal to 2 less twice the number of holes !

The surface of a piece of Gruyère cheese with N holes is made up of N sphere-surfaces plus the exterior of the sphere. So its characteristic is $X = 2(1+N)$

So to build a GRUYERE-VOLUME, we start with a full sphere ($x=1$) and we remove N ensembles SPHERE-VOLUME + SPHERE-SURFACE ($X+2-1=1$). So the characteristic of the GRUYERE-VOLUME is equal to $(1+N)$

Yeah but surely you don't think that you're going to cure poor old Amundsen of his geoneurosis with this sort of nonsense !

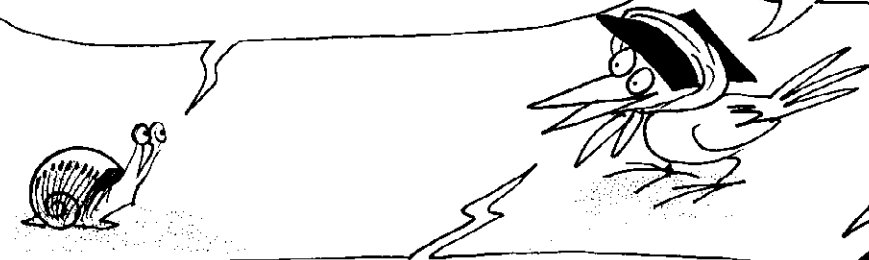
THE WORLD IN WHICH WE LIVE



We can calculate the characteristic of a sphere S^2 by considering it to be the union of two hemispheres and an equator, which gives $\chi = 1+1+0 = 2$

In "HERE'S LOOKING AT EUCLID" we presented the concept of a **HYPERSPHERE S^3** , with three dimensions, a three dimensional space completely **CLOSED ON ITSELF**

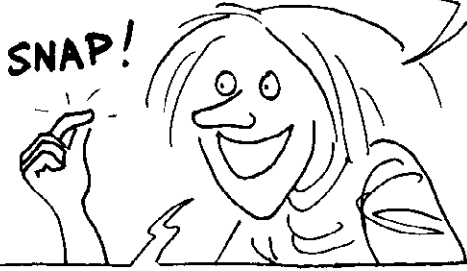
Let's calculate the characteristic of this **Hypersphere S^3** . As we saw in "HERE'S LOOKING AT EUCLID" the equator (*) is a sphere S^2 whose characteristic has a value of 2.



Are you nuts?

So our hypersphere S^3 is therefore made up of two contractile volumes, each counting for -1.

$\chi = -1 - 1 + 2 = 0$



*Which separates the object into two similar elements

So the characteristic of a hypersphere S^3 is nil !

Right, let's move on to a hypersphere S_4 , with four dimensions



That is, a hyperspheric space S_3 evolving cyclically in time (*). This hypersphere S_4 will have as equator a hypersphere S_3 , and the two hemispheres, both counting for 1

So the characteristic X in this space-time, of the hypersphere S_4 , will once again be $1+1+0 = 2$

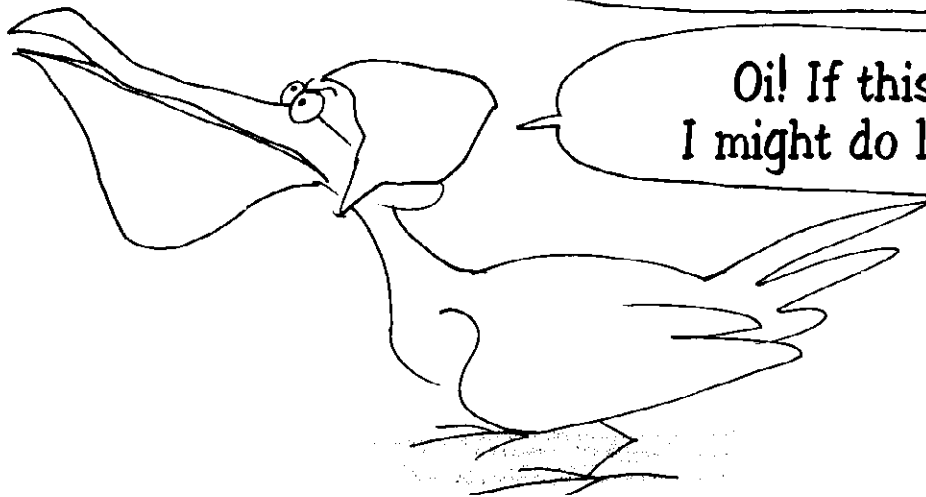
If you take an S_5 hypersphere with five dimensions, its characteristic will again be nil and its equator will be an S_4 hypersphere.



And so on...The Euler-Poincaré characteristic of a hypersphere S_n is 2 if N is EVEN, and 0 if it is ODD.



Oi! If this carries on I might do like Amundsen.

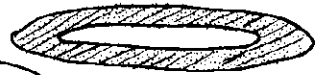


(*) See BIG BANG and FRIEDMANN's models on page 64

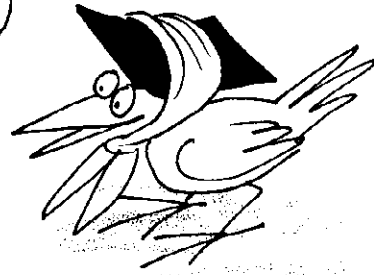
So this Euler-Poincaré characteristic has helped us put a bit of order into the jungle of geometrical objects



So the end of a cylinder is topologically identical to a disc with a hole in it, and its characteristic is nil.

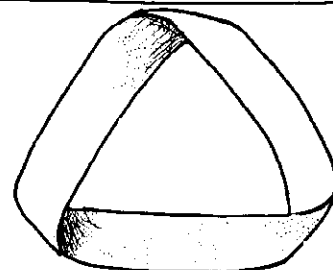
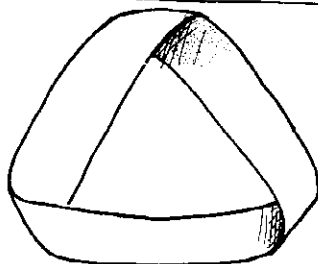


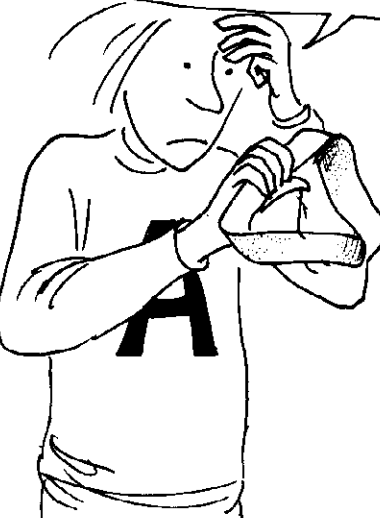
But what do you think of this object ?



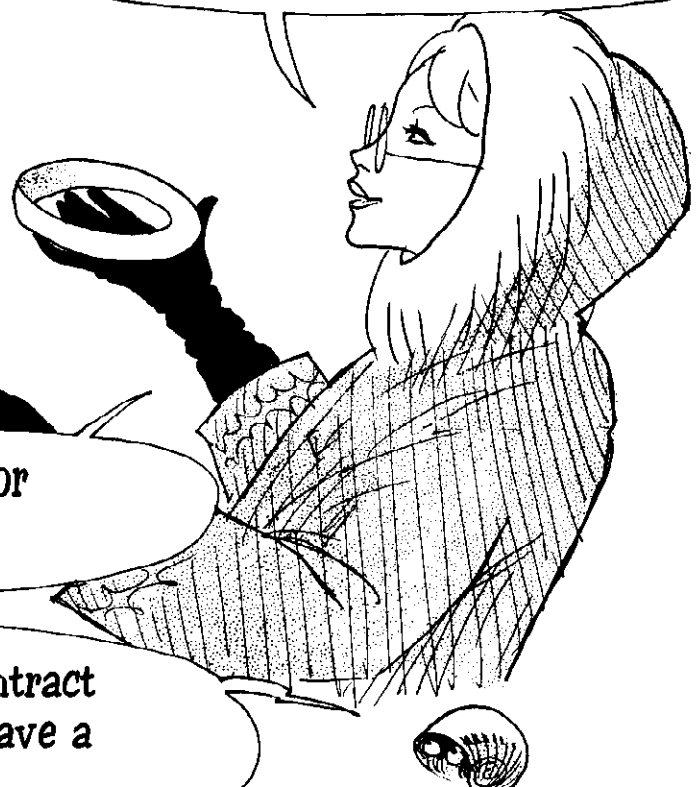
A MOEBIUS STRIP, which has only one side. As we can't give it a BACK or a FRONT we say that it is INORIENTABLE.

In fact any strip that has an ODD number of HALF-TURNS are Moebius strips and INORIENTABLE. But these two strips seem different somehow...






It doesn't matter how I turn them, I can never get them to be the same



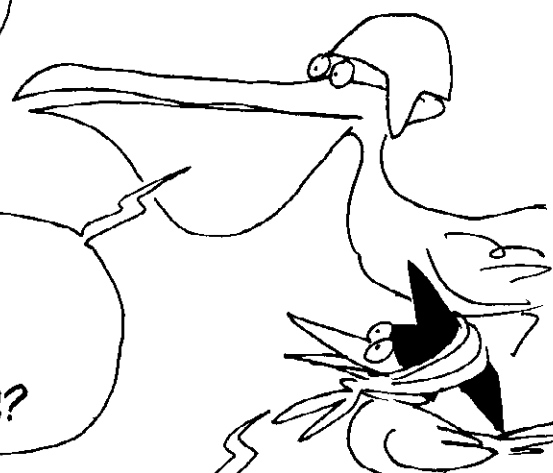
They're not **TURNUED** in the same **DIRECTION**. In fact one is the mirror of the other; we say they are **ENANTIOMORPHIC**.

Just as my left hand is a mirror image of my right hand.

All these bands, which can contract according to a closed curve, have a characteristic equal to 0.



Of course, **INORIENTABLE SPACES** with N dimensions (*) exist too.

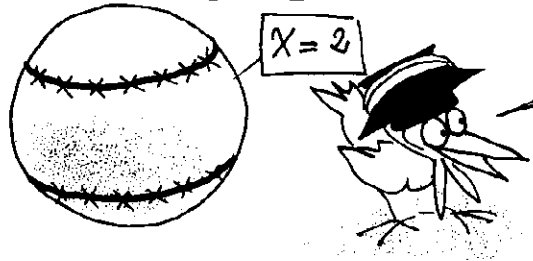
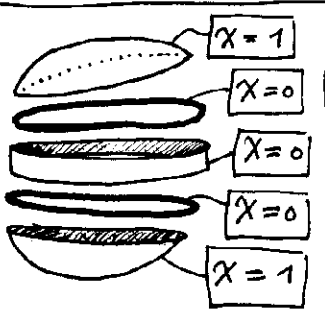


A **MOEBIUS STRIP** is an **INORIENTABLE** surface which has an **EDGE**. Are there such things as **INORIENTABLE SURFACES WITHOUT AN EDGE, CLOSED ON THEMSELVES?**

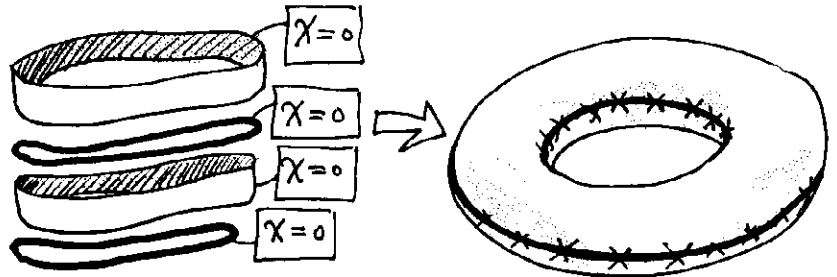
Answer in the next chapter

EDGE ON EDGE

A CLOSED CURVE (decomposable into a segment and a point) has a characteristic nil. The same for a STRIP, bilateral or unilateral, which can be contracted according to a closed curve (see theorem, page 17) When a bilateral strip is closed with two discs along two closed curves, we have made a SPHERE-SURFACE S^2 (with two dimensions)



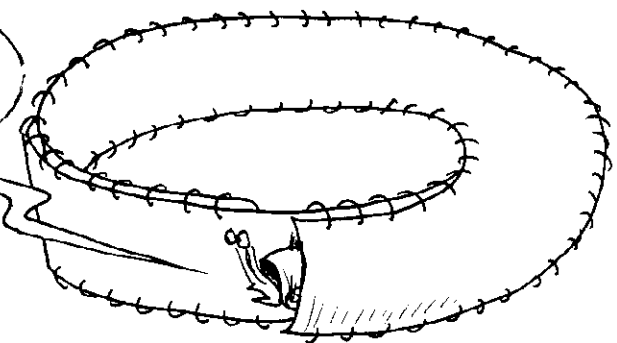
We could also stitch two bilateral strips one on the other, along two closed curves and we'll get a TORUS-SURFACE T^2



So normally I should be able to stitch two Moebius strips along just ONE CLOSED CURVE



Hey!! That's tight



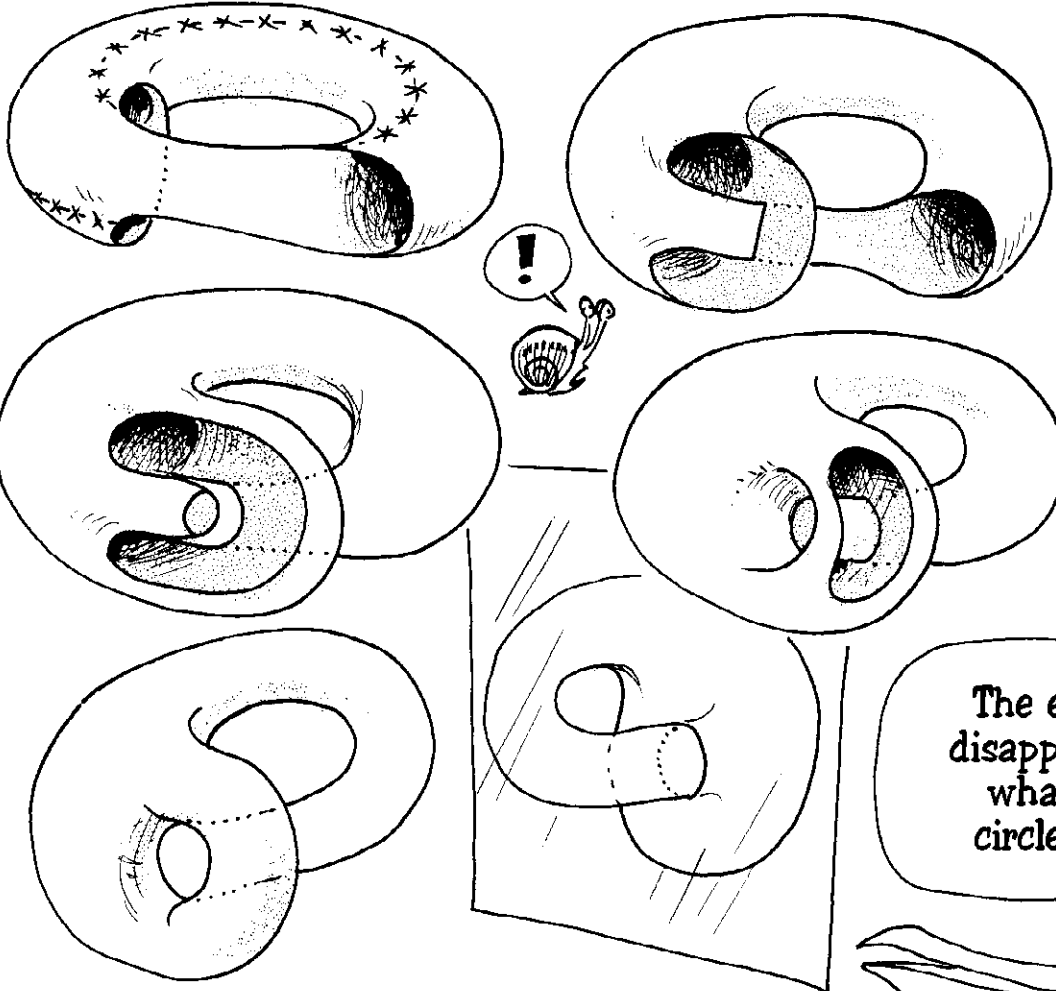
We'll have to use some TRANSVERSINE (*)

TRANSVERSINE !?



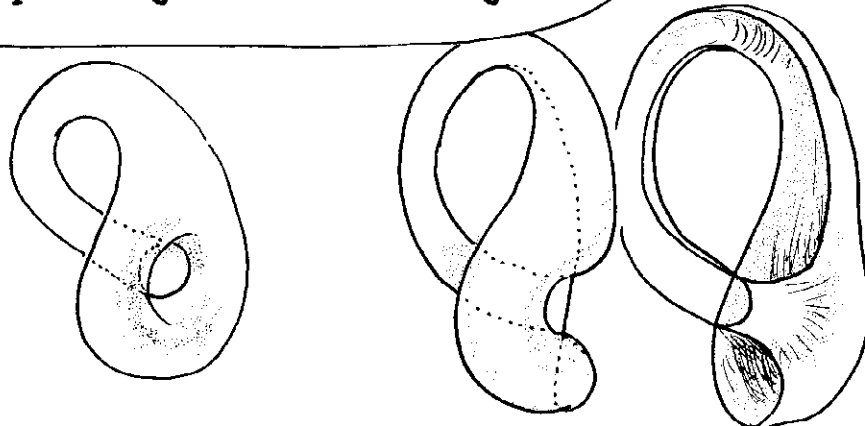
(*) TRANSVERSINE is extracted from the shells of HOMOMOLES

If we smear **TRANSVERSINE** on a shell, it starts to grow, according to its edge, tending to form a closed surface but allowing that surface to **GO THROUGH ITSELF!**



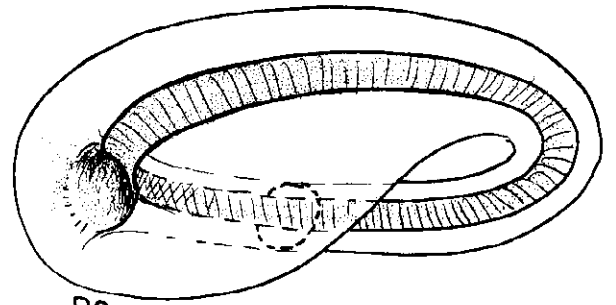
The edge has disappeared but what's that circle thingy?

It's the **AUTO-INTERSECTING CURVE**, which isn't an **EDGE**. You can verify this with this **KLEIN BOTTLE**, where the surface develops everywhere continually.

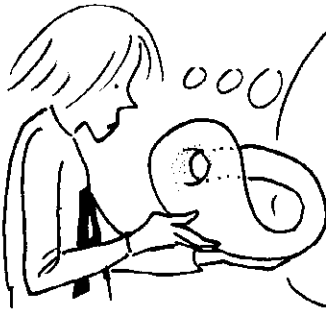


Two cross sections

Its characteristic is nil because it's made up of two Moebius strips ($x=0$) and a closed curve ($x=0$). It isn't easy to find your way round one of these.



Of course, if you find a Moebius strip on a surface it means it only has one side.



Tell me Tiresias, couldn't we find a Moebius strip on your shell somewhere?

Don't start you two.

er..!

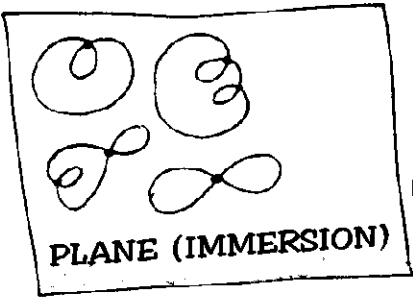
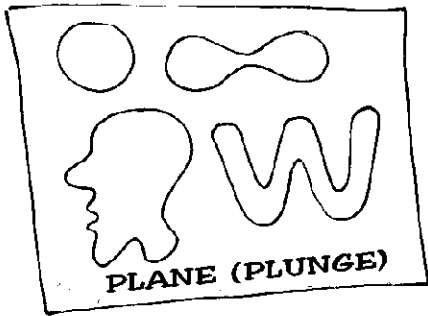
It's a pretty strange surface all the same.

Up to now we've only touched on surfaces that don't cut each other in their normal form, such as a SPHERE. Surfaces that cut each other in our space are called IMMERSIONS

Immersion's ?

PLUNGES AND IMMERSIONS

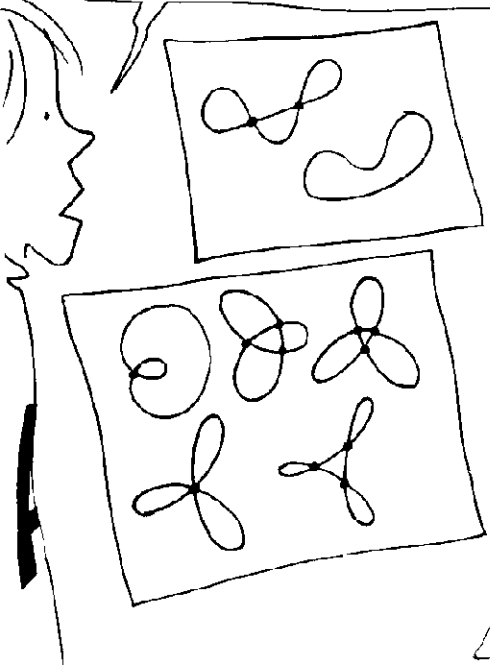
A closed curve, that is to say a unidimensional geometric, with no accidents on the way and whose only characteristic is to neither have a beginning nor an end, can be situated in an infinity of ways on a plan.



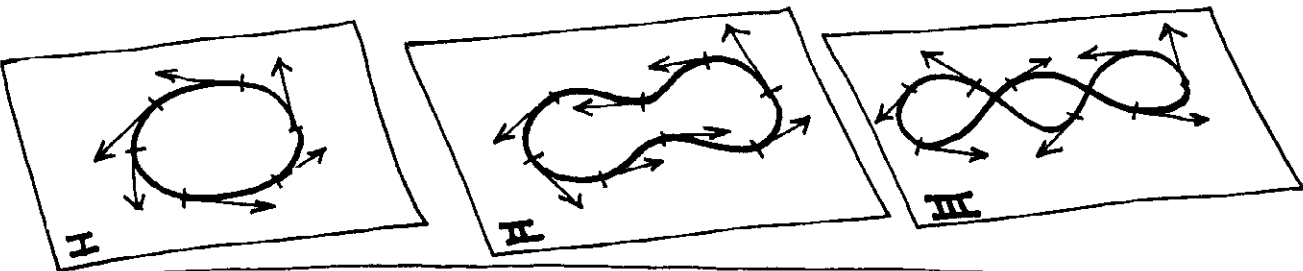
When it doesn't cut itself, I would say that it has **PLUNGED INTO THE PLANE**, otherwise I would say that it is **IMMERSED (*)**

I suppose they're characterised by the number of intersecting points

No, because if I continually deform these curves I can make the **POINT COUPLES** appear and disappear. But what will stay the same is the **NUMBER OF TURNS**.



Look, I'm making a vector remain tangent to the curve



By regular deformation (without broken lines) in the PLANE, I can go from curve I to curve III. In doing this we have the total rotation of the arrow (360°) when crossing each curve

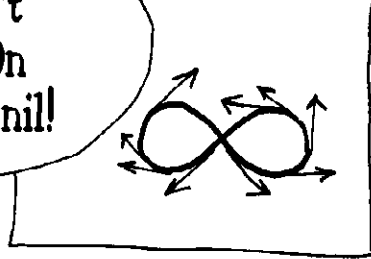
It is **REGULAR HOMOTOPIA** in a PLANE. It keeps the number of turns of the arrow tangent to the curve.



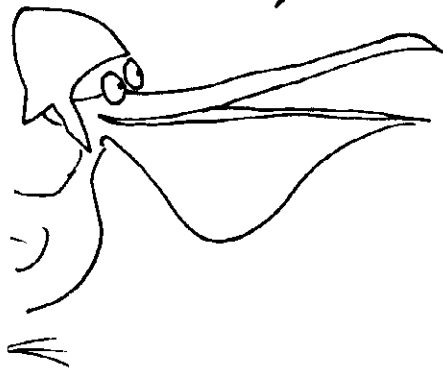
Well I've tried everything and I can't change this **EIGHT** into a **CIRCLE** !..



That's normal. The arrow doesn't do the same number of turns. On the **EIGHT**, the algebraic sum is nil!



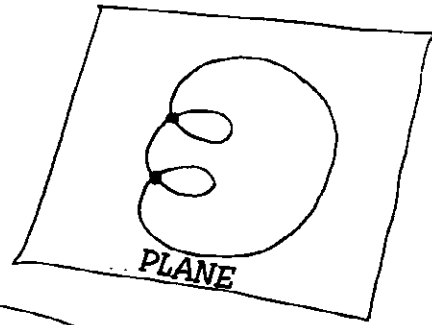
Given this rule of closed curve deformation (continuity, regularity), in a surface, some things are **POSSIBLE** and others are always **IMPOSSIBLE**



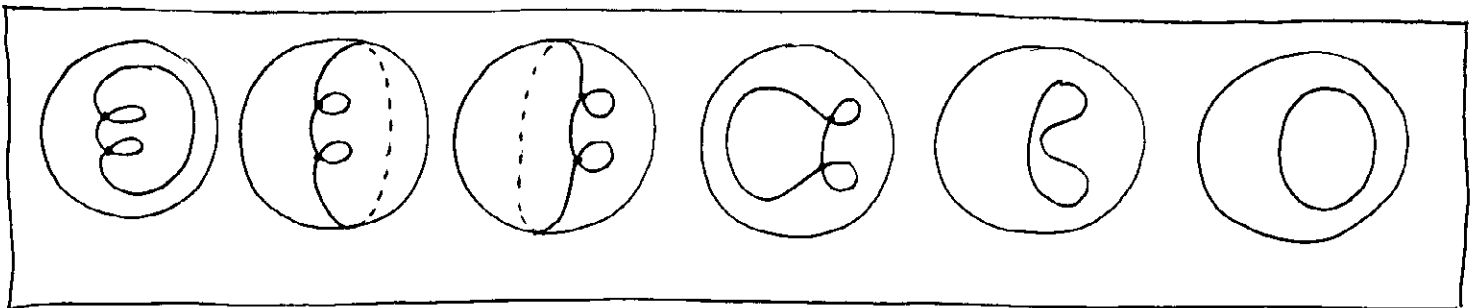
It's not so simple !



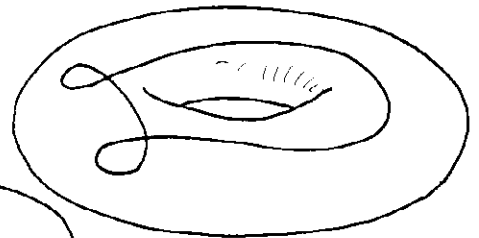
It depends on the space used to represent the object. Look at this curve for instance. On a PLANE there is no way to get rid of the two double points.



But on a SPHERE...

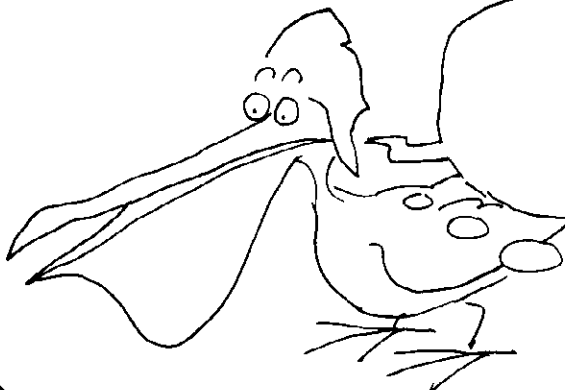


So some things that seem impossible in such a REPRESENTATIONAL SPACE (here the PLANE) become possible by changing this space, with a different topology. And vice-versa.



In this plane, the curve is easily undone but you can't do it if it is represented on a torus

But Tiresias, in our SPACE-TIME there are things that are definitely possible or definitely impossible aren't there?



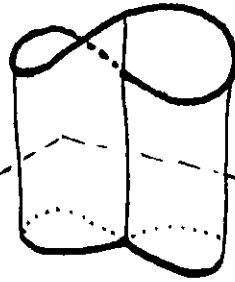
that's worrying...

Do you know the topology of our spacetime ?

Er...no...

We just live appearances... and even...

The closed curve's intersection points only hold up through their mode of representation on a surface. A bidimensional image is only a projection.



Fundamentally there is only one object in all this: **THE CLOSED UNIDIMENSIONAL CURVE**

In a space represented by 4 dimensions, the **KLEIN** bottle no longer cuts through itself !

So by changing the representational space I can do anything. Change a Klein bottle into a sphere for instance ?

No, there are characteristics that remain **INDEPENDENT OF THE REPRESENTATIONAL SPACE**

TOPOLOGY

Such as the Euler-Poincaré characteristic, orientability, closedness.

For objects of one dimension it all comes down to: A CURVE MUST BE OPEN OR CLOSED

So how's Amundsen ?

Nothing, still the same...

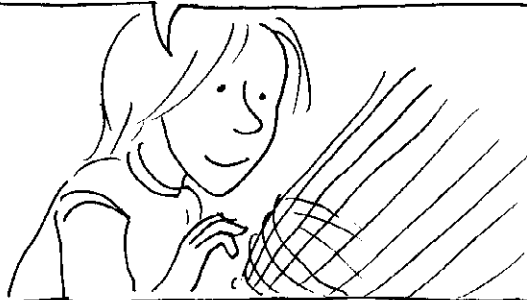
GEONEUROSIS? No, I diagnose a TOPONEUROSIS

Our mental structures, our LOGIC, our perception of the world, rest on geometrical foundations, which could give way at any moment.

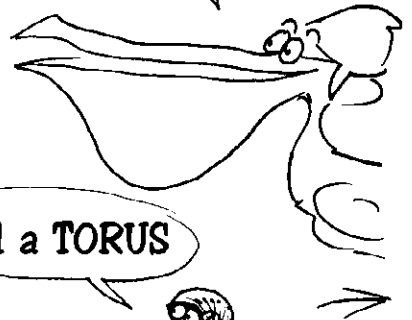
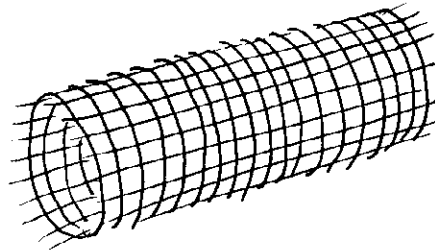
I we can't bring back a minimum of coherence to our friend's view of things he'll continue to refuse the sensorial world.

BASKET WEAVING

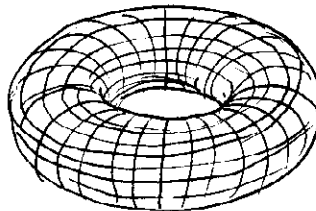
I've found another good way of representing surfaces: **BASKET WEAVING**



Well that is obviously a cylinder.



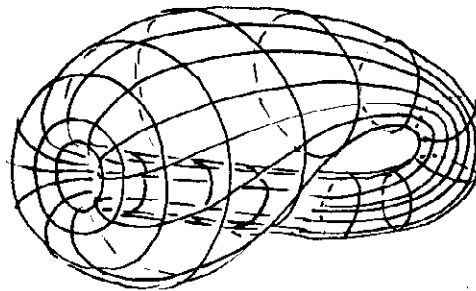
And a **TORUS**



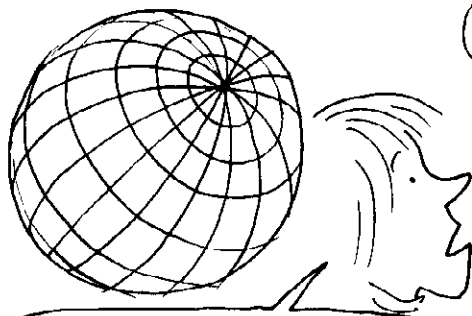
Hmm, it isn't so easy to make a sphere



A **KLEIN** bottle

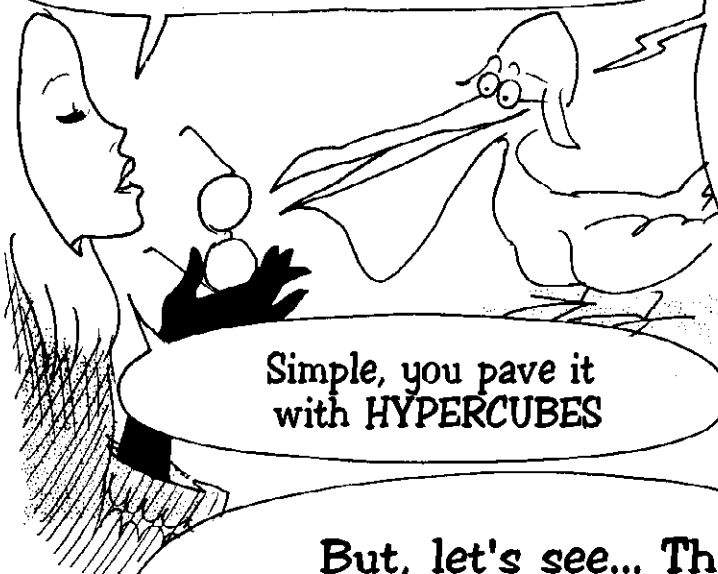


For the sphere you have to introduce 2 **POLES**.



But I don't get it, I didn't need them for the torus or the Klein bottle...

The Euler-Poincaré characteristic gives you the number of poles you need to **WEAVE** your surface. For the **TORUS** or the **KLEIN** bottle it's zero. For the sphere it's 2.



This concept can be extended to **HYPERSURFACES** of course, space with 3,4..N dimensions.

Unless we're mistaken the universe, according to the **FRIEDMANN (*)** cyclic model, is an **S4** hypersphere. So I can see that we can **PAVE** a three dimensional space using cubic structures. But what about one with 4 dimensions?

Simple, you pave it with **HYPERCUBES**


But, let's see... The characteristic of an **S4** hypersphere is 2. So our space-time, should show at least one sort of singularity then, a pole?



Hypercubes?
Really...



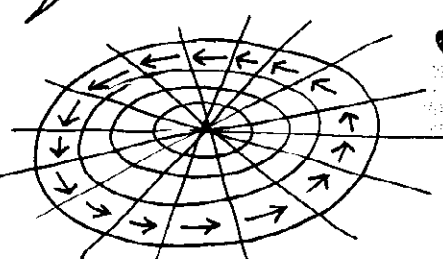
And the **BIG BANG(*)**, what's that then !?!



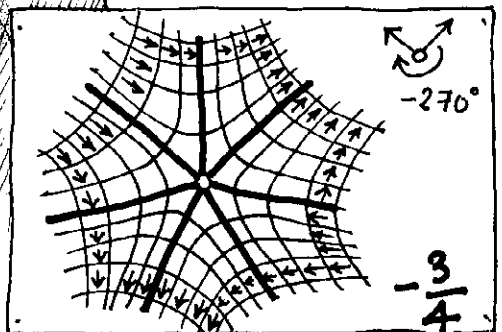
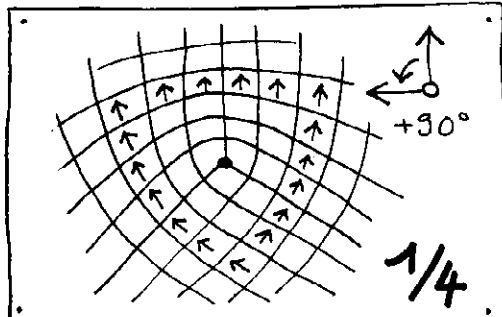
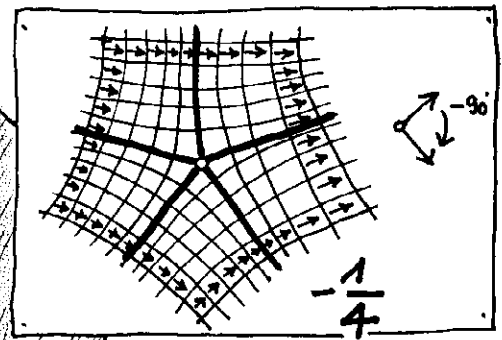
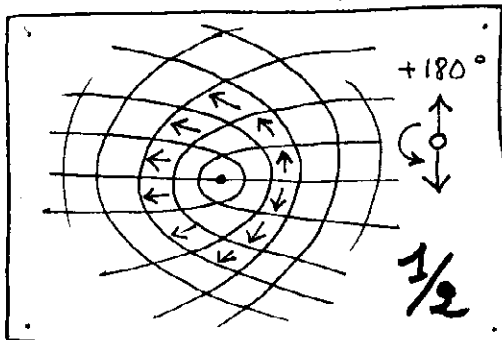
So purely geometric considerations have allowed us to perceive one of the most fantastic aspects of the history of the world, discovered at the same time as the expansion of the Universe

SINGULARITIES

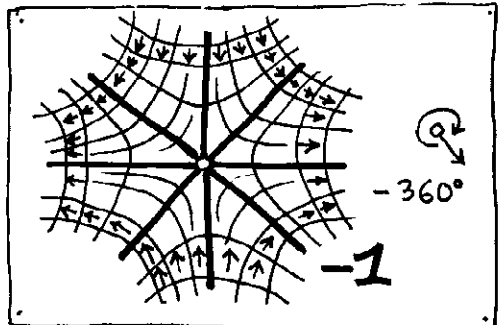
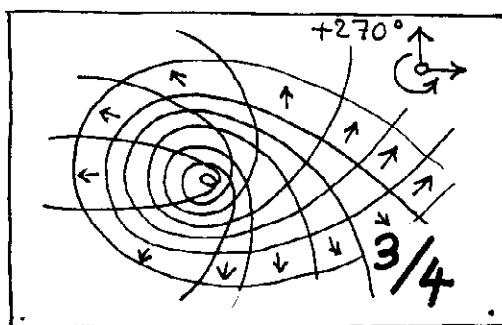
THE ORDER OF SINGULARITY OF A WEAVE is equal to the angle of the arrow's direction, positive or negative, divided by 360° (2π)



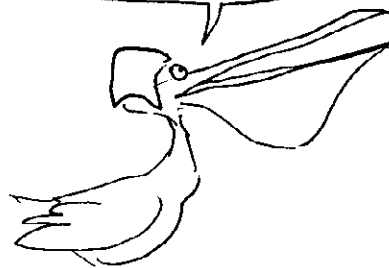
The POLE is 1.



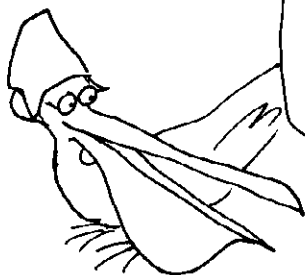
Here, on the left are singularities of a positive order and on the right, of negative order

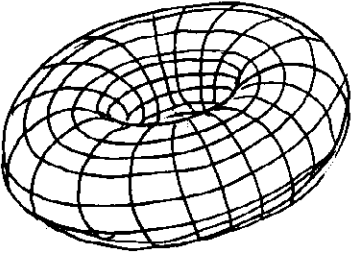


What's the point?

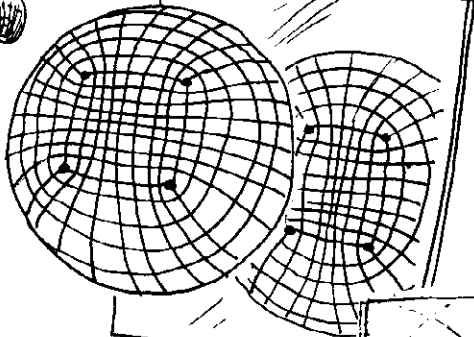


If you weave a closed surface, eventually you will have singularities. The Euler-Poincaré characteristic will be equal to the algebraic sum of the orders of singularities.

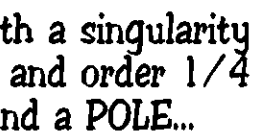




I could weave a TORUS without singularity. That's normal, its Euler-Poincaré characteristic is nil.



And here's a sphere with a grid using eight singularities of order $1/4$...



or with a singularity $3/4$, and order $1/4$ and a POLE...



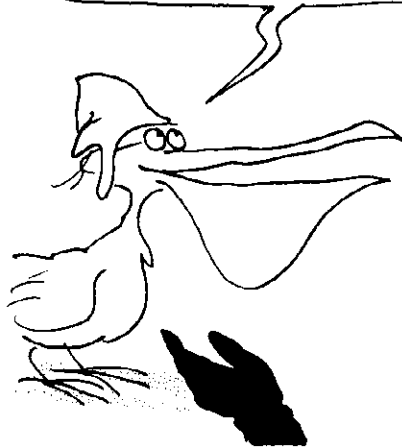
Or with four singularities of order $1/2$



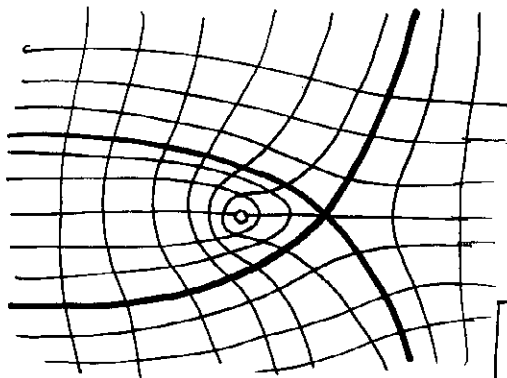
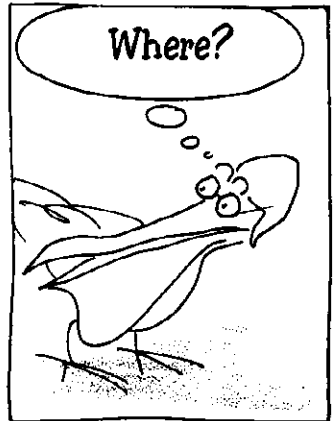
Note :

Those who have read *BLACK HOLE*, pages 14 to 36 will no doubt have noticed the similarity between the drawings of mesh singularities and those concerning POSICONES, NEGACONES and the curve. All these ideas, essentially ANGULAR, are closely linked to the TOTAL CURVATURE of a surface, represented in our space of three dimensions, which is exactly equal to the Euler-Poincaré characteristic multiplied by 360° (or by 2π)

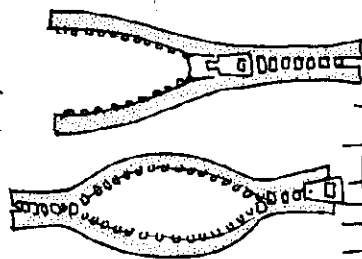
It's a pity that such things are totally useless,
like Greek and Latin.



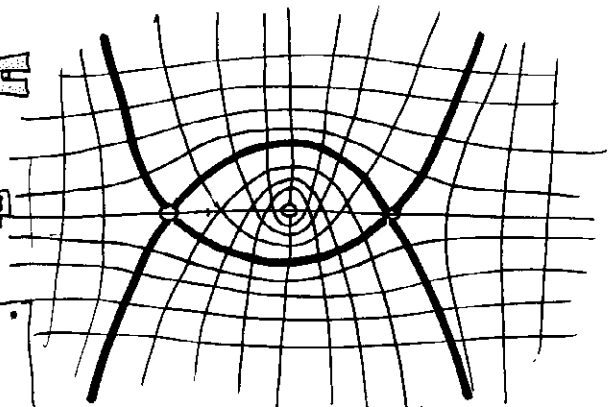
Not at all Leon!
There are lots of
singularities
in nature !



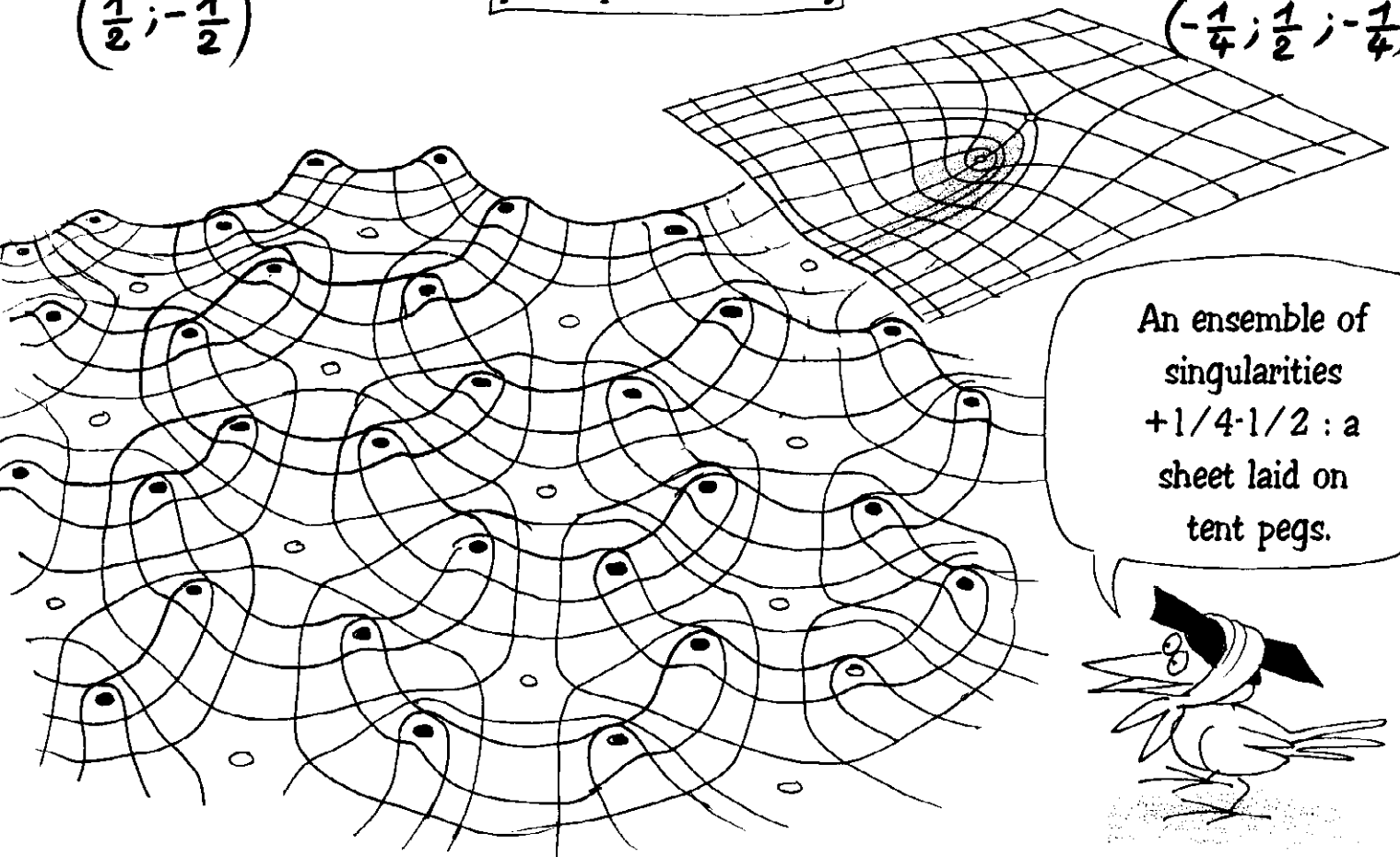
$$\left(\frac{1}{2}; -\frac{1}{2}\right)$$



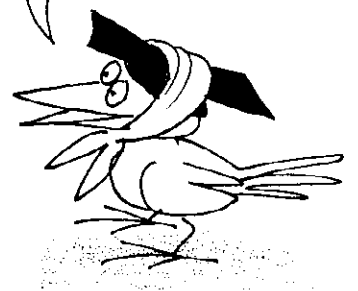
Pull apart a
zip fastener



$$\left(-\frac{1}{4}; \frac{1}{2}; -\frac{1}{4}\right)$$



An ensemble of
singularities
 $+1/4 - 1/2$: a
sheet laid on
tent pegs.

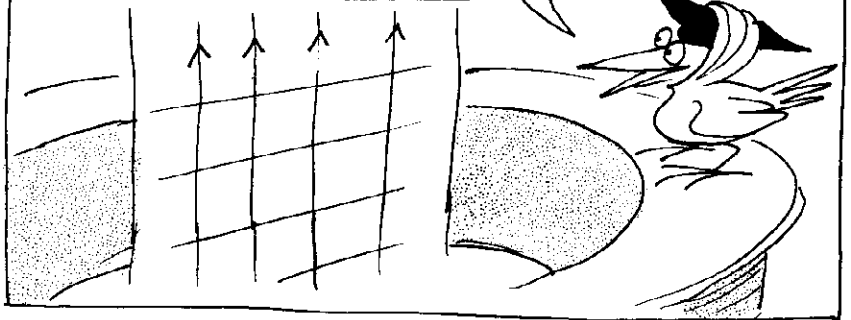


Now what are you making?

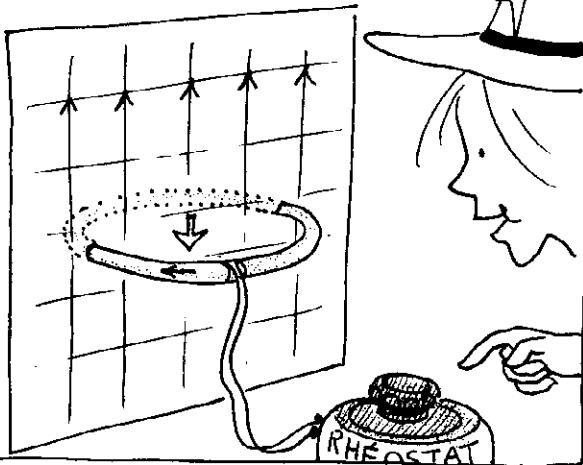


MAGNETIC FIELDS

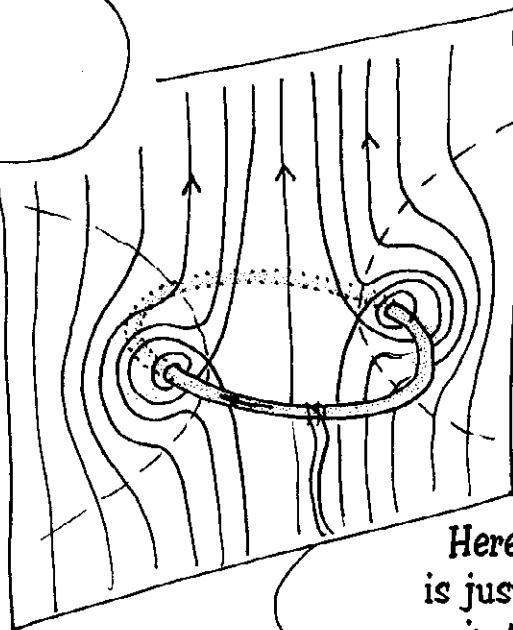
This system produces a **UNIFORM** magnetic field, its lines and fields are simple parallel straight lines.



But if I put a coil into the field it will create another field in its centre going in the opposite direction.

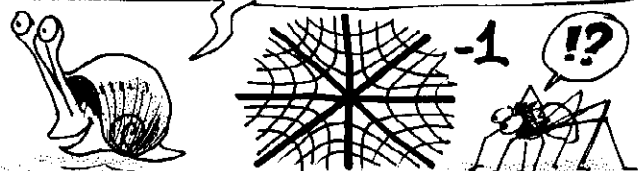
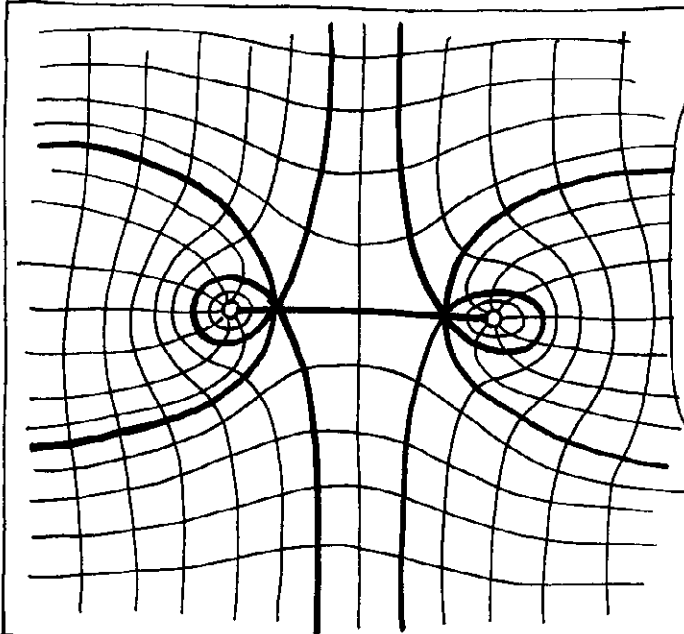


Contact!



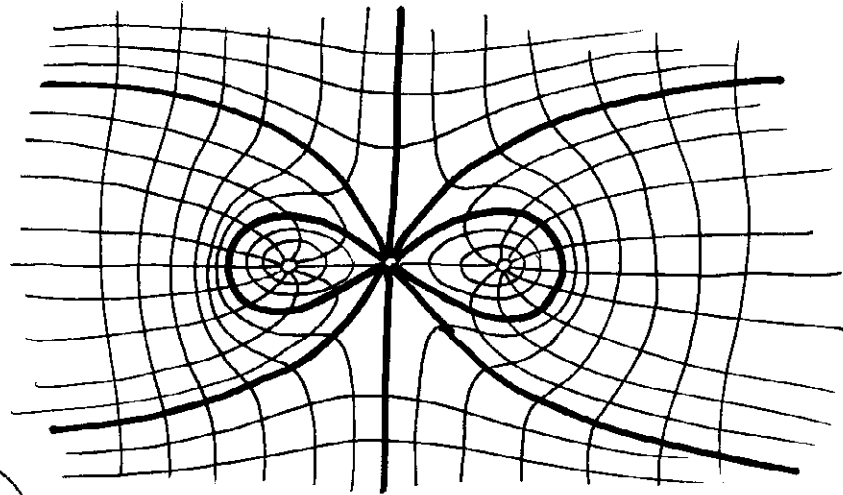
Here the field is just weakened in the centre

Oh! You've made two poles appear (the traces of the solenoid seen from the front in Figure 1) and two singularities of order -1 . The sum making zero. The negative singularities appear where the **B** field is cancelled out.

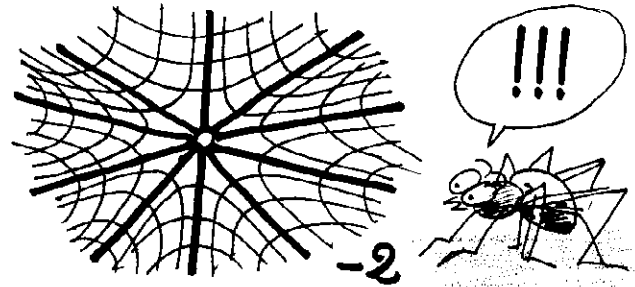


In fact the system has a symmetry of revolution and we've got an example of a mesh with lines of singularity.

I'm now going to increase the current so as to cancel out the value of the magnetic field in the centre of the solenoid.



The two points of the nil field, seen from the front in the drawing, have now joined into one, of the order -2 (an example of CONFLUENCES OF SINGULARITY)



Yes this is fun. Shall we push the field further?

It might be risky and become dangerous.

What are you afraid of Leon ?
That we create irreversible changes
in spacetime? It's only 100 Gauss
after all old fellow.

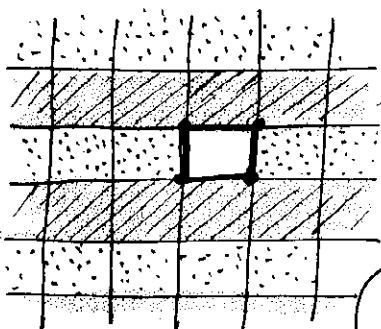
Since **THE SILENCE
BARRIER**, he's had
a real fixation on
magnetic fields.

Superb !

The magnetic field **B** has
inverted in the centre of
the coil. Its singularity
is doubled into two
singularities of order -1 .
We've created a magnetic
VORTEX with toric geometry

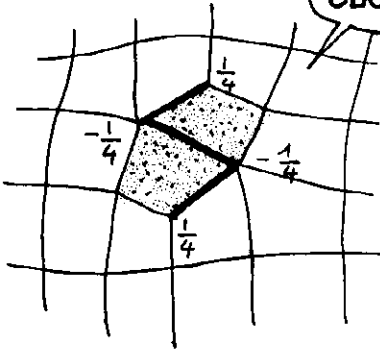
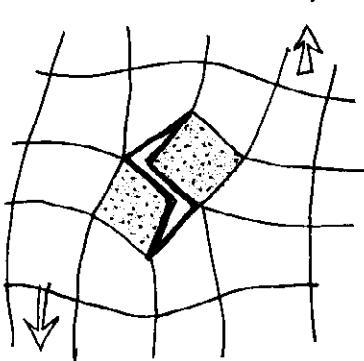
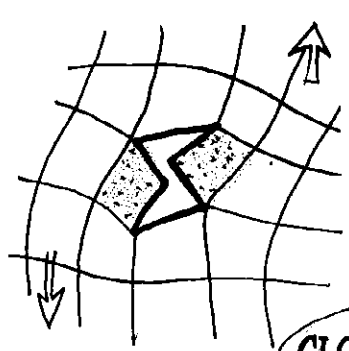
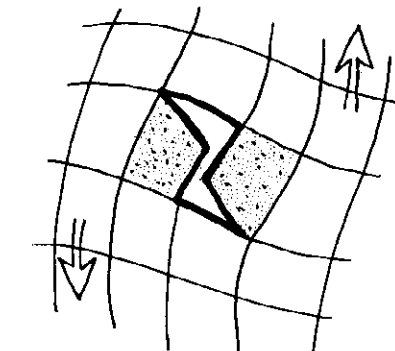
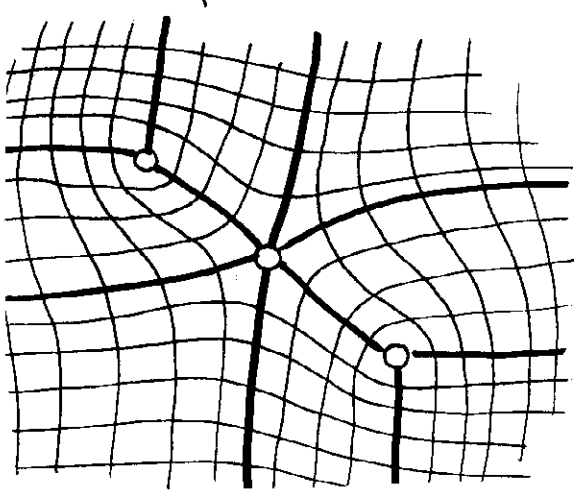
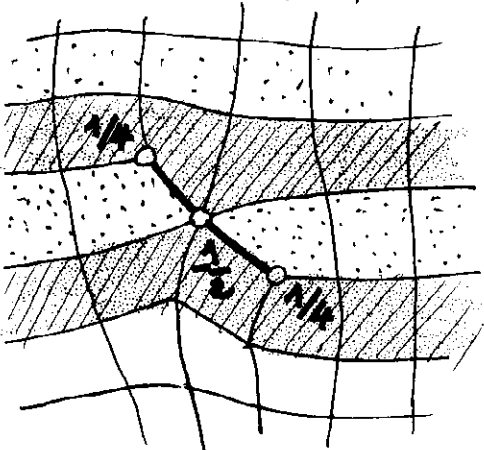
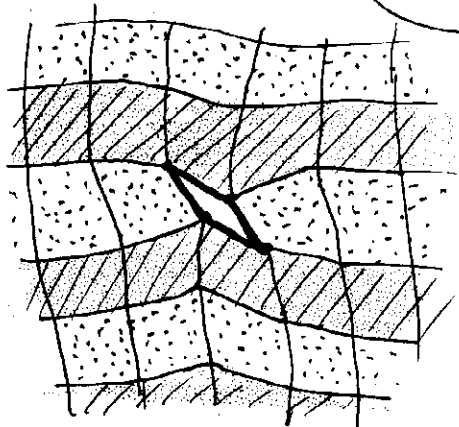
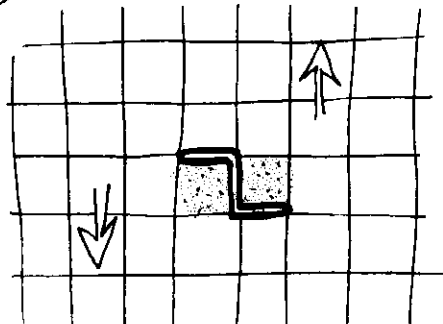
You find meshes and
singularities at all
the crossroads of physics...

CRYSTALS are a mine of singularities. In this top view of a crystal with a square mesh, if we create a FAULT, by removing an element, the hole will be made at a cost one singularity of $-1/2$ and two singularities $1/4$

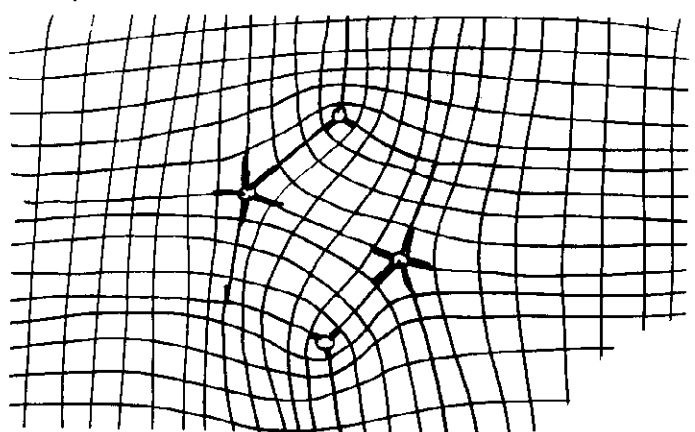


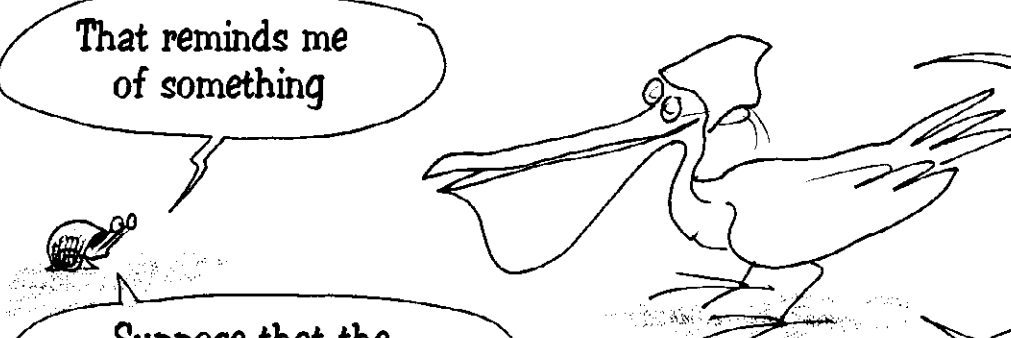
I've removed a tile

A shearing movement will cause a rearrangement of the grid, which requires two singularities of order $1/4$ and two singularities of order $-1/4$



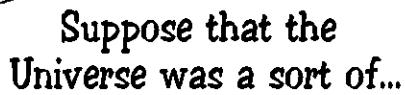
CLOMP!





That reminds me
of something


And what would
that be Tiresias ?



Suppose that the
Universe was a sort of...

...a sort of crystal ?

What if the universe was made up of sorts of slots, **ELEMENTARY PARTICLES** could be the faults or dislocations, combinations of **PAVING (*)** singularities - The movement, or the interactions would correspond to the rearrangements of the whole thing...



For a good idea
that's a good idea !



I...er...

All that follows will be illustrated using LEAFING ANIMATED CARTOONS, sorted by the letters A,B,C and D

The Management

THE BOY SURFACE

Right we've had fun but in the meantime poor Amundsen is still in the soup...

And we still don't know anything about this mad planet with no South Pole



But wait...for there to be only one pole, the Euler-Poincaré characteristic must be equal to 1. It seems to be unilateral...

A

TRANSFORMATION OF A MOEBIUS STRIP INTO A BOY SURFACE

B

DITTO: CURVE-EDGE AND AUTO-INTERSECTION ENSEMBLE

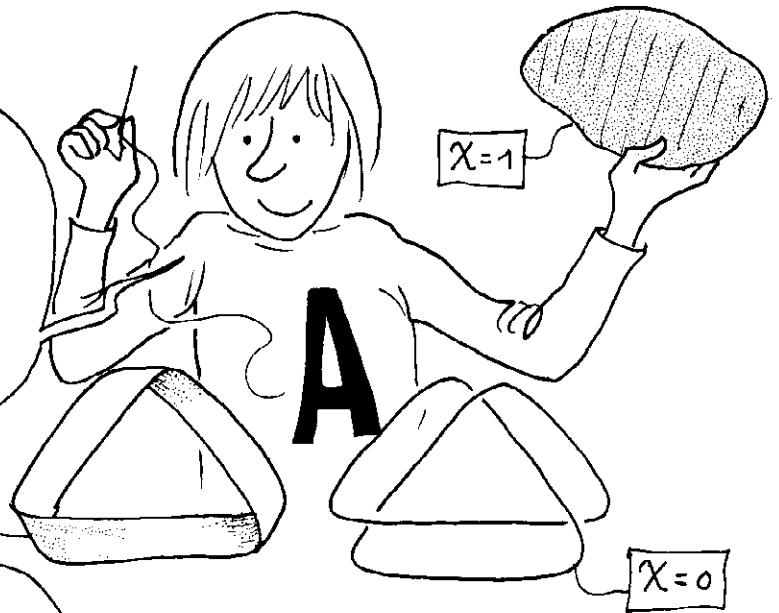
C

MAKING A CONJUNCTION OF ANTIPODAL POINTS

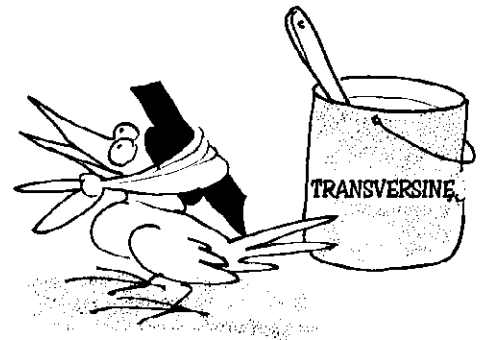
D

APPARENT INVERSION OF TIME

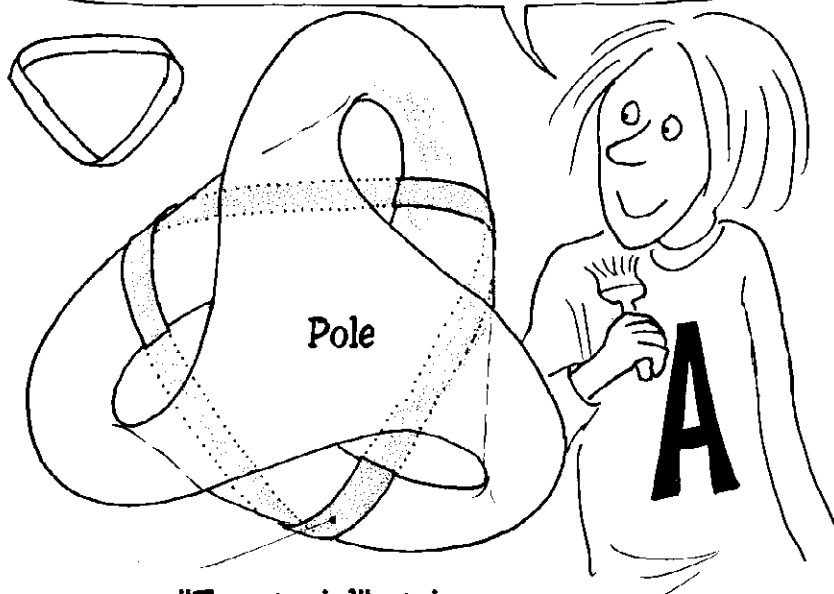
A Moebius strip has a nil characteristic. I could sew it along a closed curve, which also has a zero characteristic, a simple disc for example...



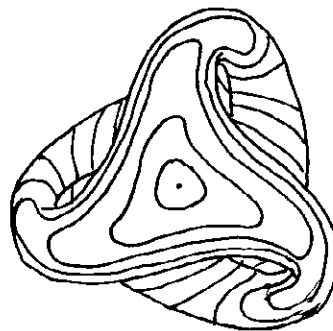
The ensemble will have a unitary characteristic and will be a closed unilateral surface. But instead of stitching it, why not use some **TRANSVERSINE**



The sequence of turning a Moebius strip into a BOY surface can be seen on the drawings A and B. Here's the final object:



"Equatorial" strip

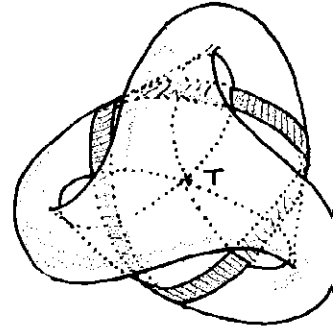


Here are the "PARALLELS" of the BOY surface. It's also the development of the edge of the Moebius strip corresponding to the sequence A.

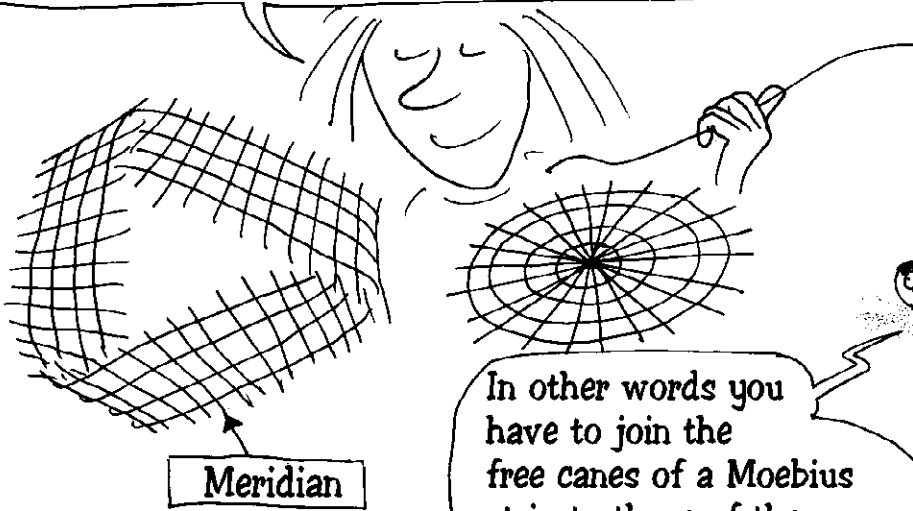


Funny sort of parallels..

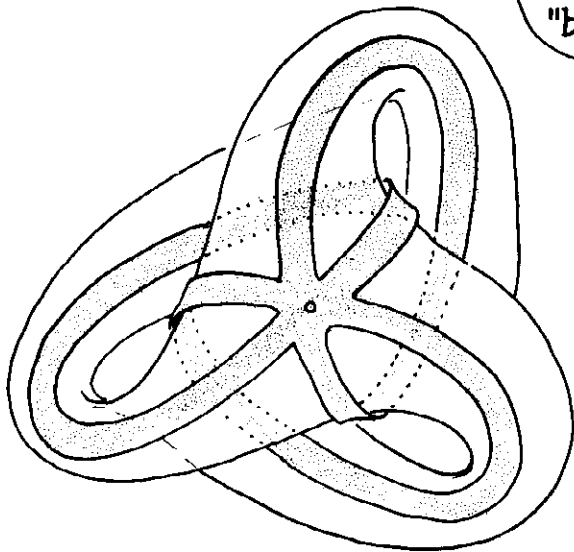
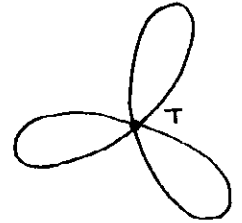
It's weaving work Leon. We just have to prolong the "meridians" of the Moebius strip to bring them to the bottom of the basket, the pole.



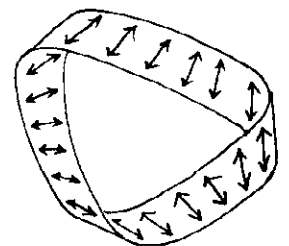
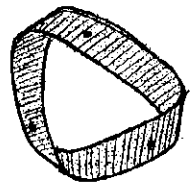
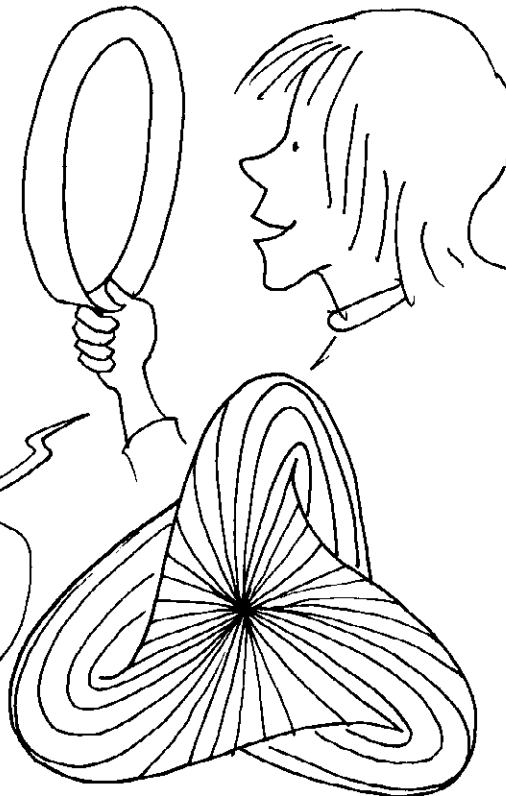
BOY SURFACE WITH INITIAL MOEBIUS STRIP



In other words you have to join the free canes of a Moebius strip to those of the "bottom of the basket".



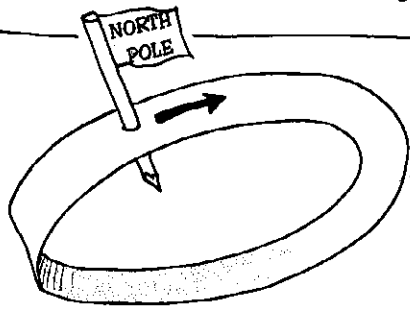
The NEIGHBOURHOODS of the "meridians" are Moebius strips with one half turn.



THE FIRST MODEL OF THE BOY SURFACE WITH ITS ENSEMBLE "MERIDIANS"+"PARALLELS", WAS IMAGINED BY THE AUTHOR. A FINE MODEL WAS THEN MADE BY THE SCULPTOR MAX SAUZE WHICH IS VISIBLE IN THE "TROOM" OF THE PALACE OF DISCOVERY in PARIS, FRANCE.

The Management

We've moved along one of these strips, leaving the "NORTH POLE" to look for the "SOUTH POLE"



And of course we came back to Perry's flagpole!



But if we moved along a Boy surface, how come we didn't detect the auto-intersection regions?

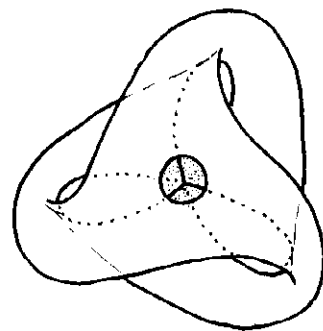
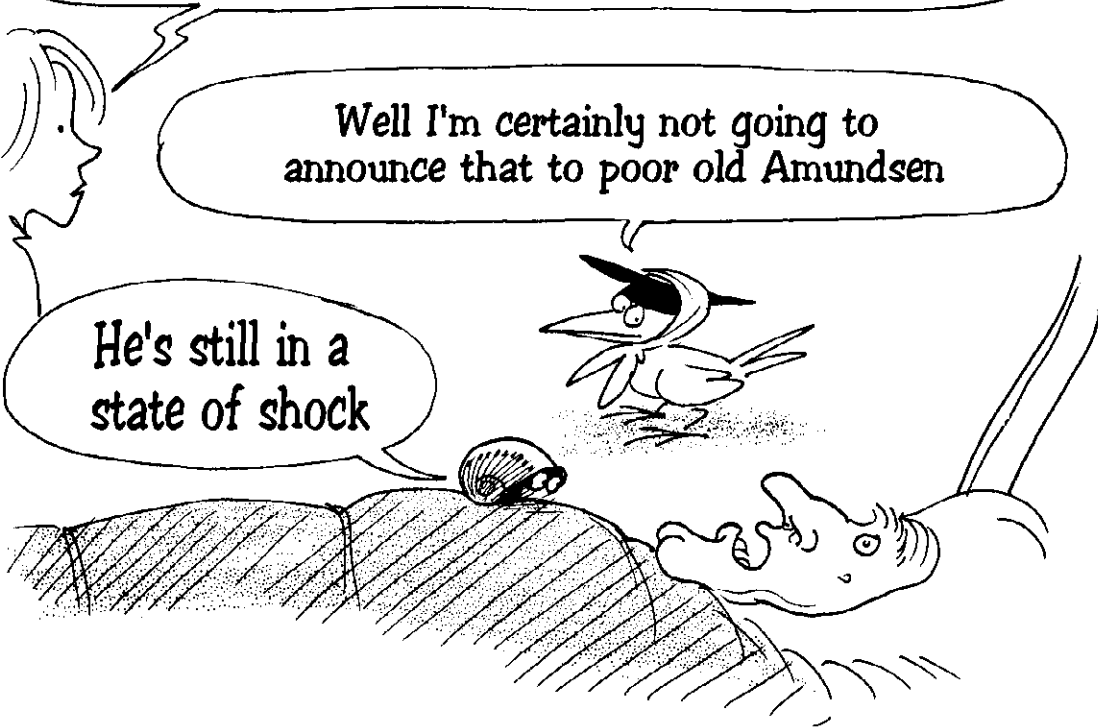
Remember, this IMAGE of auto-intersection is just an effect of immersion of the BOY SURFACE into the REPRESENTATIONAL THREE-DIMENSIONAL SPACE. In fact, the Boy surface and the KLEIN bottle exist as TWO DIMENSIONAL OBJECTS INDEPENDENTLY OF THE SPACE IN WHICH THEY ARE REPRESENTED.

Here's a good method to forget about the idea of auto-intersection.

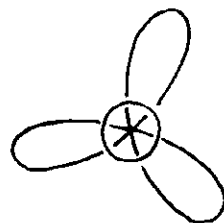
One thing is certain : The planet is a Boy surface and only has one pole.

Well I'm certainly not going to announce that to poor old Amundsen

He's still in a state of shock

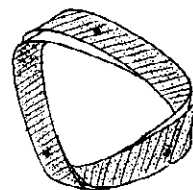
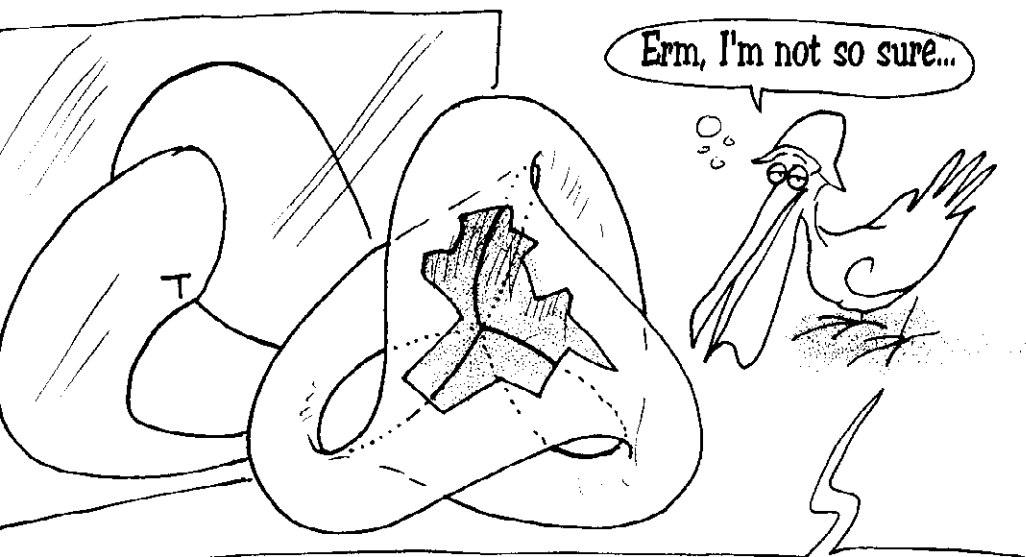


MOEBIUS STRIP WITH A CIRCULAR EDGE

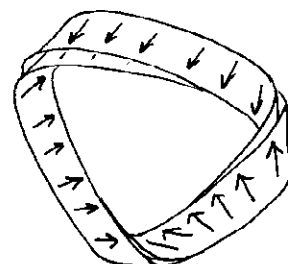


THE BOY CUBE

Erm, I'm not so sure...



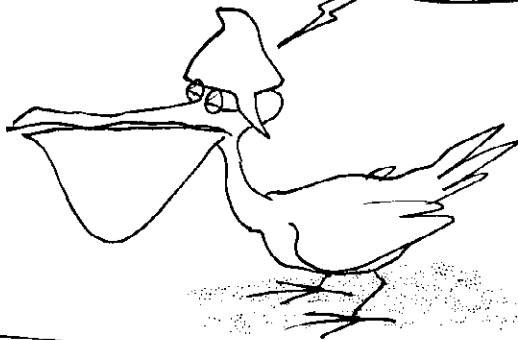
I might seem a bit nuts to you but I must admit, even with the drawings, the cross sections, various views, I still haven't understood the Boy surface...



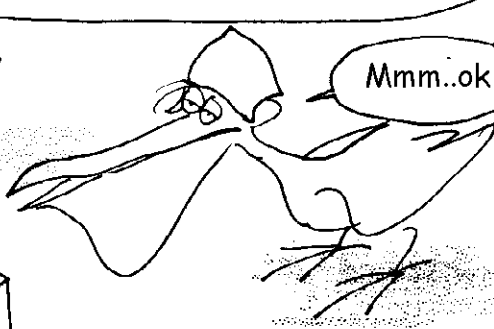
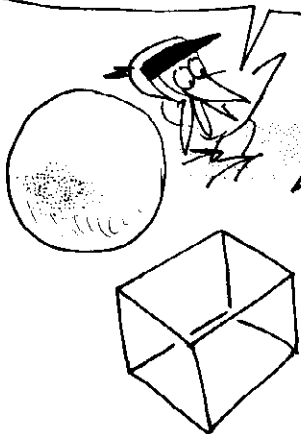
are you having trouble understanding its topology?

It's.. erm... yes... it must be that

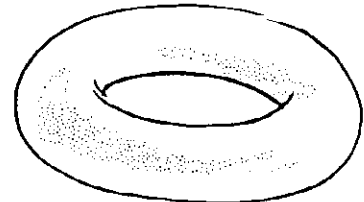
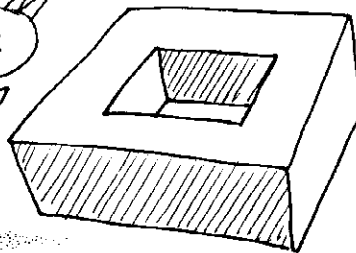
wait, Léon, I've found something that will help you



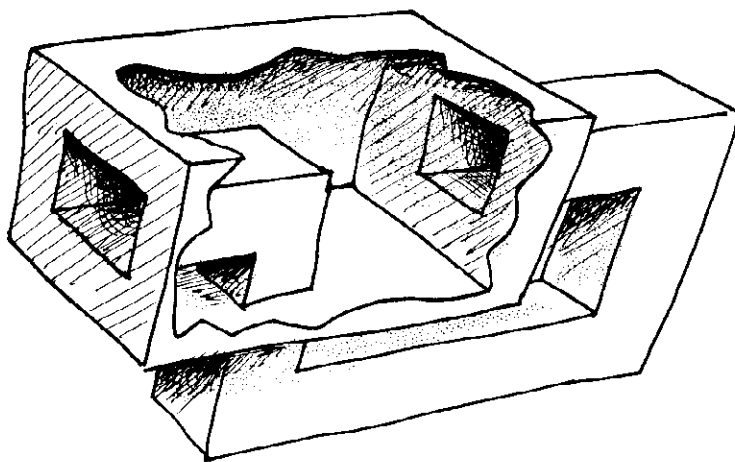
Léon, a sphere or a cube it's the same thing. The same topology, the Same Euler-Poincaré characteristic, the same total curvature



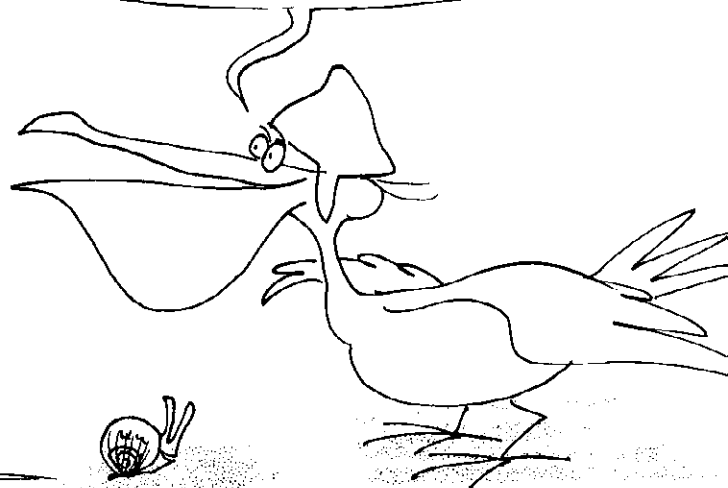
Mmm..ok



and that's a torus!

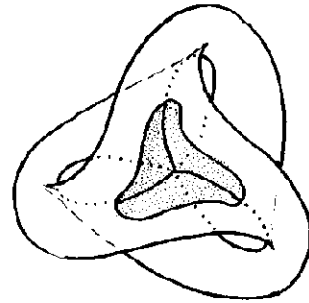
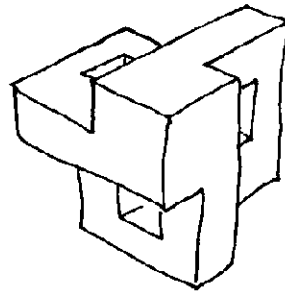
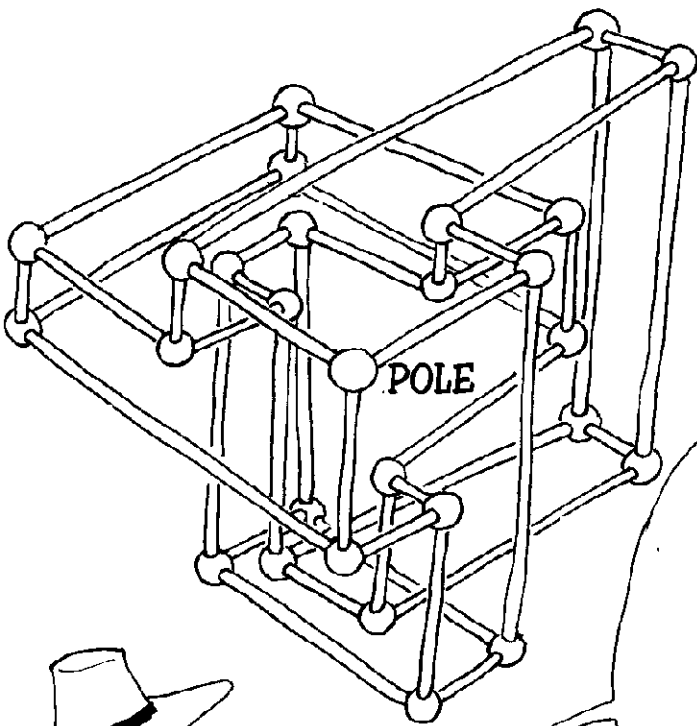


And so, that's a **KLEIN - CUBE** ?

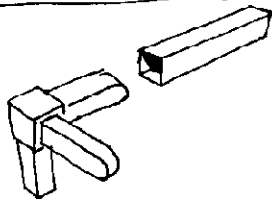
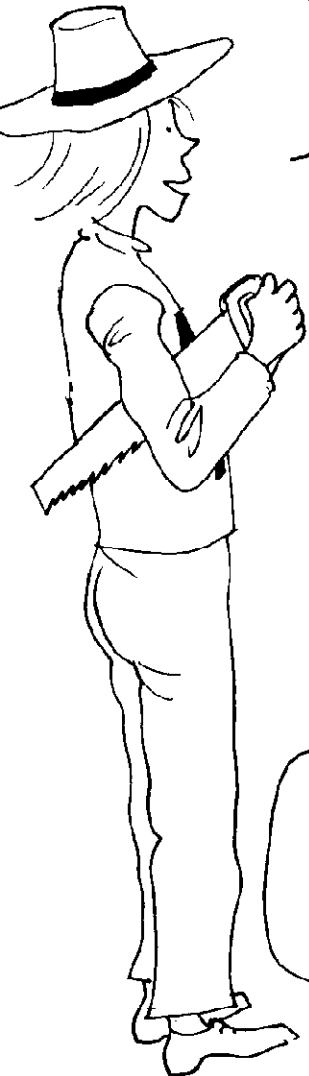
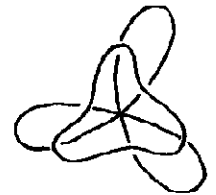


Exactly!

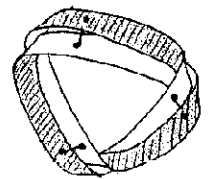




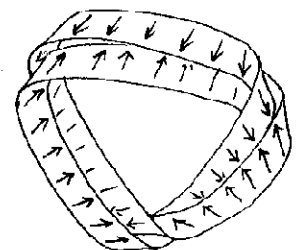
And here's a
BOY-CUBE patented
 by Archibald.
 28 summits
 43 edges
 16 faces
 $X=28-43+16 = 1$

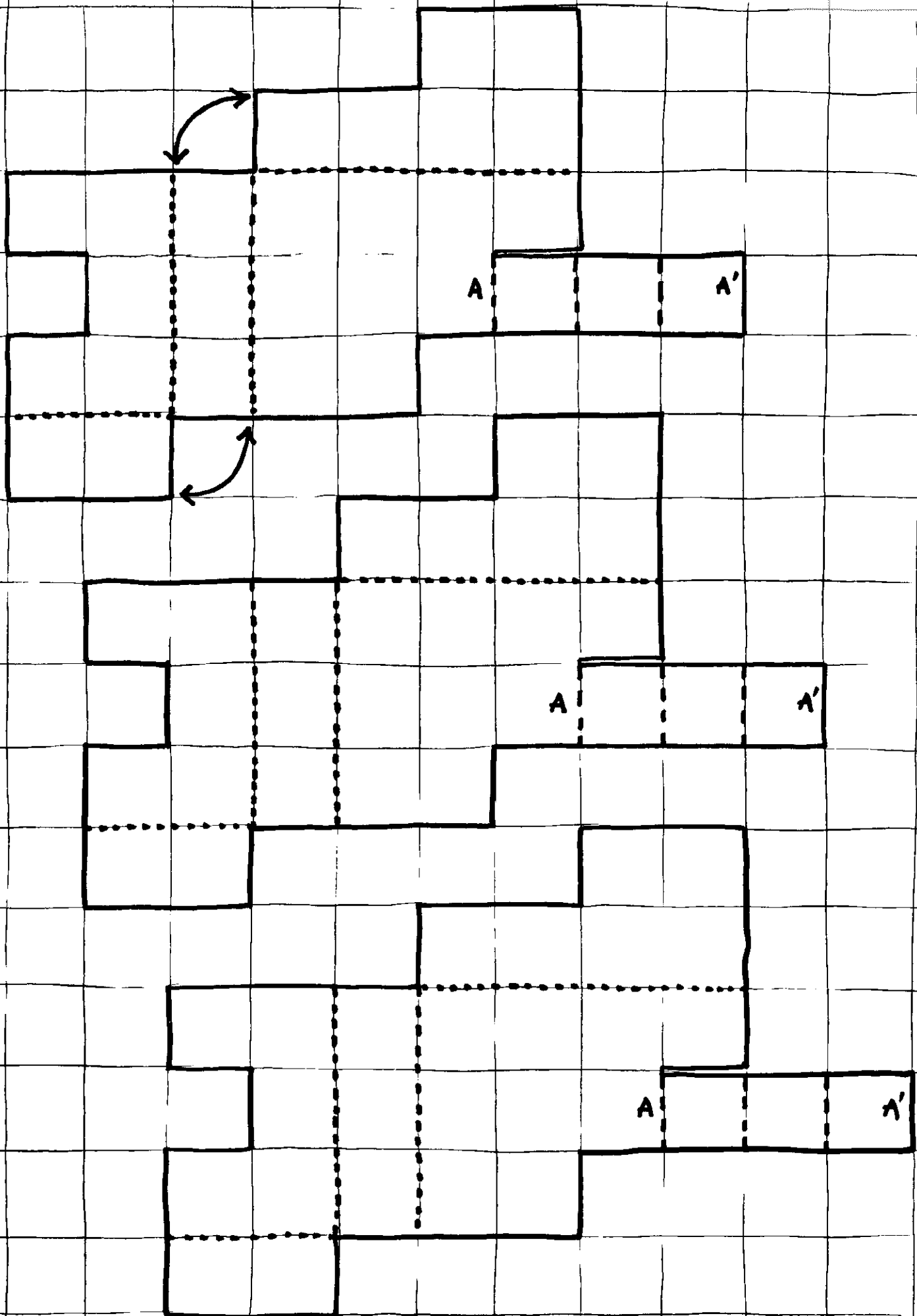


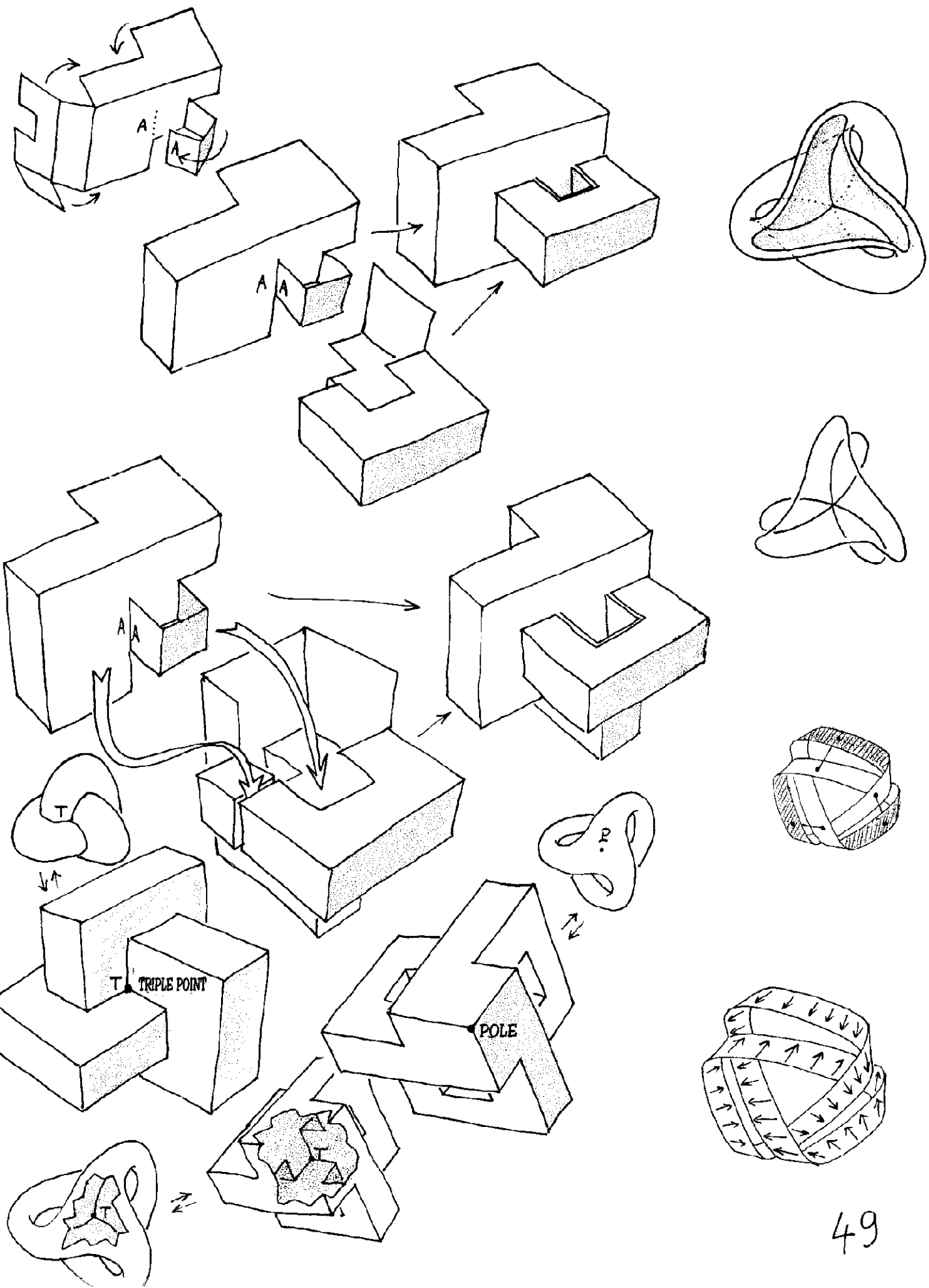
Nice models can be
 made using **REYNOLDS**
 shelves (square
 Dural tubes and
 angle pieces
 in plastic).



On the following
 page there are
 drawings to cut
 out and make your
 own **BOY-CUBE**



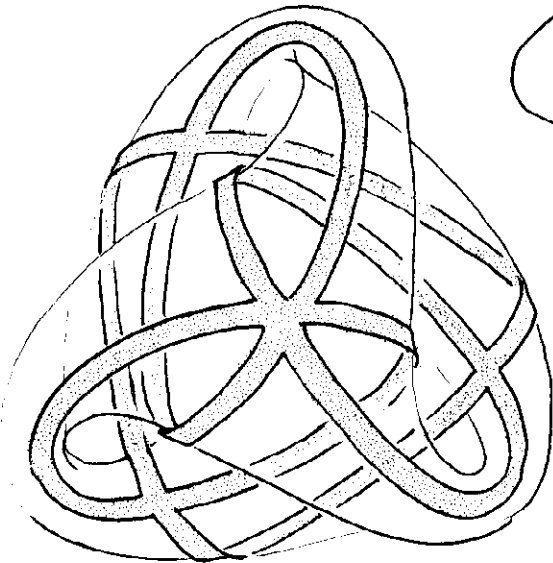




COVERINGS

So that's the end of the story then?

No, there's a sudden surprise...

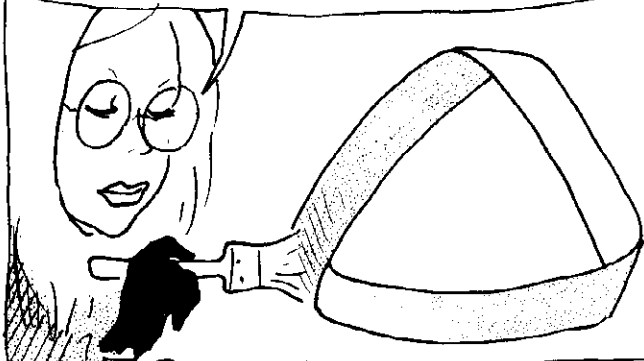


The **TWO-LEAFED COVERING** of a **UNILATERAL, NON ORIENTABLE OBJECT** is **BILATERAL, ORIENTABLE** and has a double characteristic.

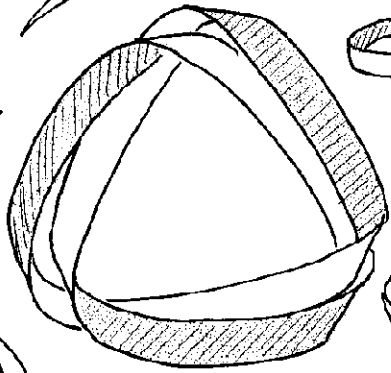
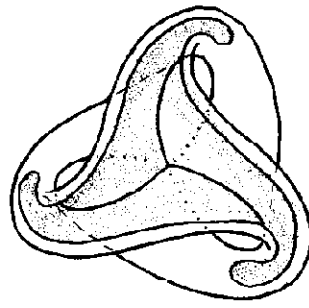
What is all this nonsense ?

It's simple. Take a Moebius strip and cover it with paint on its **UNIQUE** side, then take the strip away...

...and just keep the paint !



This new strip, closed on itself, has two faces because it was in contact with the Moebius strip. You can see the sequence in the images

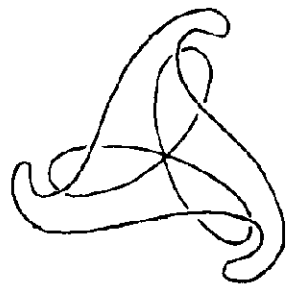


$$\text{[Strip]} = \text{[Strip]} + \text{Segment}$$

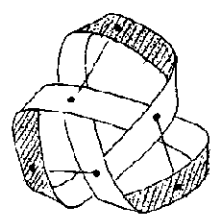
$$\text{[Strip]} = \text{[Strip]} + \text{Segment}$$

$$\text{[Strip]} = \text{[Strip]} + \text{Segment}$$

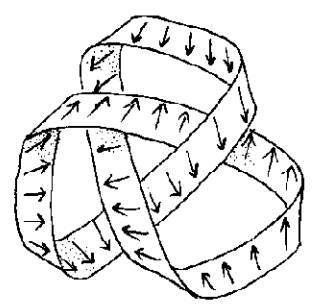
Both its characteristic and that of the Moebius strip are nil.



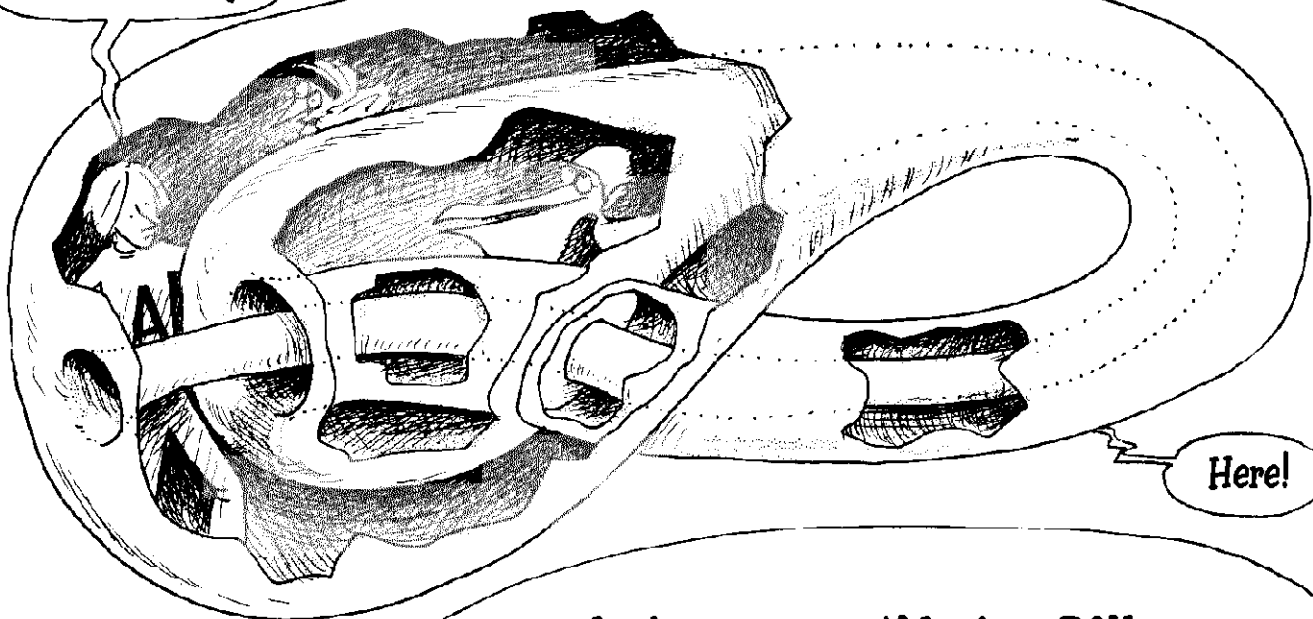
Look...If I paint a **KLEIN BOTTLE** on its unique face then take away the bottle to just leave the paint, I will obtain a **CLOSED, REGULAR SURFACE**, with **TWO FACES** and possessing a Euler-Poincaré characteristic of $2 \times 0 = \text{ZERO}$



That is to say an immersion of a **TORUS!**

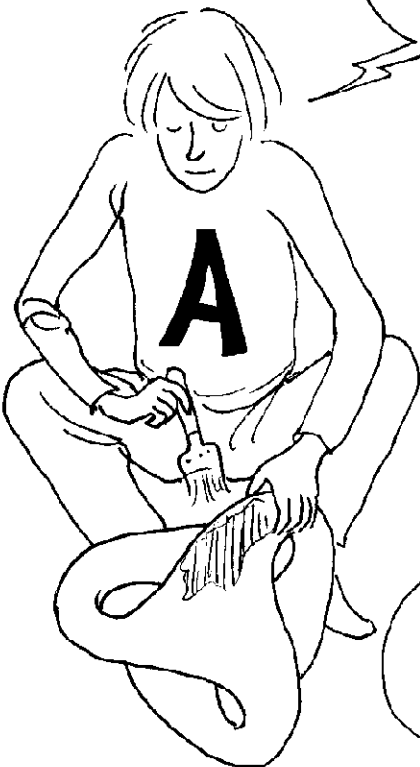


Tiresias, where are you?

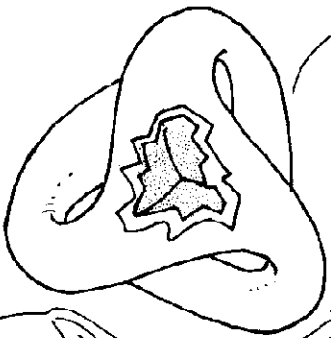


Here!


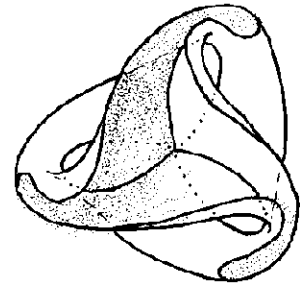
In the same way if I take a **BOY** surface and cover it with paint then remove the **BOY** and keep the paint I will obtain a **CLOSED, REGULAR** surface **WITH 2 FACES** and having a Euler-Poincaré characteristic of $2 \times 1 = 2...$




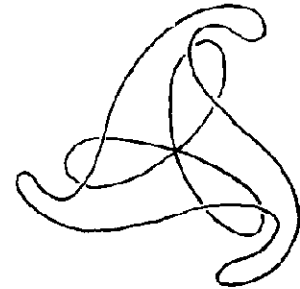
...in other words,
an **IMMERSION OF
THE SPHERE !**



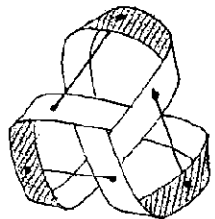
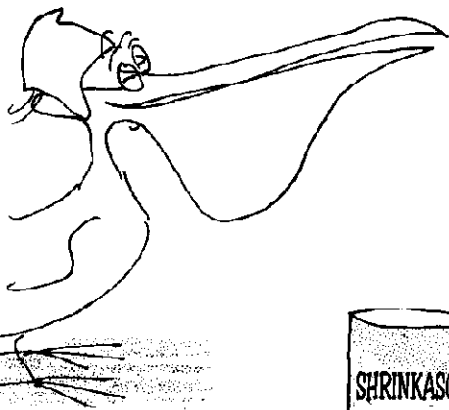
Can I REALLY "unfold"
this weird sphere and
turn it into an
"ordinary" sphere?



With **TRANSVERSINE**,
no problem, the
same for a **TORUS**



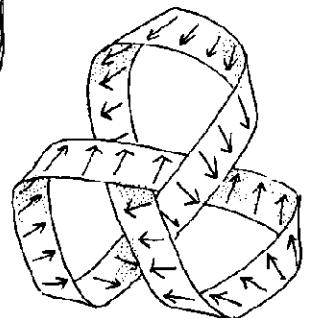
Lets go in the opposite
direction...suppose that
I want to "refold" a
sphere without any folds !

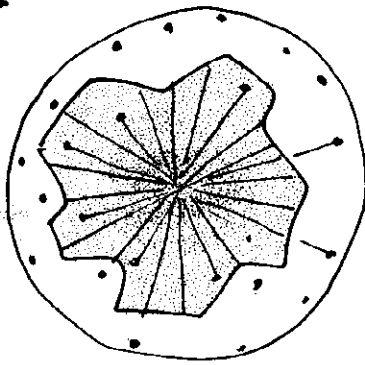


**CROSSED
STRIPS
RESULT**



You need some
SHRINKASOL





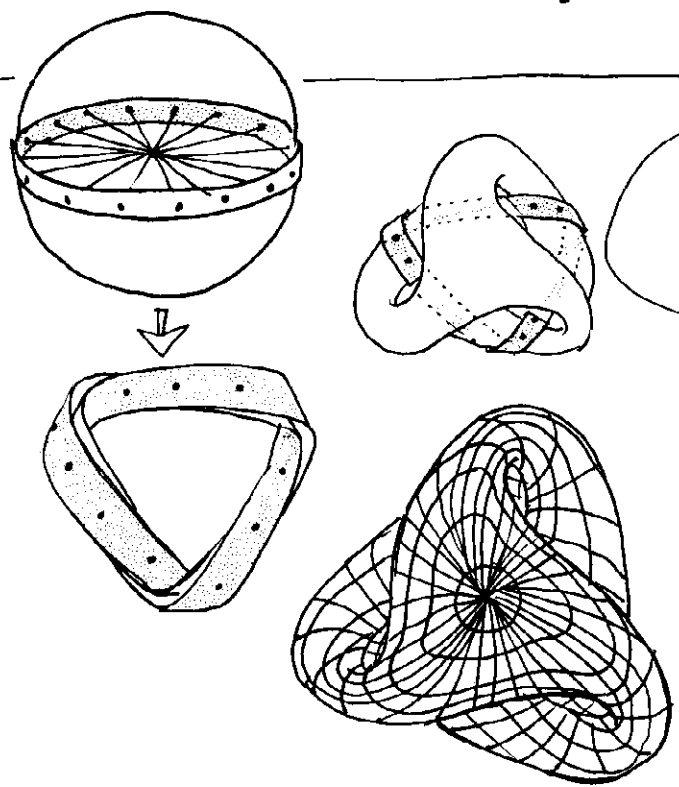
We begin by joining each point of the sphere to its antipode using strings soaked in SHRINKASOL



These strings contract to the point where they have zero length, while the surface of the sphere remains constant. We bring each point into CONJUNCTION with its ANTIPODAL.

But as you'll see all that in another album, dedicated to turning a SPHERE inside out. In the meantime, the series of images in the 'filmstrip' G show how the EQUATOR of the SPHERE folds in on itself, becoming the EQUATOR of the BOY. The NORTH pole then, obviously, sticks itself next to the SOUTH pole.

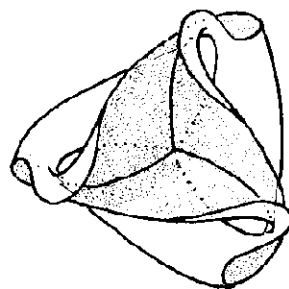
The Management



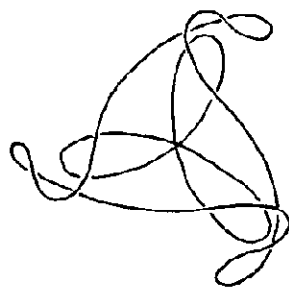
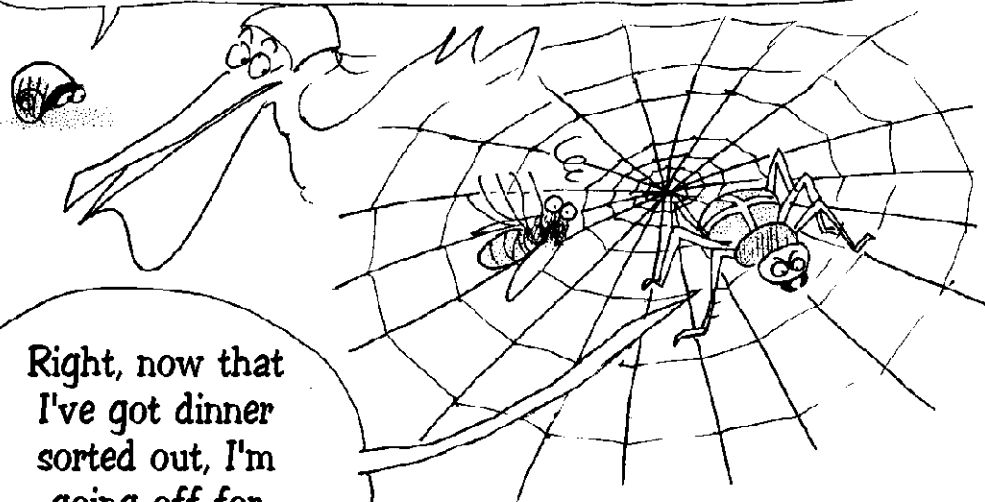
All the meridians and parallels of the sphere cover each other.



Imagine a spider living on a **BOY** surface whose mesh is made of its parallels and meridians. It would think it lived...on a sphere!

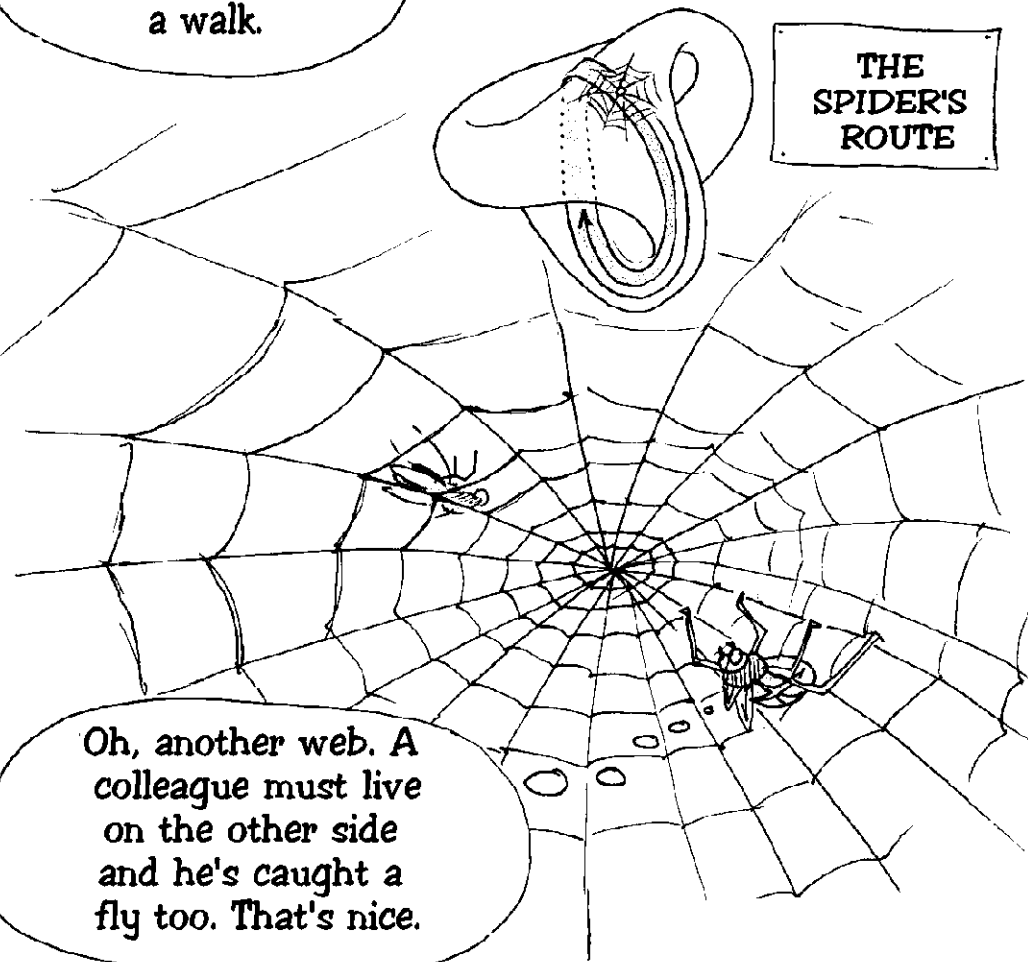


Closure of the three "tympani"

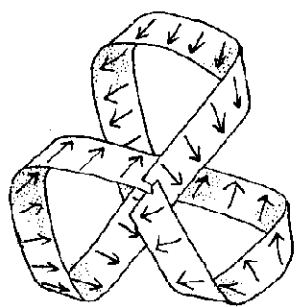
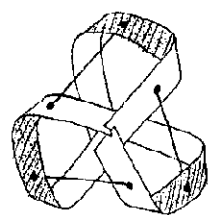


THE SPIDER'S ROUTE

Right, now that I've got dinner sorted out, I'm going off for a walk.



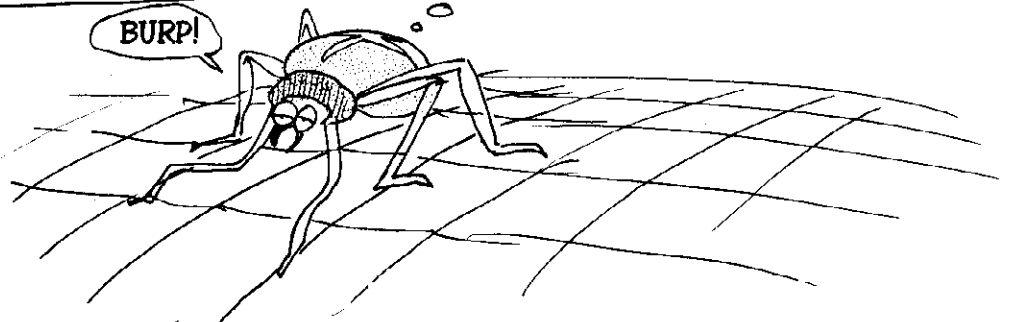
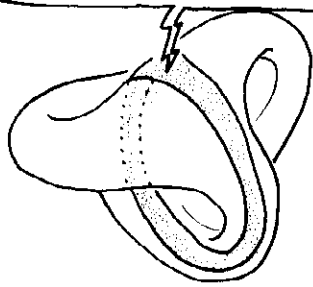
Oh, another web. A colleague must live on the other side and he's caught a fly too. That's nice.



Ah, nobody looking,
I'm going to eat its fly.

Mmm, let's go home.

BURP!



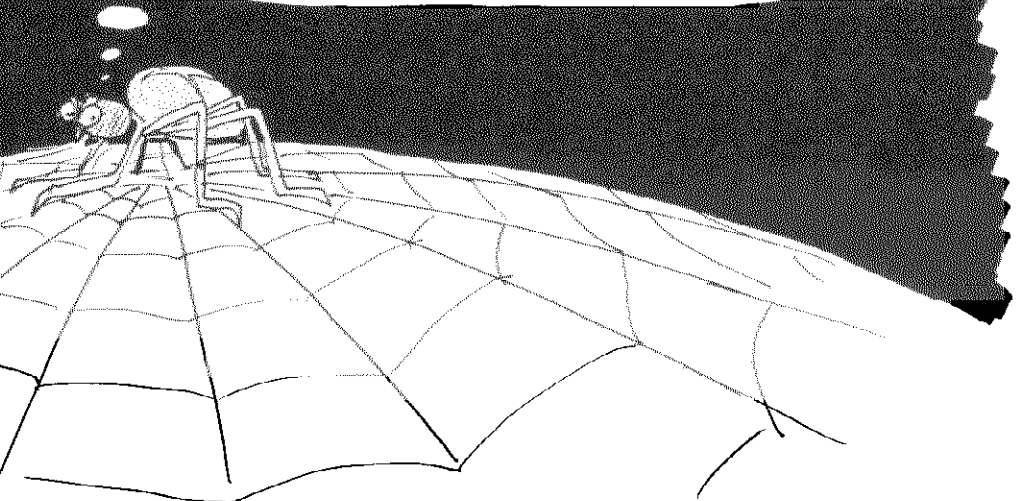
Oi! While I was away
the other spider has
been here and eaten
MY fly!

Ha Ha Ha



In fact, there was only
one fly and one spider.

I'll get you even if I wait all night,
and you'll see when I catch you...



But the spider story...
that makes me think of
something. We've got a
solution for Amundsen.

Mr Amundsen, we've
sorted everything out,
we've found your
south pole.

How's that?

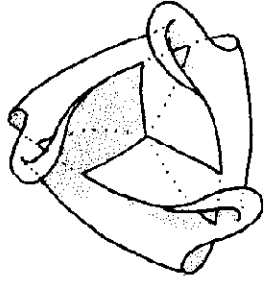
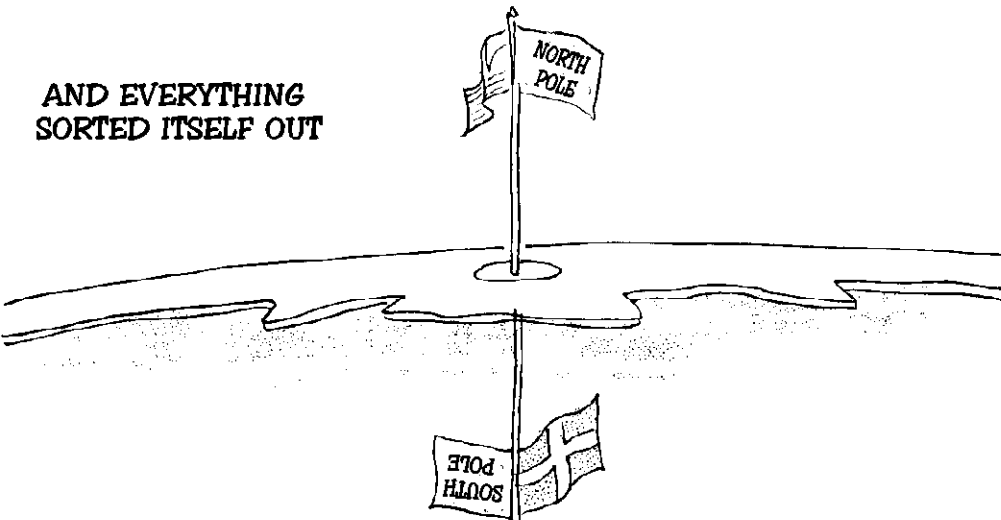
Ah...

You can go but take
this with you...

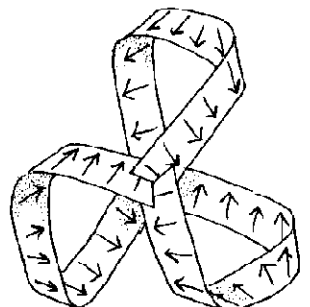
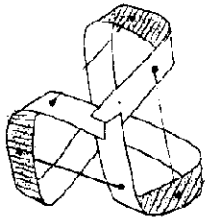
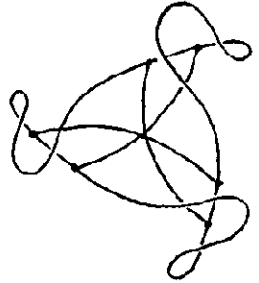
They gave the same
thing to Perry

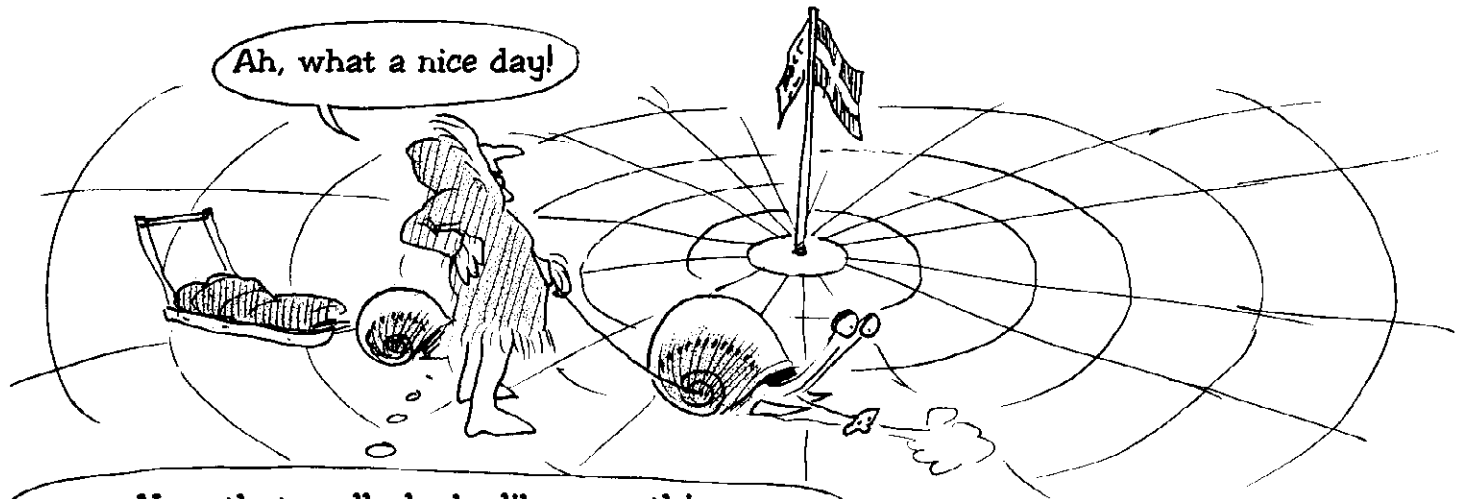
...and you just
PUT IT DOWN

**AND EVERYTHING
SORTED ITSELF OUT**

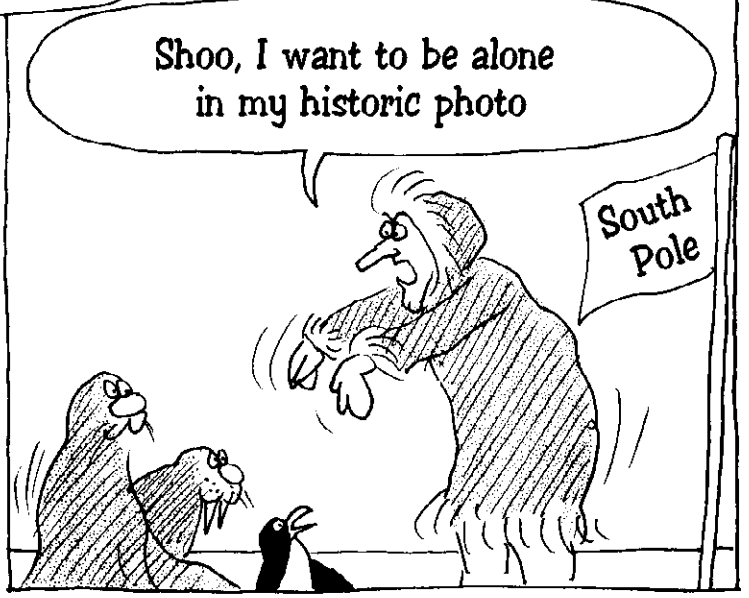
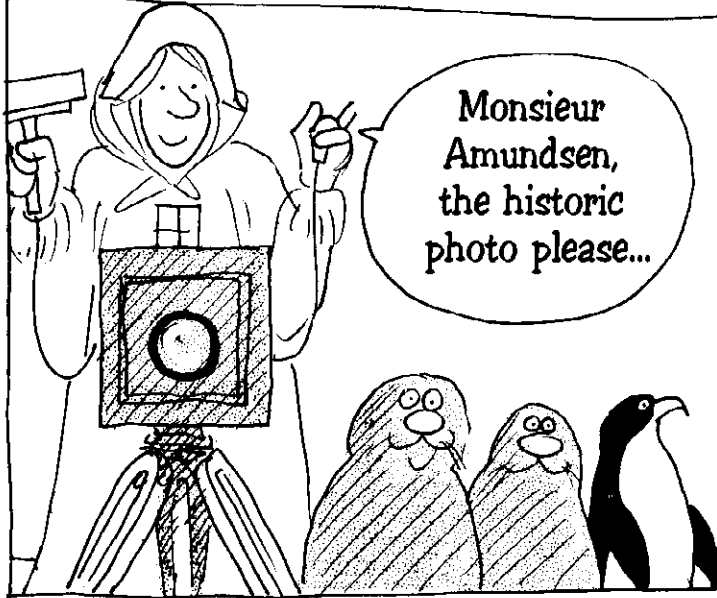


APPEARANCE
OF "EARS"





Now that really looks like something



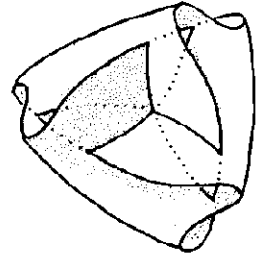
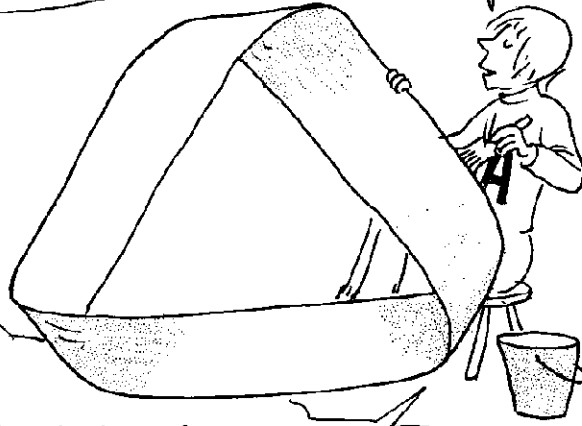
In science it's like in anything else, sometimes you shouldn't dig too far...

...each pole has its place and the stable doors are properly bolted.

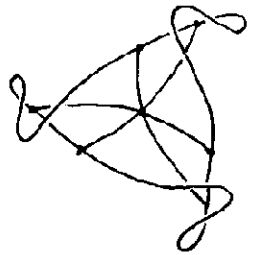
Not only that but if we dug under the North pole we might get some nasty surprises.

And someone here might get very upset about that.

Right, that's one thing done then. What's Archie up to?

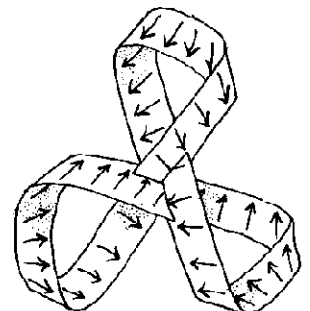
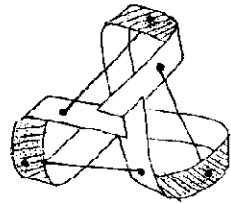
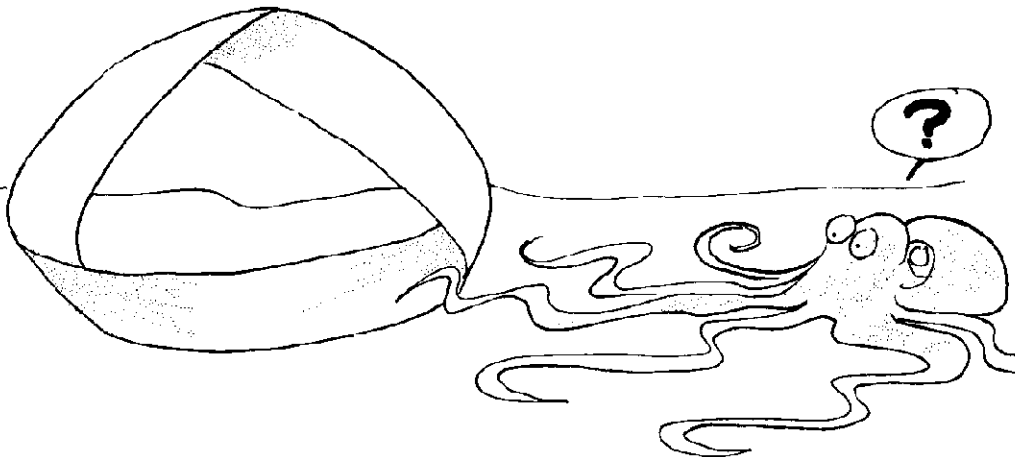


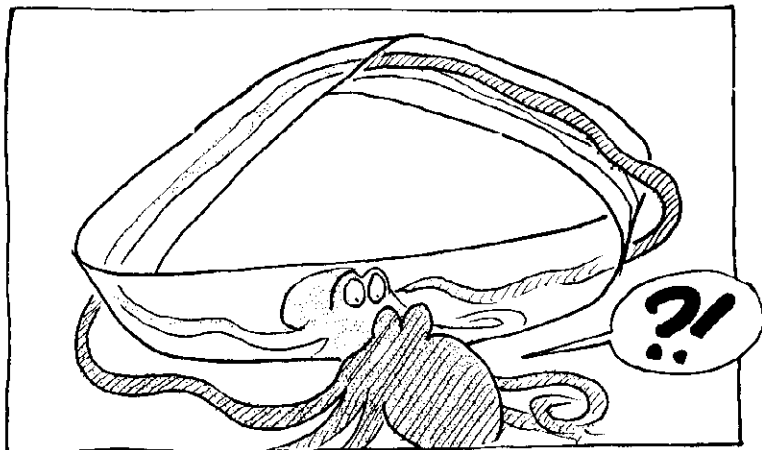
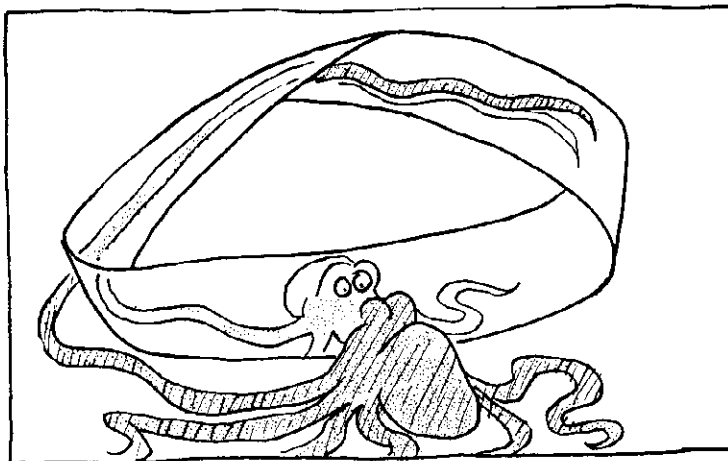
Do you know what a two-way mirror is? You can see a reflection in it and look through it at the same time. Well I'm changing a Moebius strip into a two-way mirror.



THE MIRROR STAGE

To catch squid.



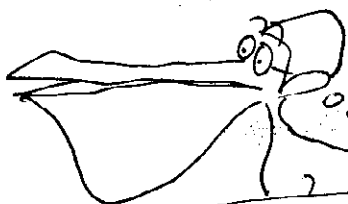


What's happening !?! The squid seems to be stupefied

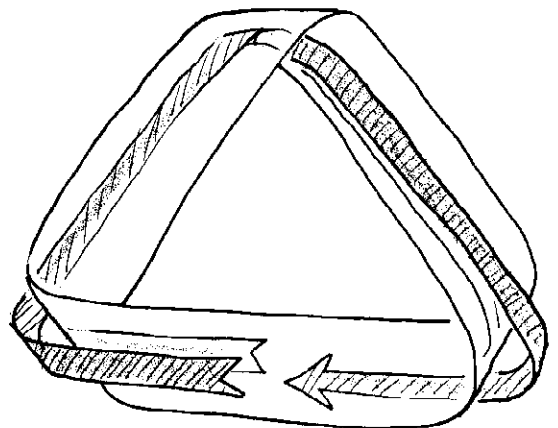
And it isn't feeling anything because its real arm is scratching the image of its head while its "image arm" is scratching its real head.



It's scratching its head desperately



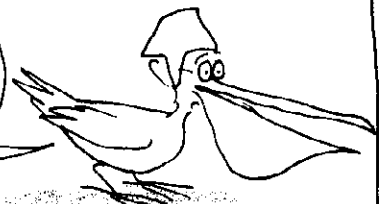
Poor thing..



As the mirror is unilateral, by going round it, its arm has "passed to the other side"

And as the mirror is perfectly semitransparent it can't manage to work it out !!!

It's looking pretty freaked out !



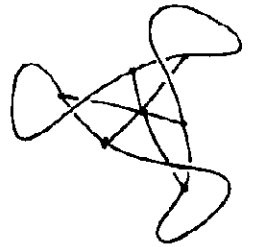
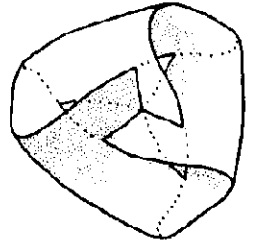
Put yourself in its place!



You see, if one day you scratch your ear in front of a mirror and feel nothing, it means that the mirror is unilateral (*)

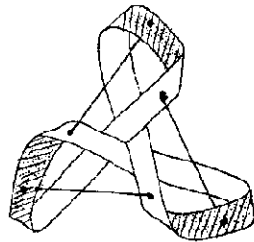
If we transformed a BOY surface into a seethrough mirror the universe would be indisassociable from its own image.

Wouldn't that be dangerous?
I don't know...the universe seized by a sort of logical contradiction, it might make it disappear (*)



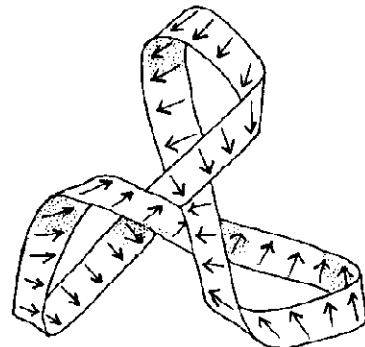
SPACE-TIME GONE MAD

We can study the topology of spacetime using twordimensional models, one for space and one for time



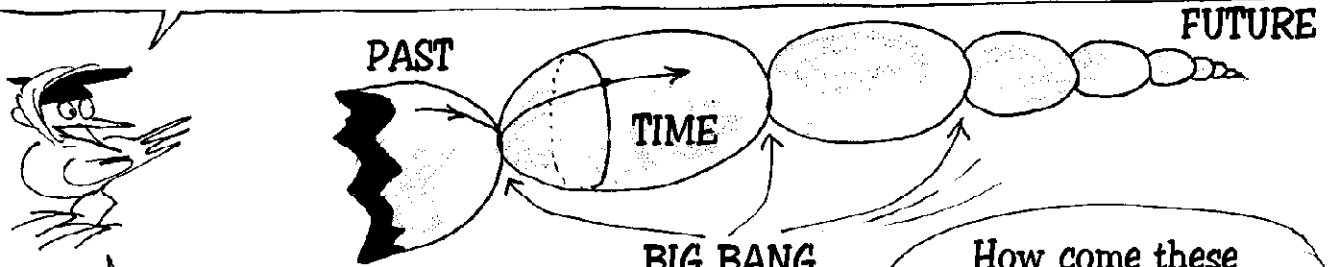
CREATION OF A TRIPLE POINT

That makes a grid or mesh



(*) NO ONE HAS EVER TRIED THIS

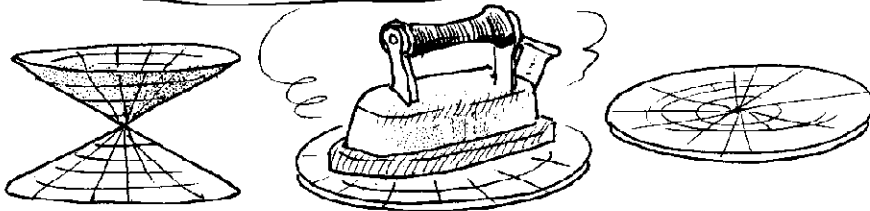
We saw in the **BIG BANG** that **FRIEDMANN's CYCLIC** universe model could be represented by an image of an infinite string of sausages, each tied point a new **BIG BANG**



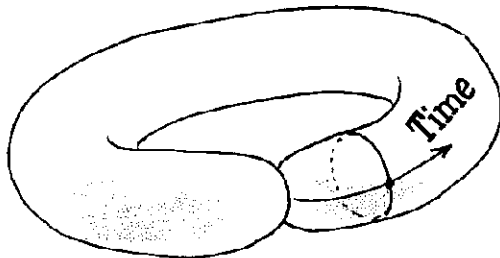
Each **BIG BANG** being a **POLAR** type singularity

How come these singularities are **CONNECTED** ?

BIG BANG



Take a cone and flatten it.



Would could also imagine that these events could repeat themselves infinitely, in which case we'd have this...

Or we can suppose that **TIME** is simple a **BEGINNING** and an **END**, like this



time



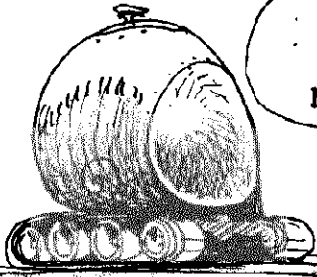
BIG BANG

YOU ARE HERE

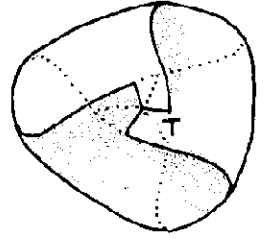
SPACE **TIME**

In this classic model of **SPHERICAL SPACETIME**, one of the poles is the **BIG BANG** and the other the **ANTI BIG BANG**. Space can be considered to be parallel curves, the equator being the maximum extension of the "time lines" corresponding to the meridians.



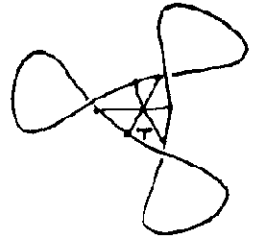
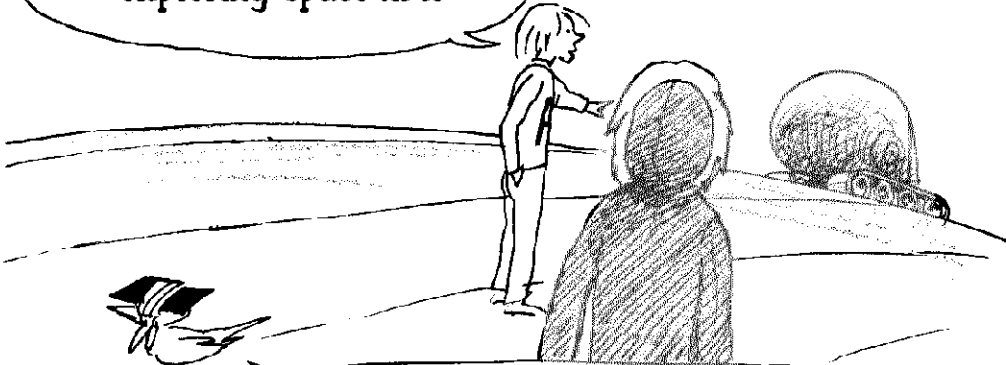


To travel along these meridians, these **UNIVERSE LINES**, there's nothing better than a **CHRONOSCAPE**

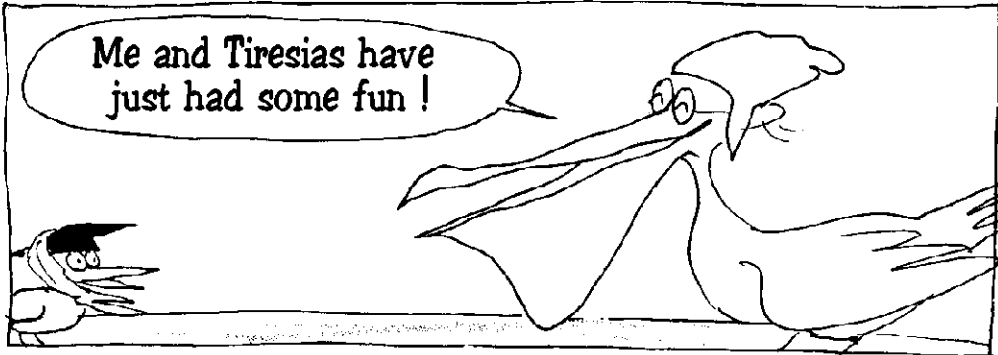


CREATION OF A TRIPLE POINT

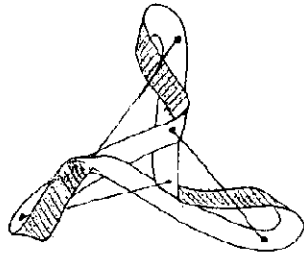
We could borrow one of these machines. I wouldn't mind exploring spacetime



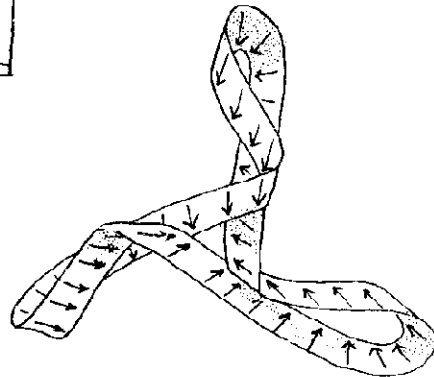
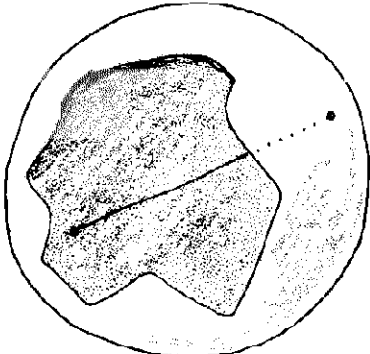
Where are Leon and Tiresias?



Me and Tiresias have just had some fun!



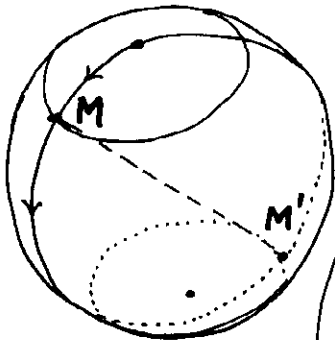
We took all the points of this spacetime and joined them to the **ANTIPODALS** with strings...



Then we soaked the strings in SHRINKASOL. Tiresias reckoned that it would be an interesting spatio-temporal experiment

You're completely mad, both of you. You can't imagine the consequences !!!

Why, what will happen ?



Because of of what Tiresias did, SPACE-TIME is now collapsing on itself. All EVENTS corresponding to its EXPANSION phase, that is to say since the BIG BANG and to the point of MAXIMUM

EXTENSION, will find themselves in conjunction with the corresponding events of the CONTRACTION phase, because of the coincidence of the ANTIPODAL REGIONS.

You mean the BIG BANG and the ANTI BIG Bang are going to get mixed together ?

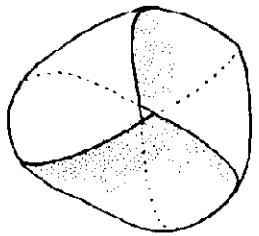
It strange, weird and a real coincidence

I suppose someone has already thought about this ? (*)

I should never have listened to Tiresias



The conjunction phenomenon will bring spacetime regions face to face with their antipodes and so in **TEMPORAL OPPOSITION** to them.

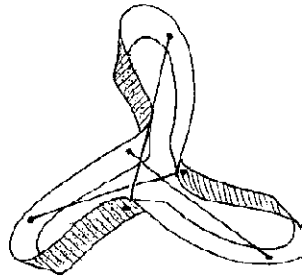
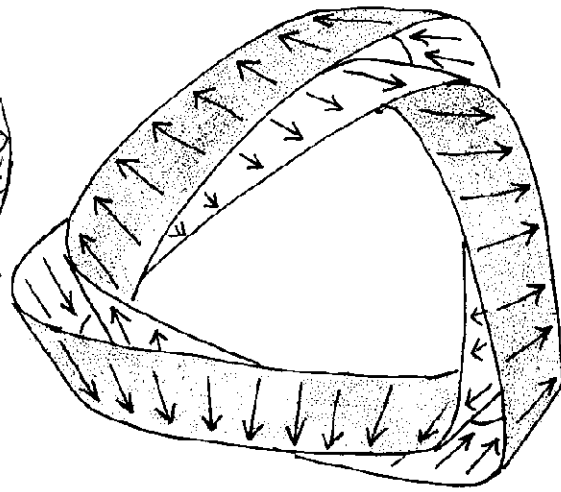
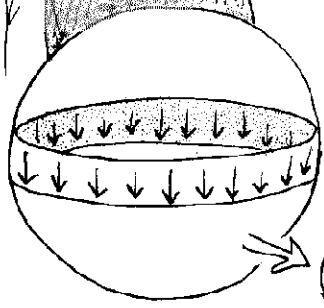
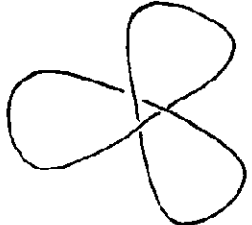


Impossible !

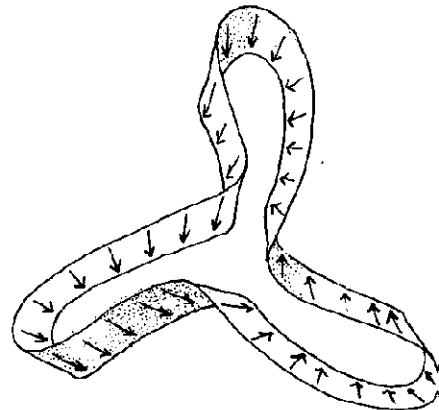
Not at all. Take the region near the equator of this spherical spacetime for example, which corresponds to the state of maximum extension. We can see clearly how it folds in on itself in the filmstrip D.



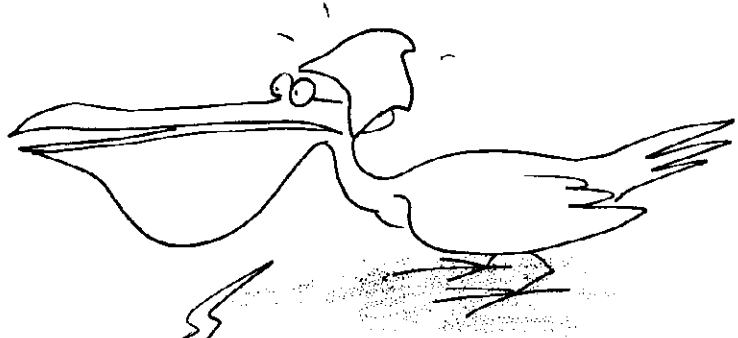
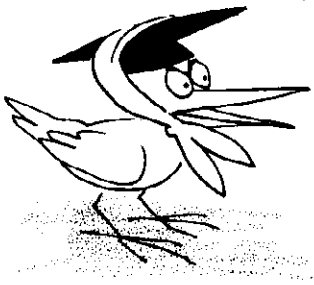
The time arrows put themselves in **OPPOSITION**.



You mean that what is the **PAST** for some, is the **FUTURE** for their **ANTIPODEANS** ?

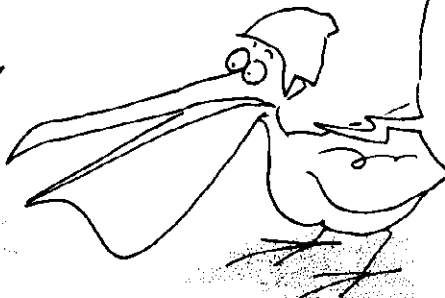


Well done Leon,
good work



You mean that this will probably plunge the universe
into a situation of unsupportable contradiction ?

A sort of logical dead end.

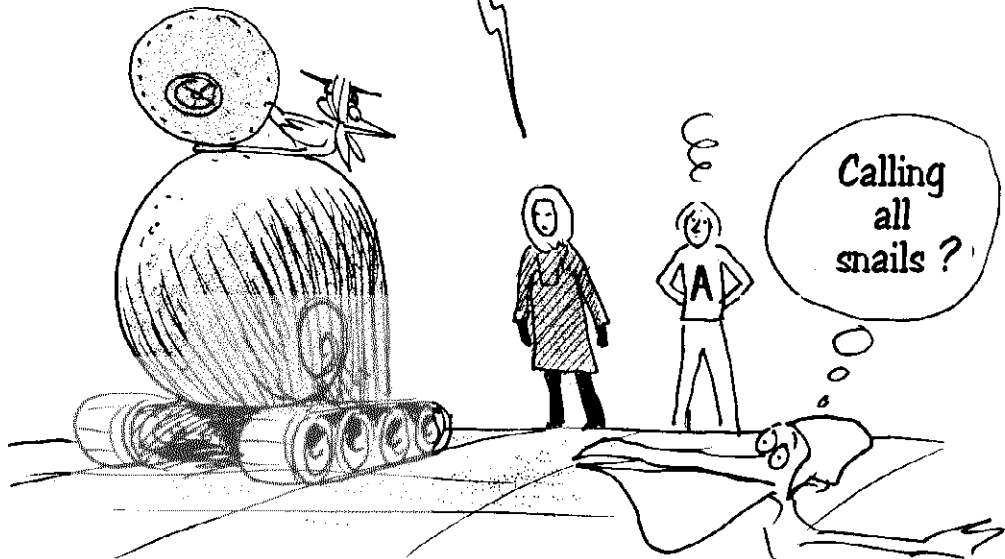


When the SHRINKASOL has had
its effect, the universe will
telescope in on itself and
we'll find time going
backwards very fast.

Where's Tiresias
by the way ?



Lets get into the Chronoscape.
We can try and call him.



Hello, Tiresias,
can you hear me ?

But wait, if Tiresias has
become **RETROCHRONIC** for us
and if we manage to get in
contact with him he'll know
already what we are going to say.

Even worse, he'll be the
one transmitting this
message in his **PROPER TIME!!**

Good heavens!...

Anyway, if we do come
across him it'll be
even worse still !

Feynmann thought
that antimatter
lived in inverted time !

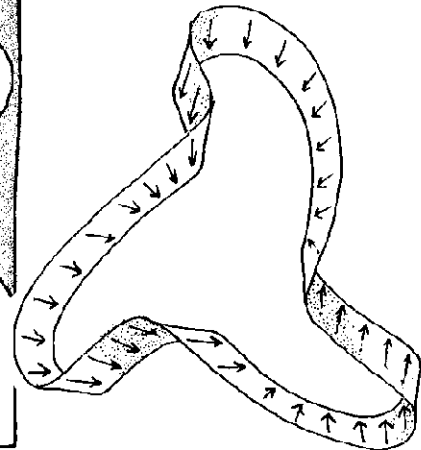
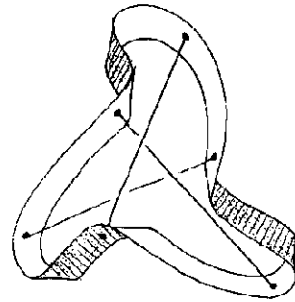
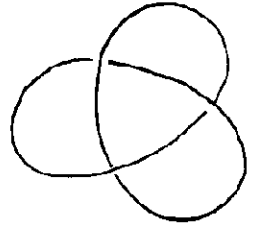
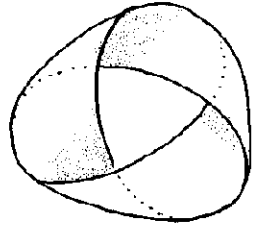
Why ?

And the Abbé **LEMÂITRE (*)**
thought that antimatter
was matter seen **BACK
TO FRONT (*)**

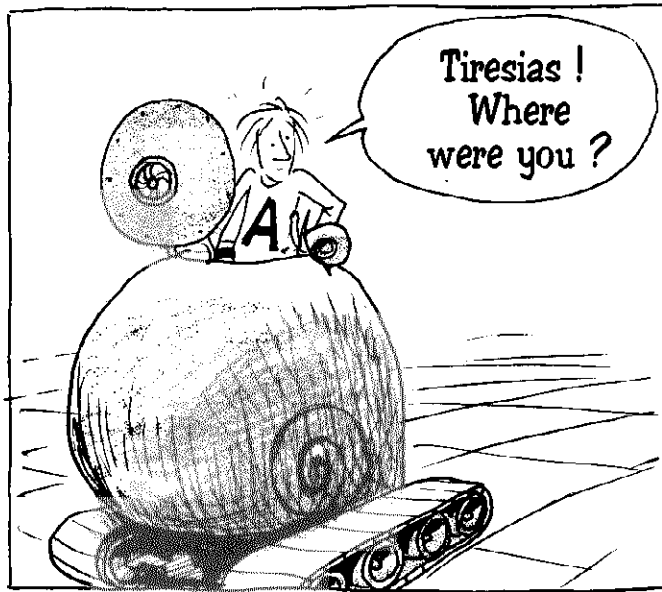
If we had the bad luck
to come across Tiresias
he would have become
an **ANTI-TIRESIAS**

What do you
mean, **BOOM ?**

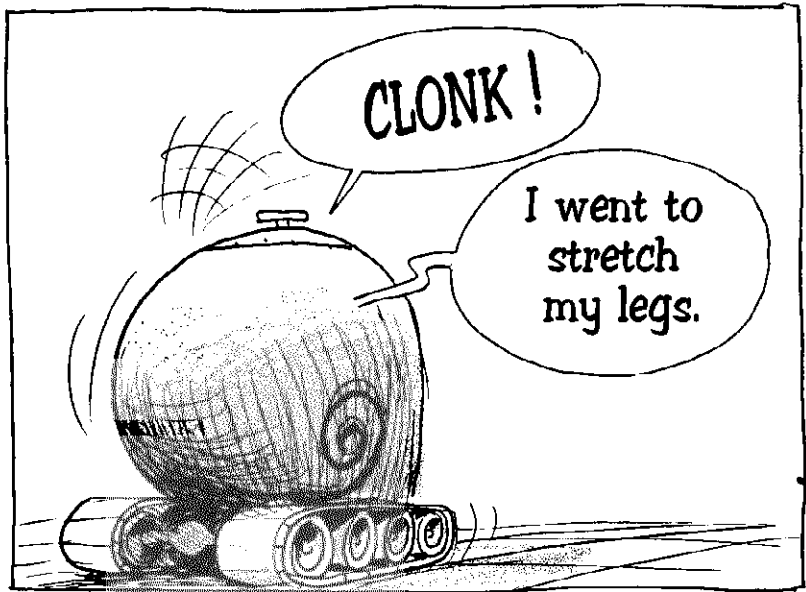
And so,
BOOM !.



(*) See the **BIG BANG**



Tiresias!
Where were you?



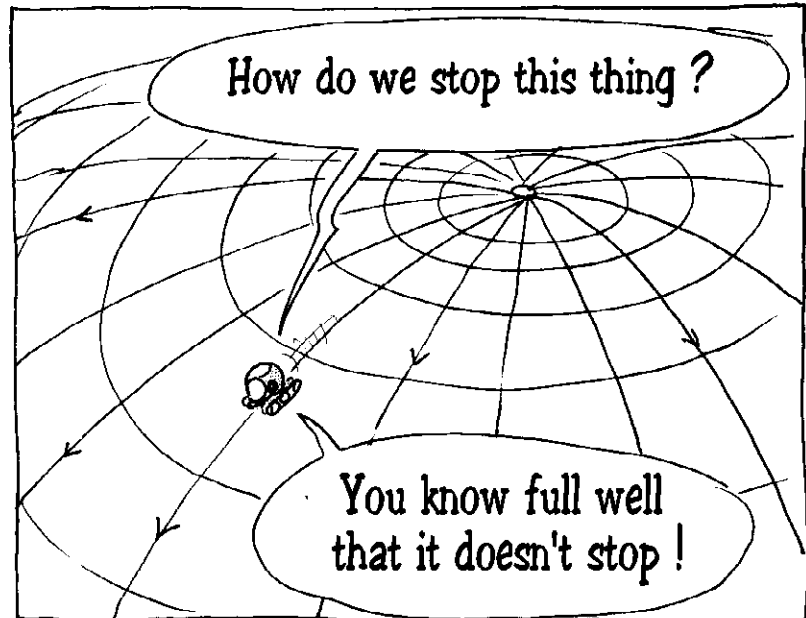
CLONK!

I went to stretch my legs.



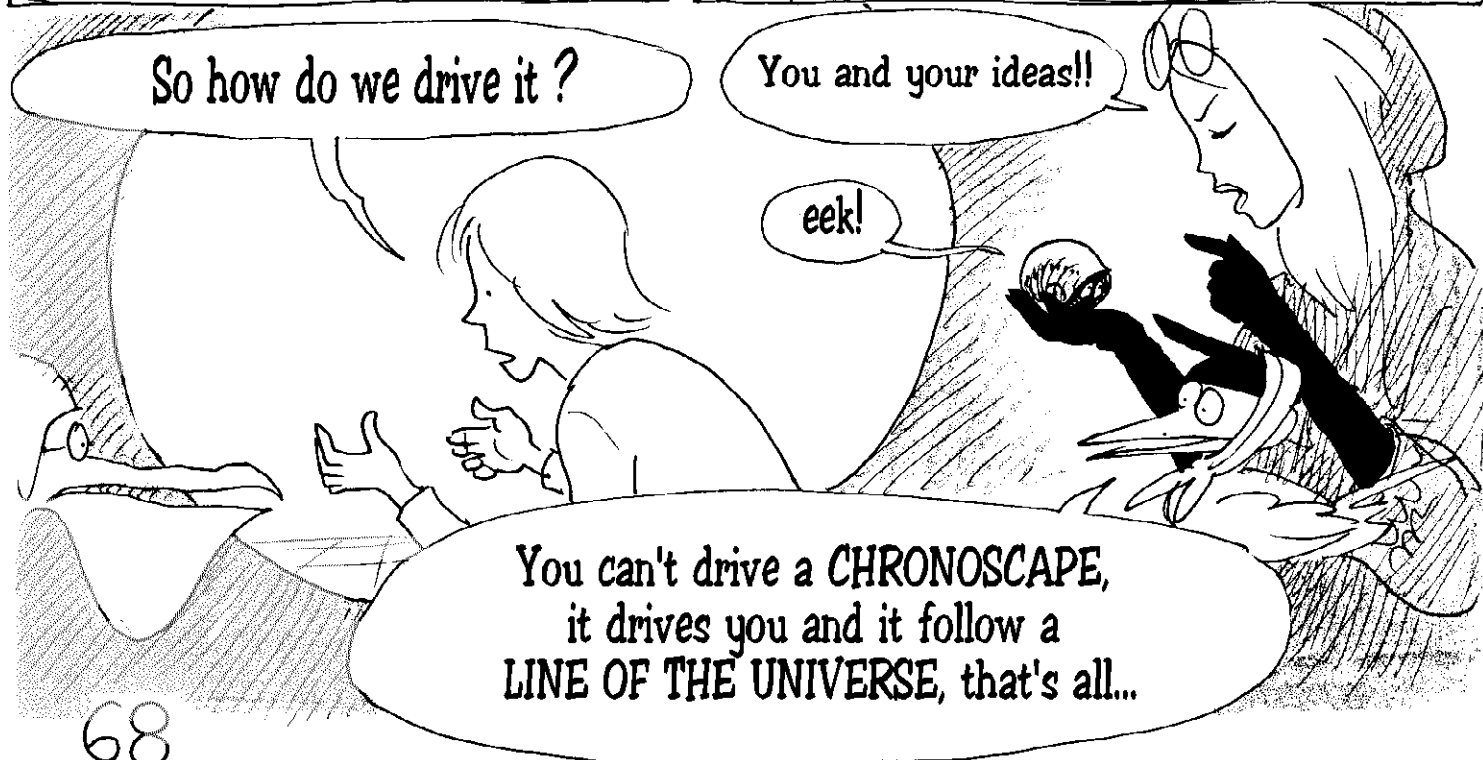
Huh! The CHRONOSCAPE has started up by itself...

You shouldn't have slammed the door so hard!



How do we stop this thing?

You know full well that it doesn't stop!



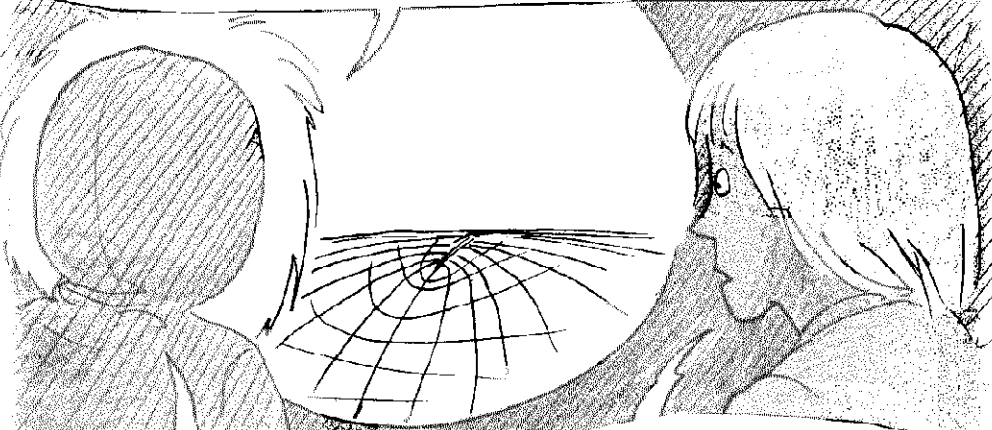
So how do we drive it?

You and your ideas!!

eek!

You can't drive a CHRONOSCAPE, it drives you and it follow a LINE OF THE UNIVERSE, that's all...

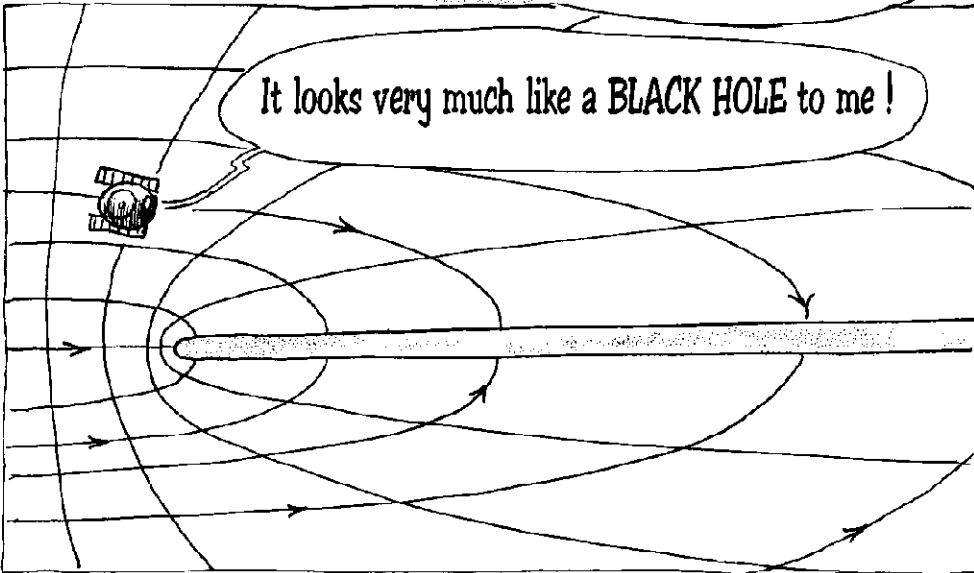
Hey, look at that ! Straight ahead !



It looks like a navel

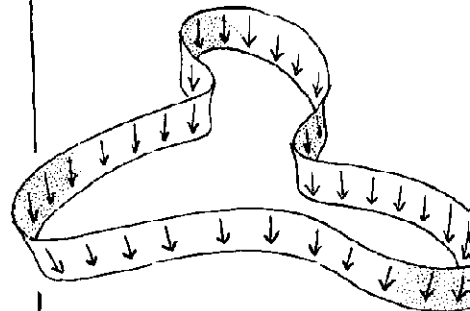
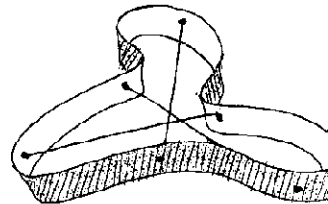
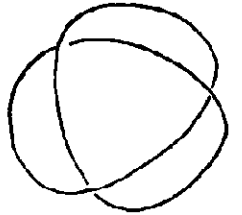
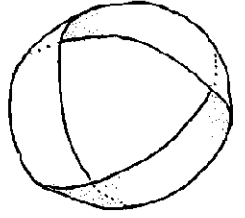
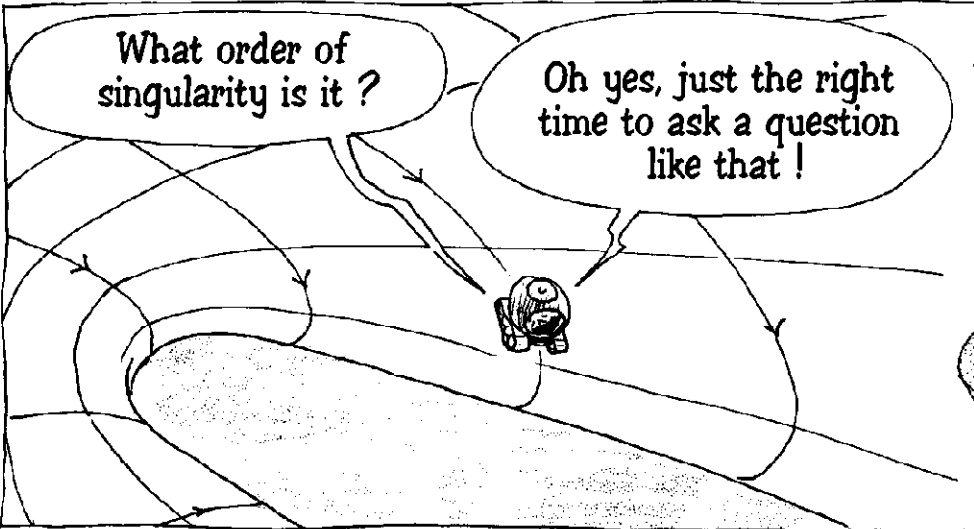
Our Universe line is going straight towards it !

It looks very much like a BLACK HOLE to me !

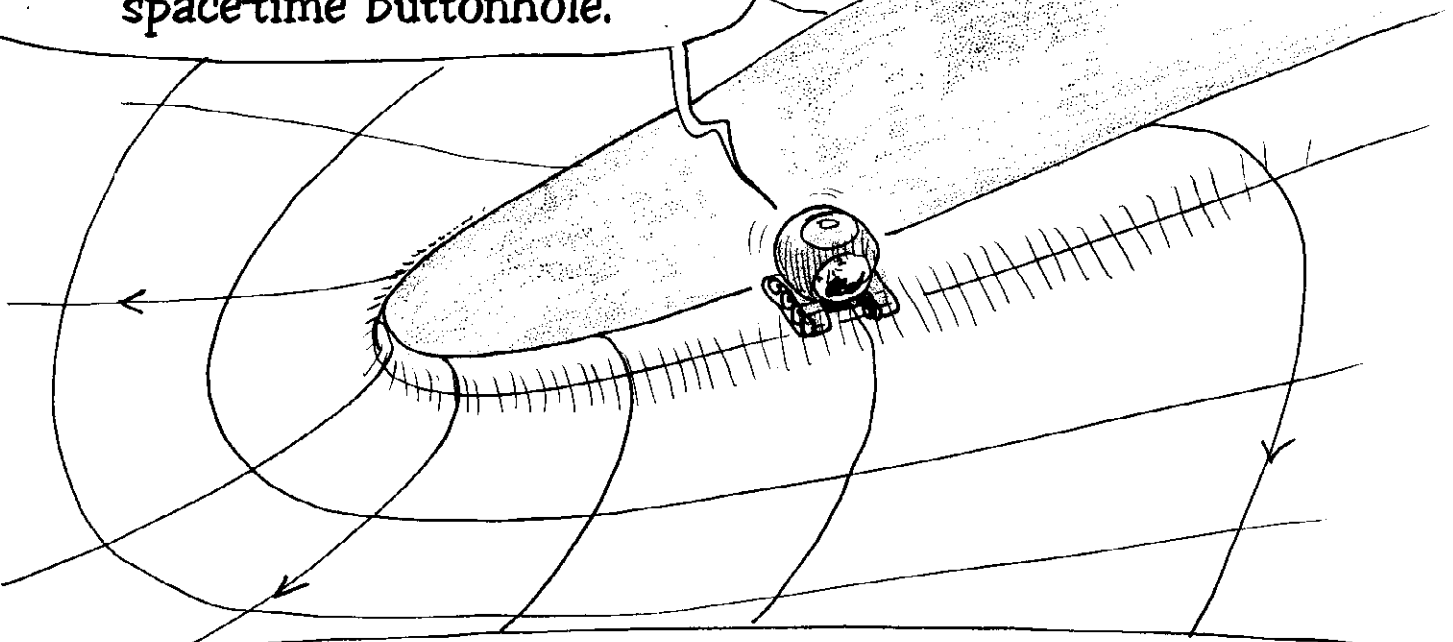


What order of singularity is it ?

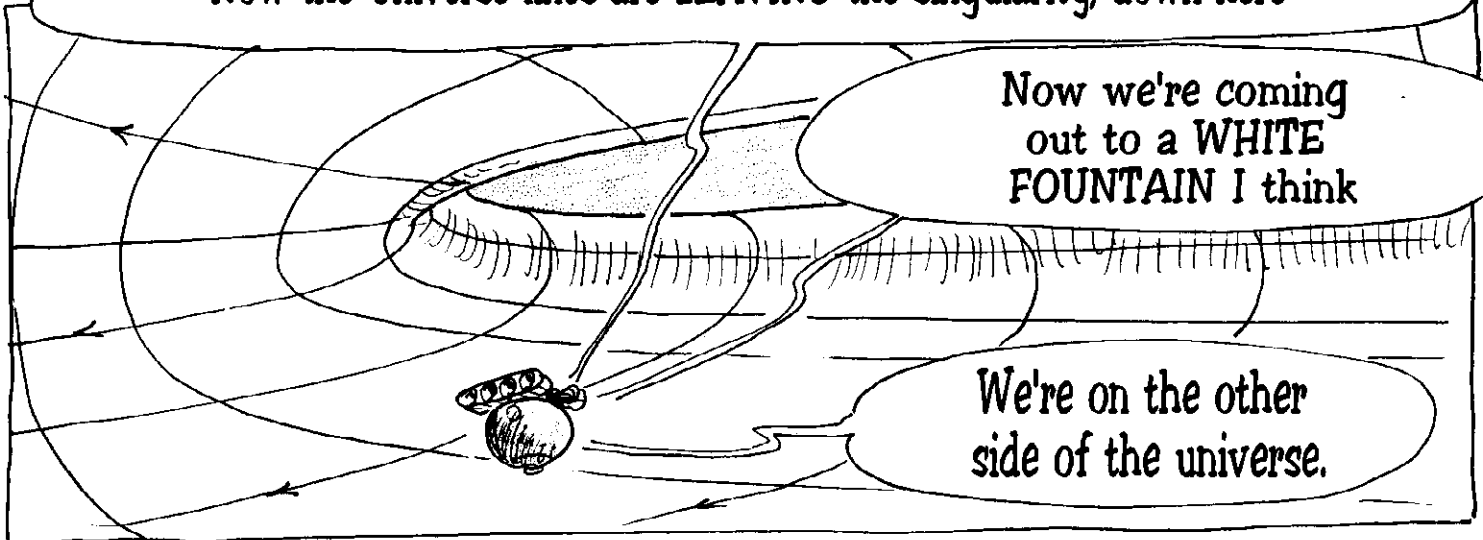
Oh yes, just the right time to ask a question like that !



It looks like a sort of spacetime buttonhole.



Now the Universe lines are LEAVING the singularity, down here



Now we're coming out to a WHITE FOUNTAIN I think

We're on the other side of the universe.

It looks very much like the other side except that it goes the opposite way. And I have a distinct impression of 'déjà vu' don't you ?

Ah, I'm getting it,
the MIRROR !...

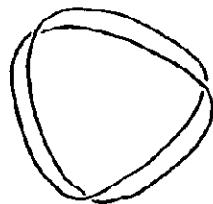
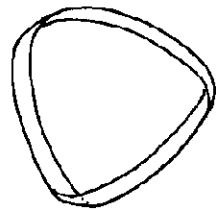
What mirror ?

The two halves of the universe are mirrored
in relation to each other but it's a
SPATIO-TEMPORAL mirror. On the other side of
the black hole everything is inverted in
relation to time, the laws of physics:
singularity repels matter instead of
attracting it !!(*)

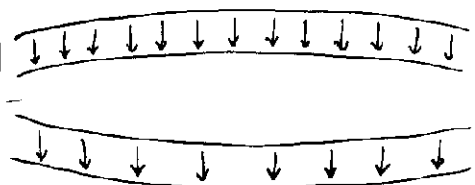
Does that mean that we're
going to relive this book
in the other direction ?

Yes. The CHRONOSCAPE will stop,
then Archie will open the door,
then Tiresias will go out for
a crawl, then...

FIN



BILATERAL STRIP
ANTIPODAL POINTS JOINED



(*) THE SAME STRUCTURE CAN EXIST IN 4 DIMENSIONS.

SCIENTIFIC ANNEX

BOY, a pupil of Hilbert, discovered his surface in 1902. The first analytical representation of it was given in 1981 by Jérôme Souriau, son of the mathematician J.M. SOURIAU, and the author of this book. The semiempirical method used assimilates the meridians of the surface to ellipses which are then given parameters. The current point is given by :

$$\begin{cases} x = X_1 \cos \mu - Z_1 \sin \alpha \sin \mu \\ y = X_1 \sin \mu + Z_1 \sin \alpha \cos \mu \\ z = Z_1 \cos \alpha \end{cases} \quad \begin{cases} X_1 = \frac{A^2 - B^2}{\sqrt{A^2 + B^2}} + A \cos \theta - B \sin \theta \\ Z_1 = \sqrt{A^2 + B^2} + A \cos \theta + B \sin \theta \end{cases}$$

$$\alpha = \frac{\pi}{8} \sin 3\mu \quad \begin{cases} A(\mu) = 10 + 1,41 \sin(6\mu - \pi/3) + 1,98 \sin(3\mu - \pi/6) \\ B(\mu) = 10 + 1,41 \sin(6\mu - \pi/3) - 1,98 \sin(3\mu - \pi/6) \end{cases}$$

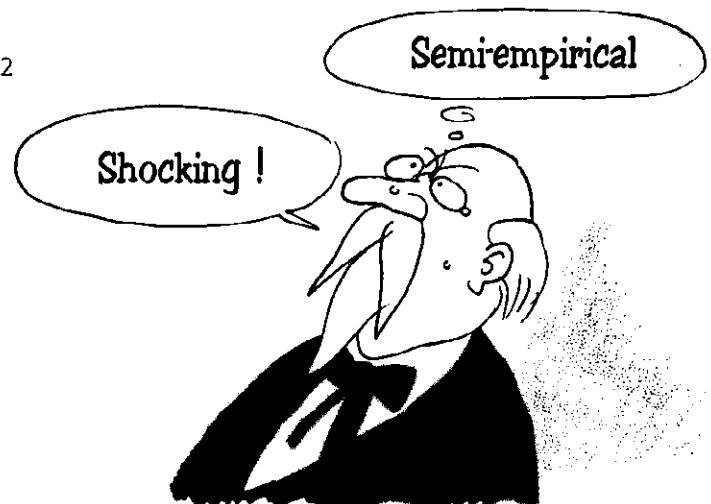
Meridians: curves $\mu = cte$, θ variant of 0 to 2π , μ variant of 0 to π

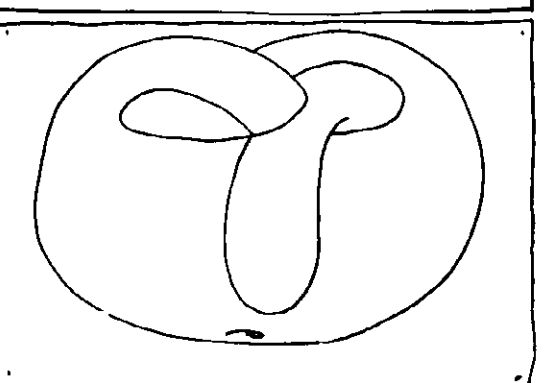
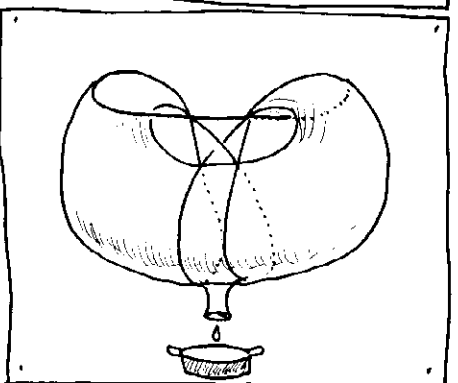
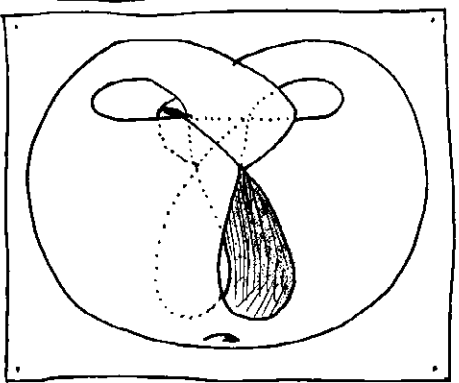
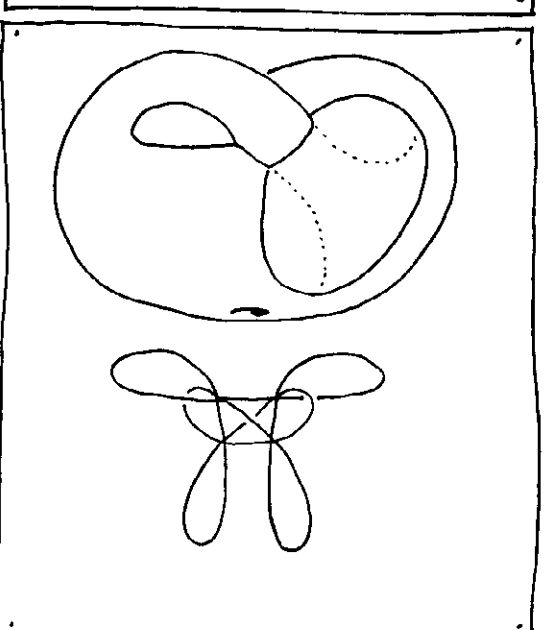
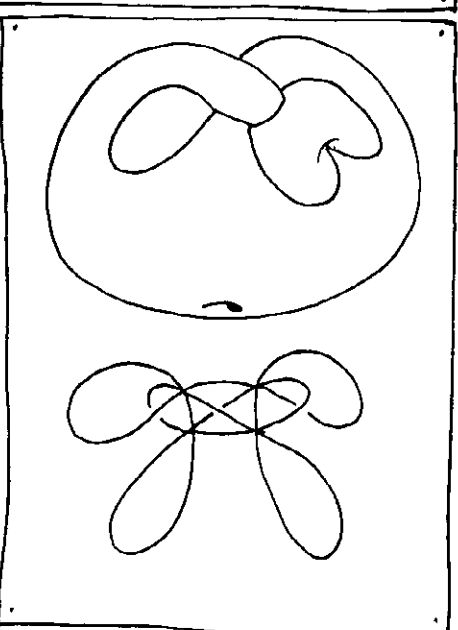
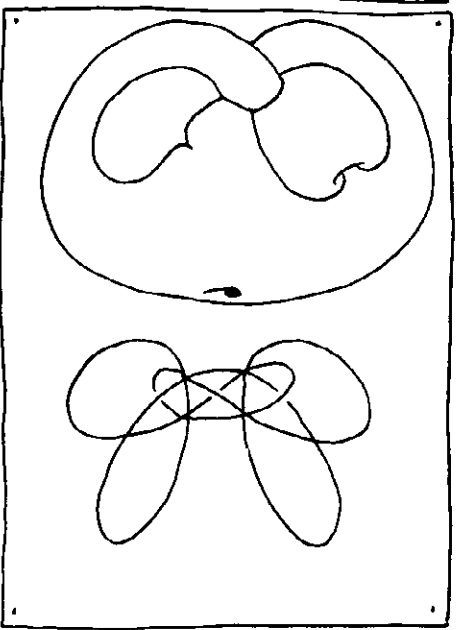
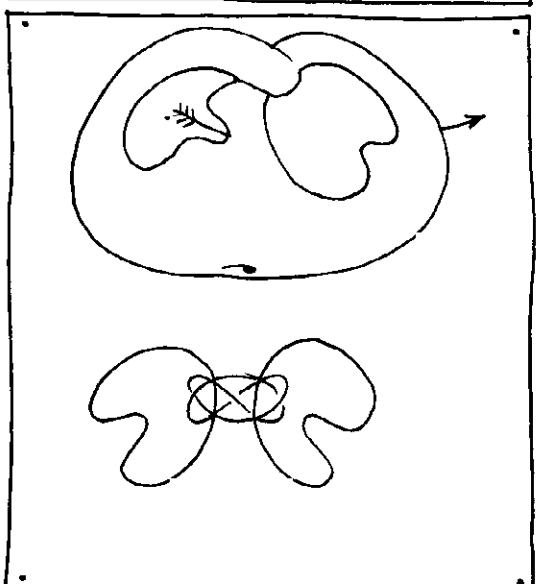
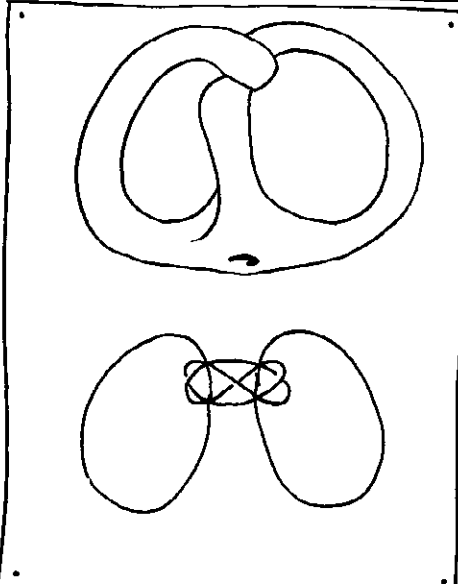
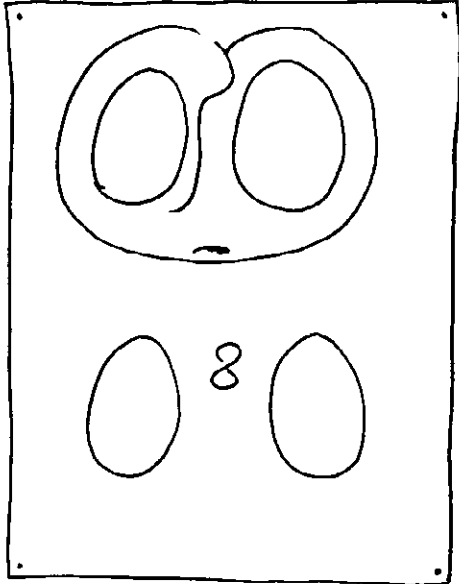
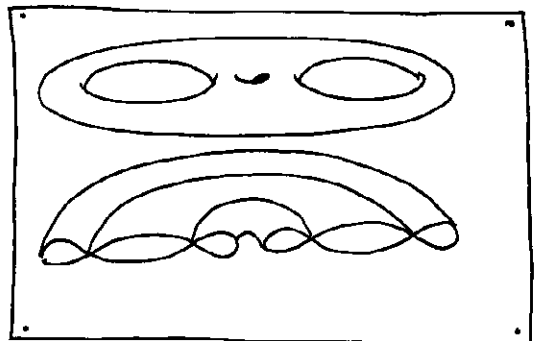
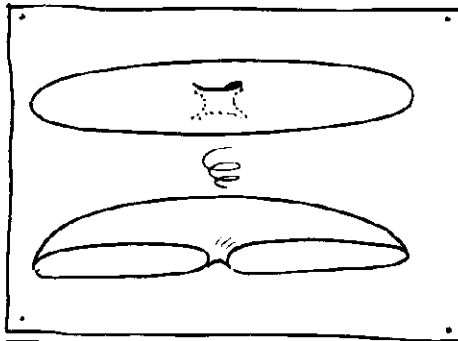
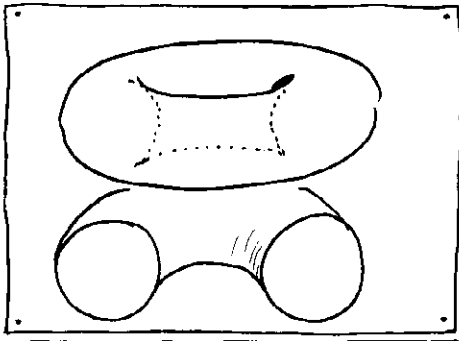
The following programme in BASIC traces the drawing on the cover pages

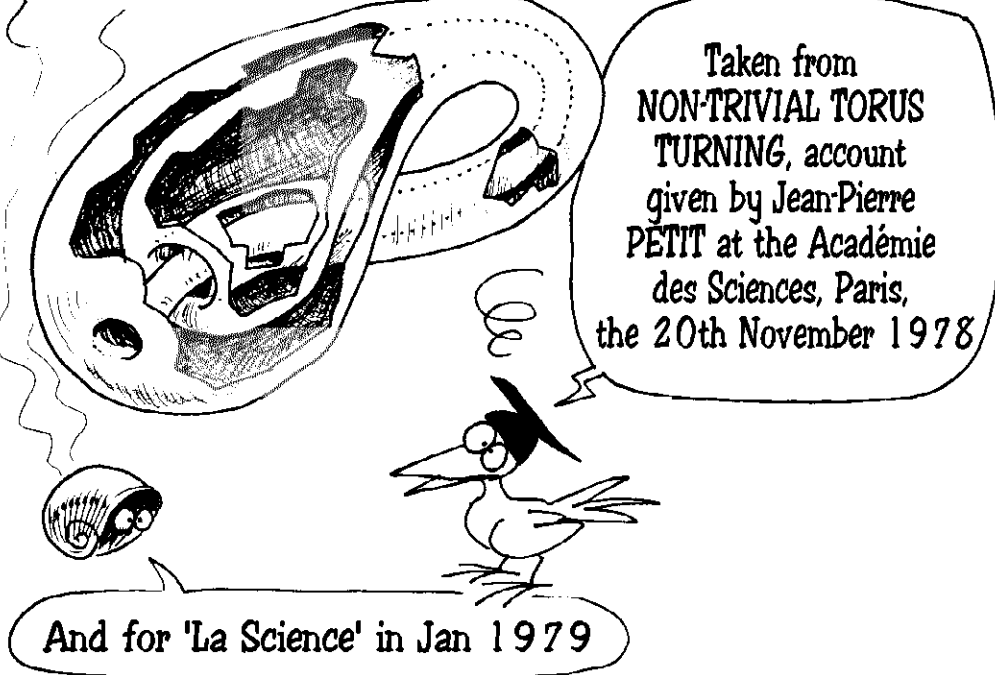
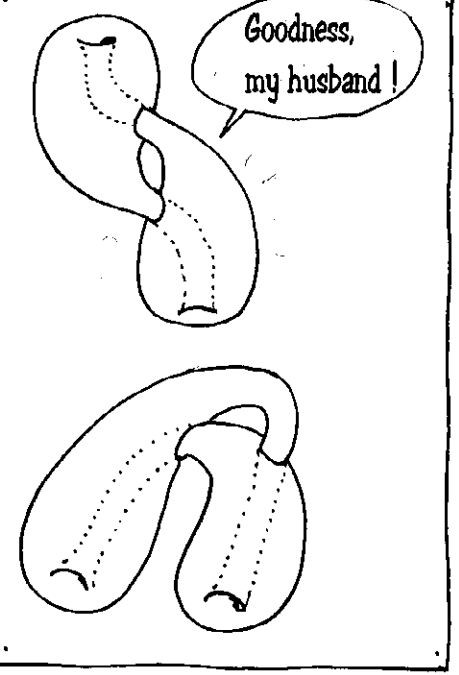
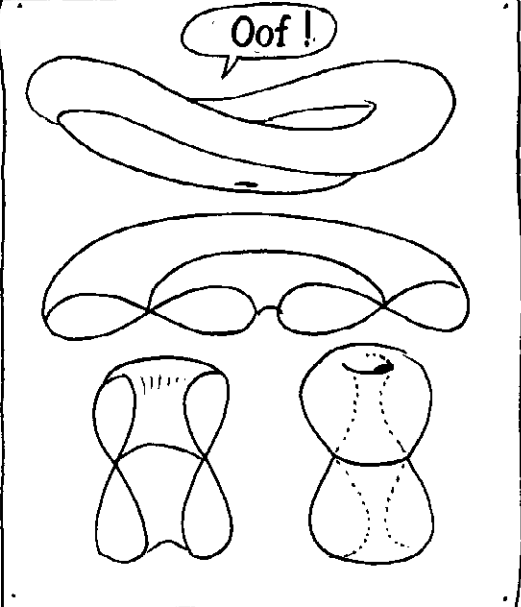
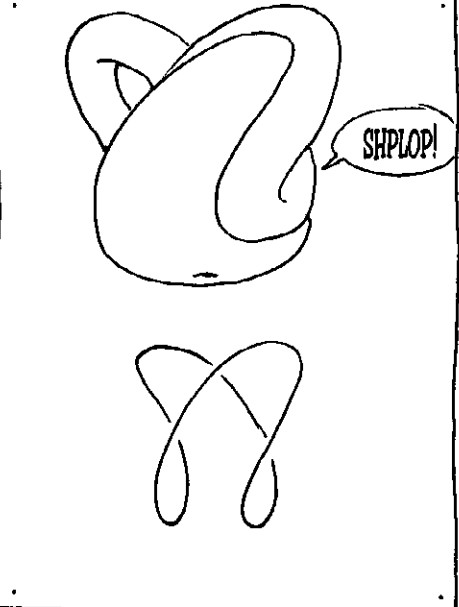
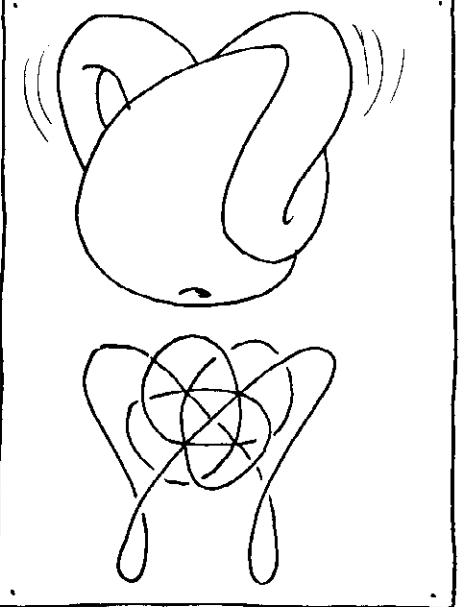
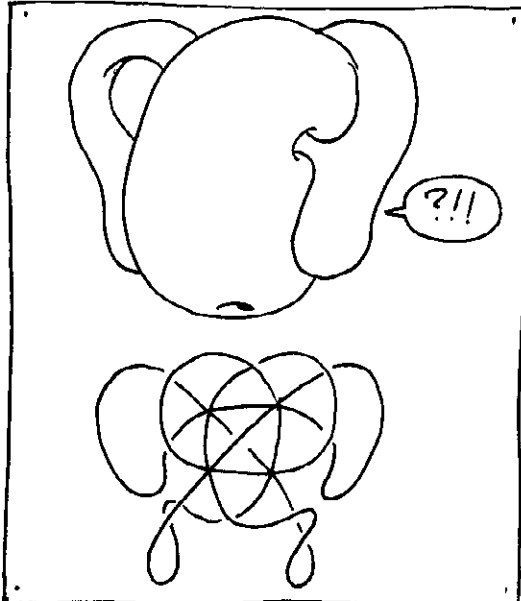
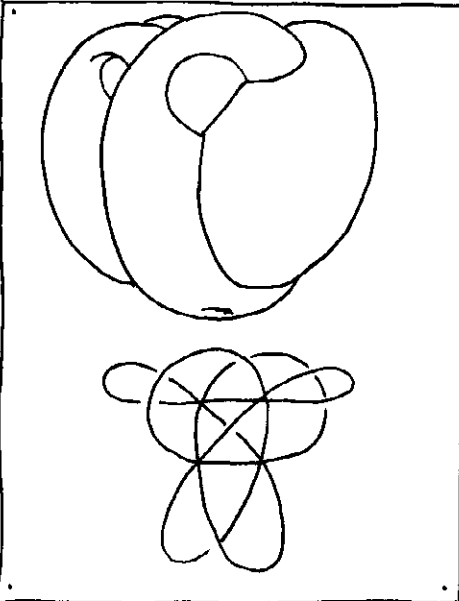
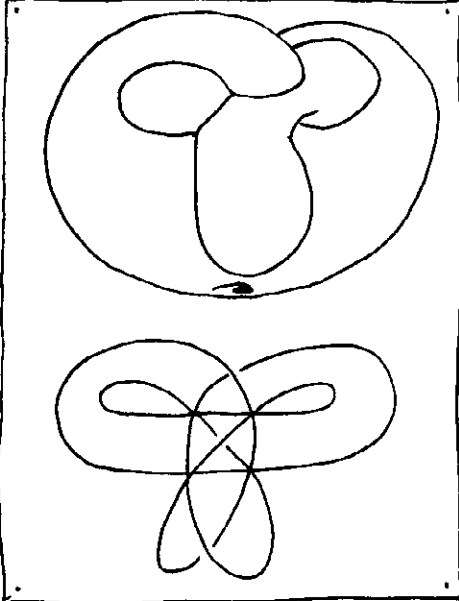
```

1 REM TRACE MERIDIENS DE LA SURFACE DE BOY
3 HOME : TEXT
50 PI = 3.141592:P3 = PI / 3:P6 = PI / 6:P8 = PI / 8
60 HGR : HCOLOR= 3
90 FOR MU = 0 TO PI STEP 0.1
95 P = P + 1
100 D = 34 + 4.794 * SIN (6 * MU - P3)
110 E = 6.732 * SIN (3 * MU - P6)
120 A = D + E:B = D - E
130 SA = SIN (P8 * SIN (3 * MU))
140 C2 = SQR (A * A + B * B):C3 = (4 * D * E) / C2
160 CM = COS (MU):SM = SIN (MU)
180 FOR TE = 0 TO 6.288 STEP .06
190 TC = A * COS (TE):TS = B * SIN (TE)
200 X1 = C3 + TC - TS
210 Z1 = C2 + TC + TS
250 REM VOICI LES 3 COORDONNEES
300 X = X1 * CM - Z1 * SA * SM
310 Y = X1 * SM + Z1 * SA * CM
350 REM PROGRAMME DE DESSIN
360 HPLOT 130 + X,80 + Y
400 NEXT TE: NEXT MU

```







Taken from
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