Spatial Indexing
GMP 24/25

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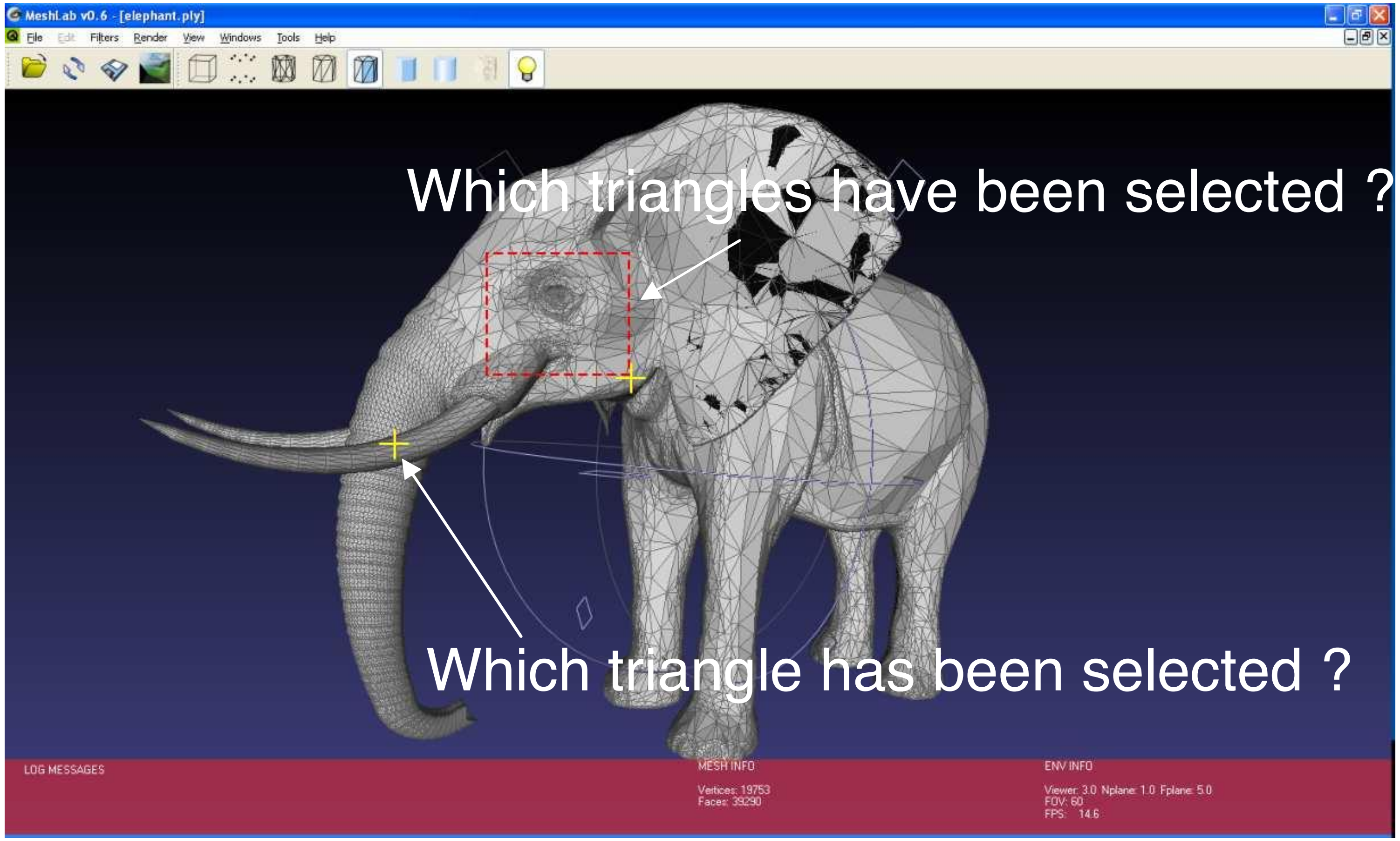
Problem statement

- Let ***m*** be a mesh:
  - Which is the mesh element closest to a given point ***p***?
  - Which are the elements inside a given region on the screen?
  - Which elements are intersected by a given ray r?
- Let m’ be another mesh:
  - Do ***m*** and **m’** intersect? If so, where?

A **spatial search data structure** helps to answer efficiently to these

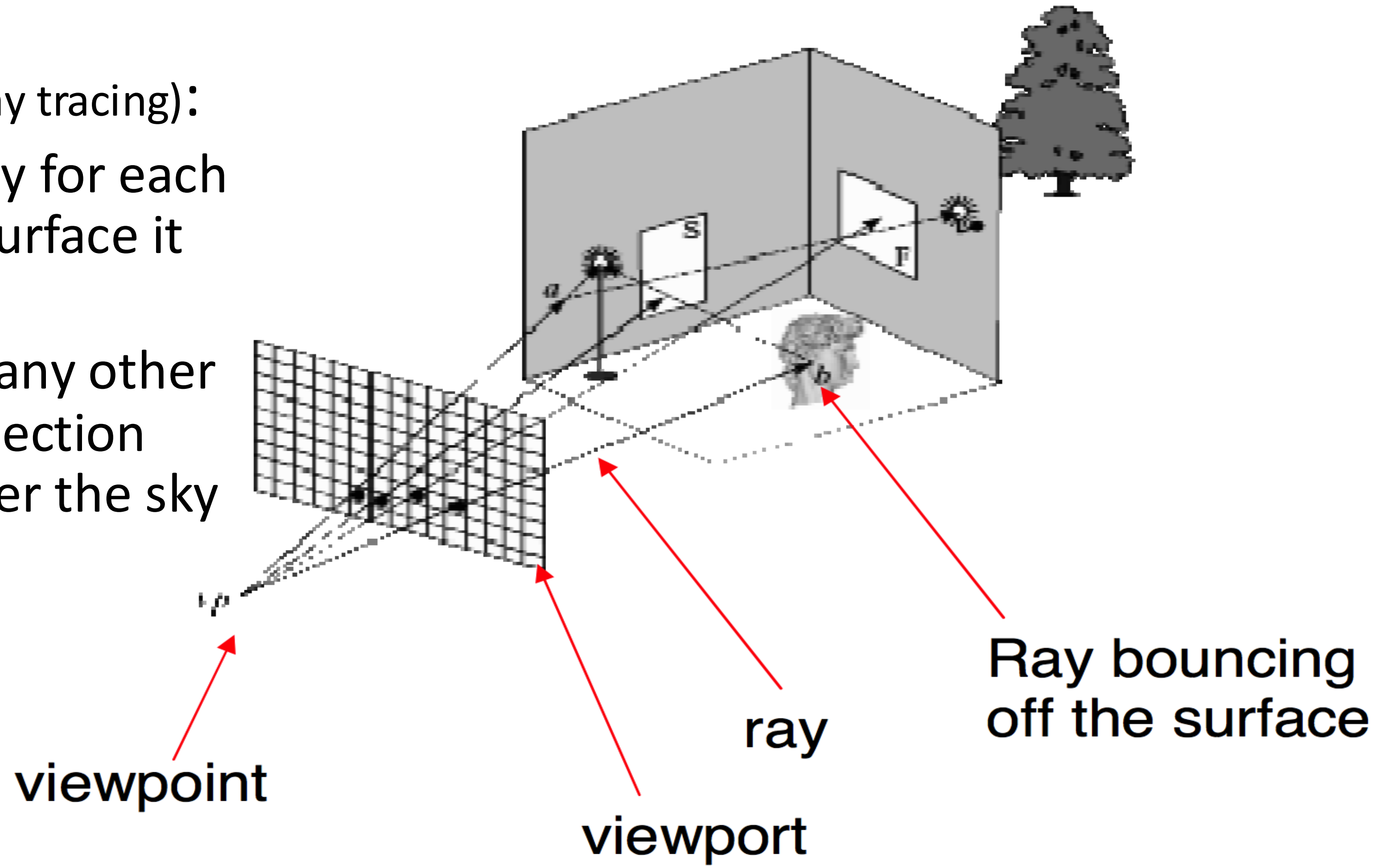
Problem statement

- Picking on a point
- Selecting a region



Problem statement: Rendering

- Path tracing (aka unbiased ray tracing):
  - From the eye, shoot a ray for each pixel, and find the first surface it encounters.
  - From this point shoot many other rays and find their intersection Recur until you find either the sky or an emissive surface

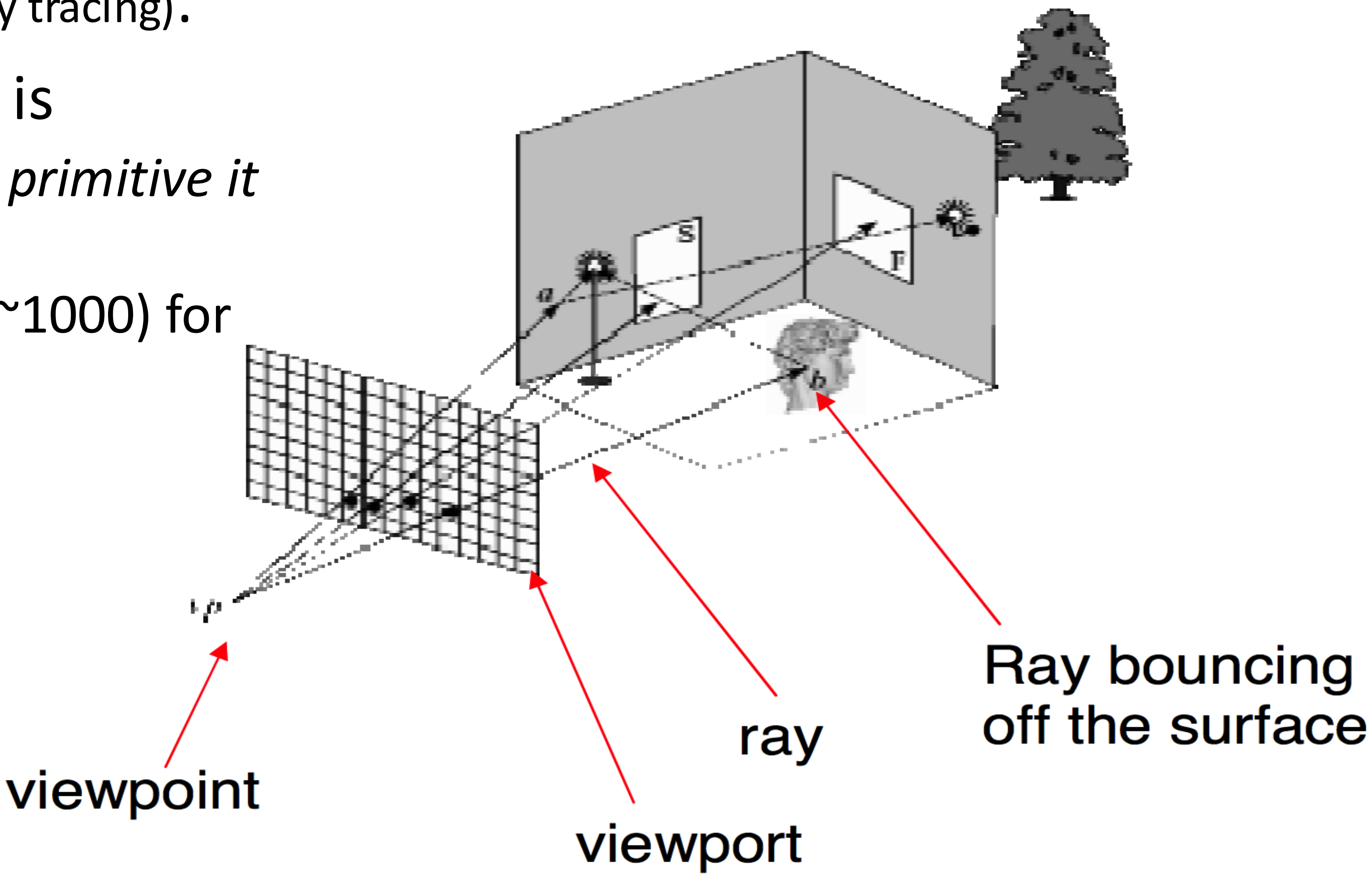


Problem statement: Rendering

- Path tracing (aka unbiased ray tracing):
- The ***core*** of the problem is

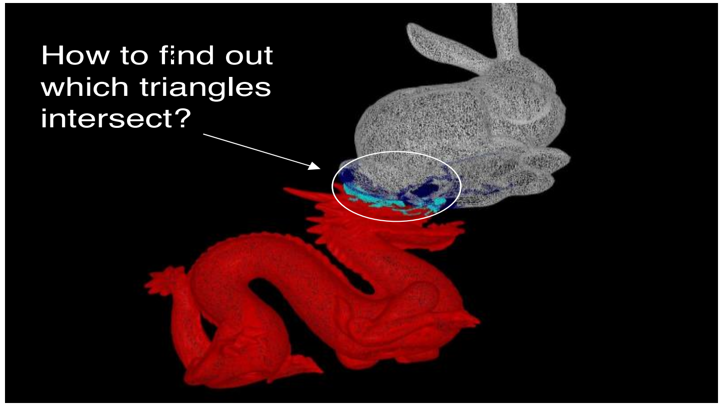
  Given a ray find the first primitive it encounters.

  - You shoot many rays (10~1000) for each hit surface
  - Primitives can easily be O(10^5) ~ O(10^9)



Problem statement: Dynamics/Simulation

- Simulating rigid body dynamics requires mainly two tasks:
  - Computing the position according to current forces
  - Computing what are the new forces according the current positions
    - Reaction forces after collision



Problem statement

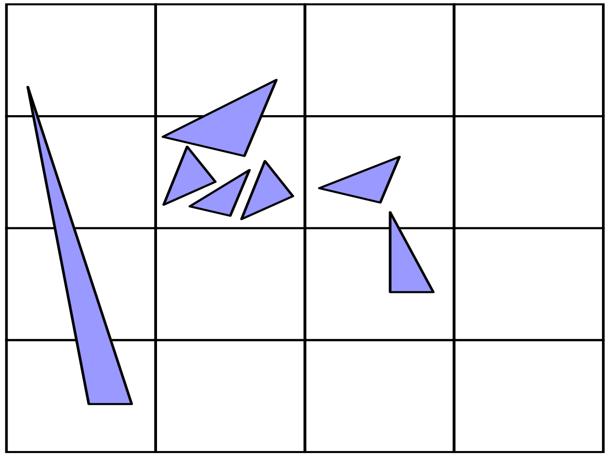
- Without any spatial search data structure, the solutions to these problems require O(n) time, where n is the numbers of primitives ($O(n^2)$ for the collision detection)
- Spatial data structure can make it (average) almost **constant** or expected logarithmic.
- Strong complexity lower bound (worst case log) are possible only for restricted (often not-practical) settings.
  - Hard to be proved, reasonable heuristics are the the standard

Indexing Structures

- Two Class of structures
- **Non-Hierarchical** / Flat space subdivision
  - It would seem trivial, but there are reasons for them
- **Hierarchical**
  - Divide et impera / adaptive subdivision

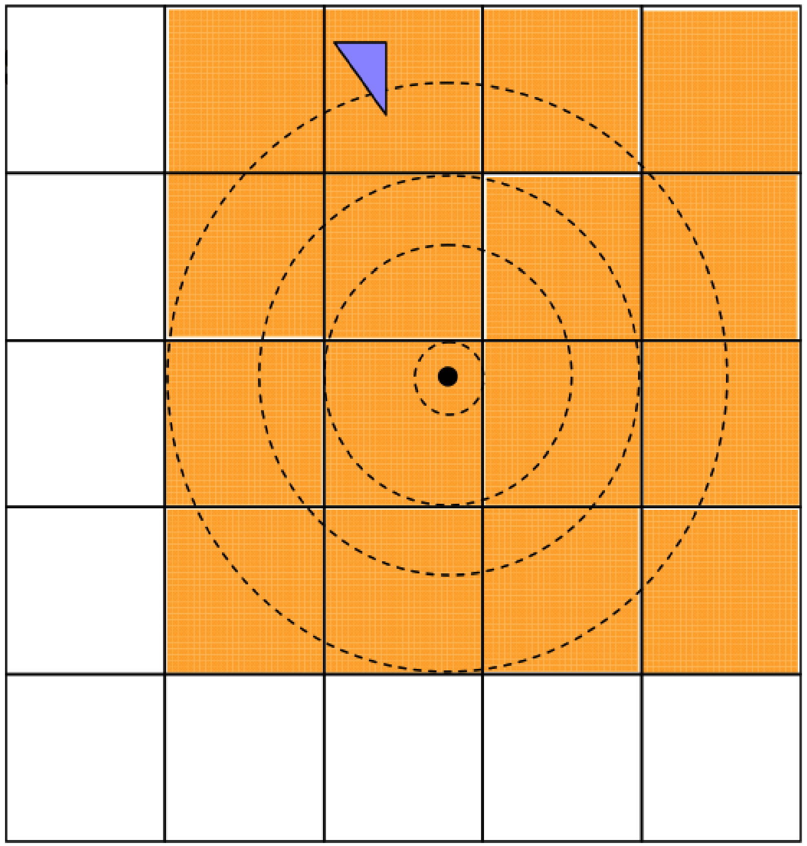
Uniform Grid

- **Description**: the space including the object is partitioned in **cubic cells**; each cell contains references to “primitives” (i.e., triangles)
- **Construction**.
  Primitives are assigned to:
  - The cell containing their feature point (e.g., barycenter or one of their vertices)
  - All the cells spanned by each primitive
- **Regular grids access by position is trivial:**
  - If you want to know if something is at (x,y,z)
    just use integer division…



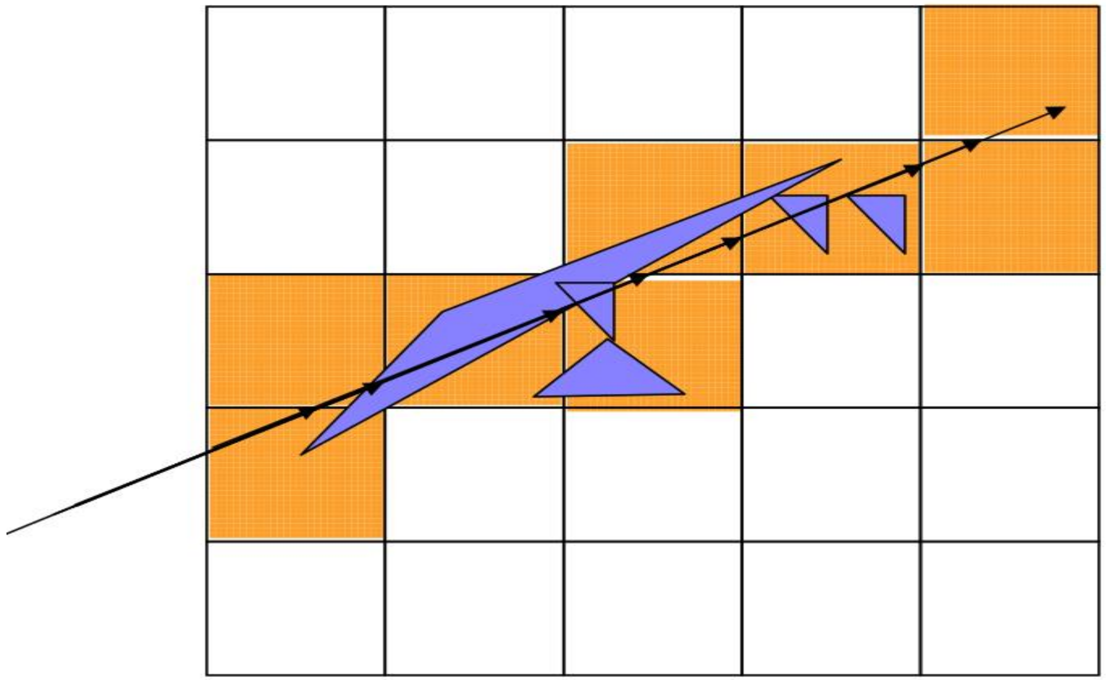
Uniform Grid

- **Closest element** (to point p)**:**
  - Start from the cell containing p
  - Check for primitives inside growing spheres centered at p
  - At each step the ray increases to the border of visited cells
- **Cost**
  - Worst**: O(#cells+n)**
  - Average**: O(1)**



Uniform Grid

- **Intersection with a ray:**
  - Find all the cells intersected by the ray
  - For each intersected cell, test the intersection with the primitives referred in that cell
  - Avoid multiple testing by flagging primitives that have been tested (mailboxing)
- **Cost:**
  - Worst: $O(\#cells + n)$
  - Aver: $O(\sqrt[d]{\#cells} + \sqrt[d]{n})$



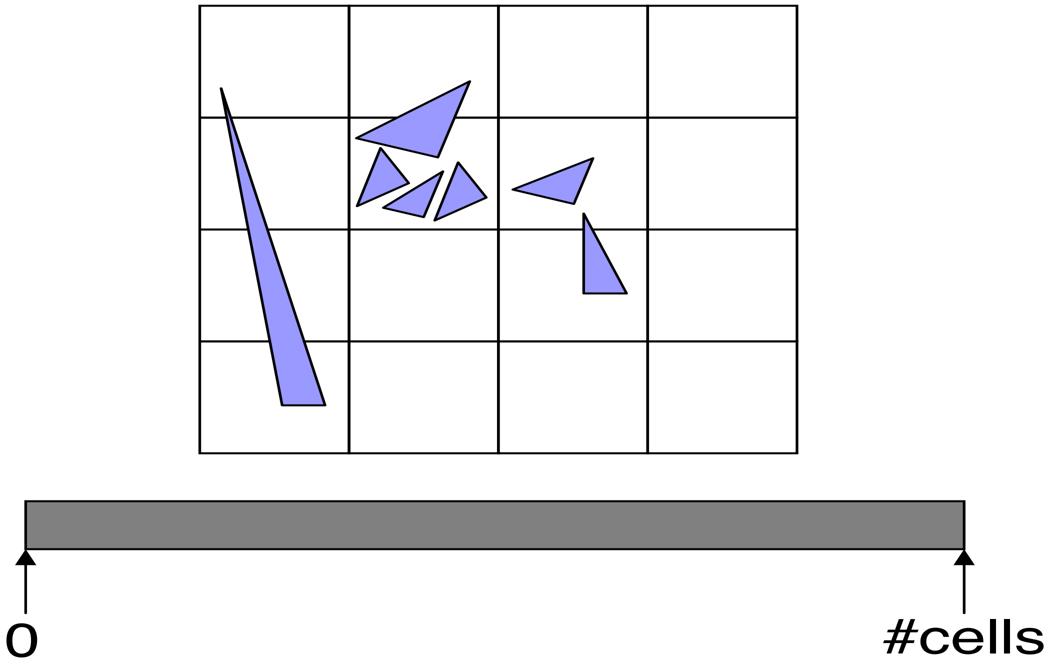
Uniform Grid

- **Memory occupation:** $O(\#\ cells + n)$
- **Pros:**
  - Easy to implement
  - Fast query
- **Cons**:
  - Memory consuming
  - Performance **very** sensitive to distribution of the primitives.

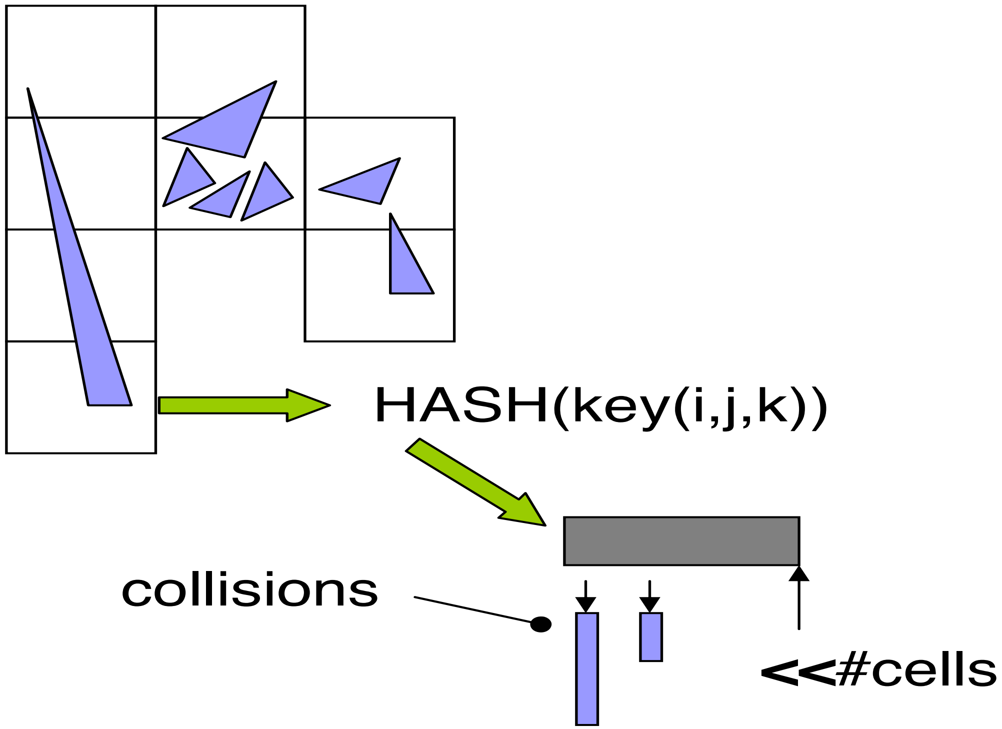
Spatial Hashing

- The same as uniform grid, except that only non empty cells are allocated

Uniform grid



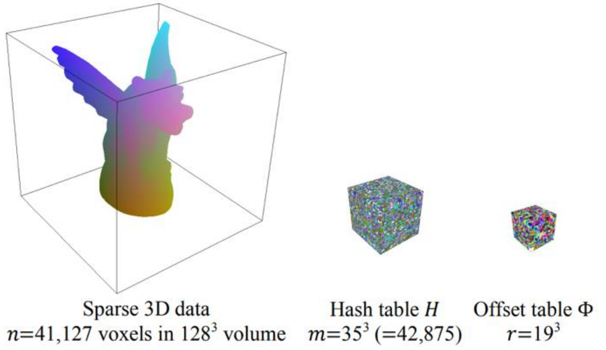
Spatial hashing



Spatial Hashing

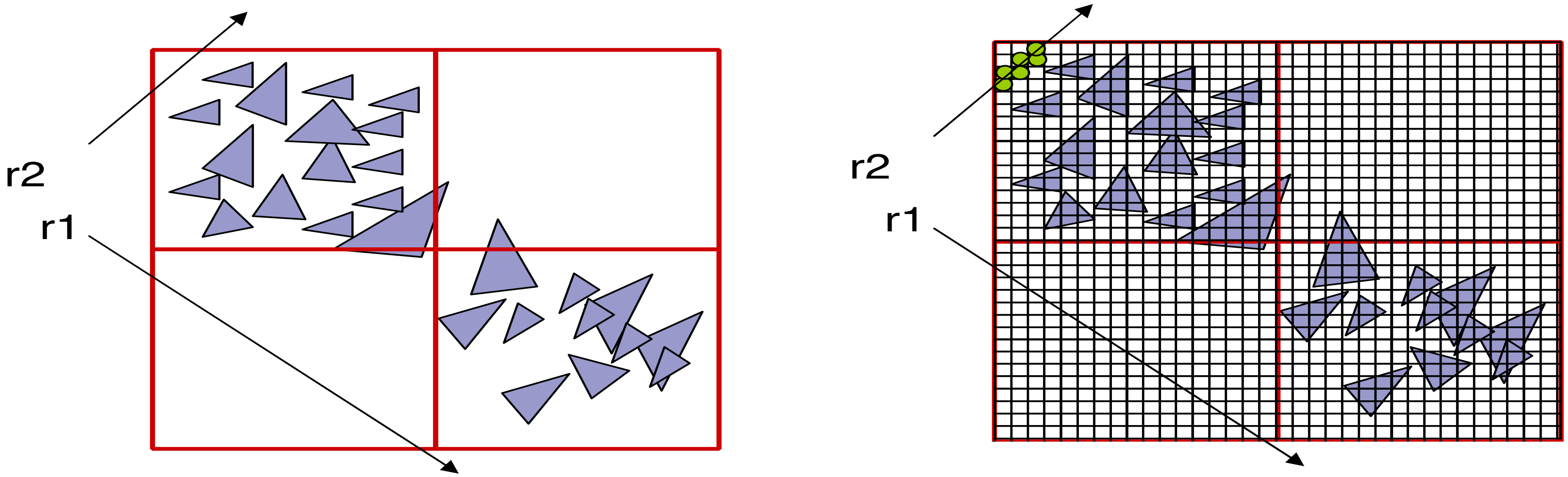
- **Cost:** same as UG, except that in worst case the access to a cell is *O(#cells)* because of collisions
- **Memory occupation:**
  - **Worst.:** *all volumetric cells are used*
  - **Aver. :** *only a few surface* **intersecting** cells are allocated
- **Pros**
  - Fast query if **good hashing** is done
  - Easy to implement
  - Less memory consuming
- **Cons**:
  - Performance **very** sensitive to distribution of the primitives.

Perfect spatial hashing [Lefebvre,Hoppe 06]



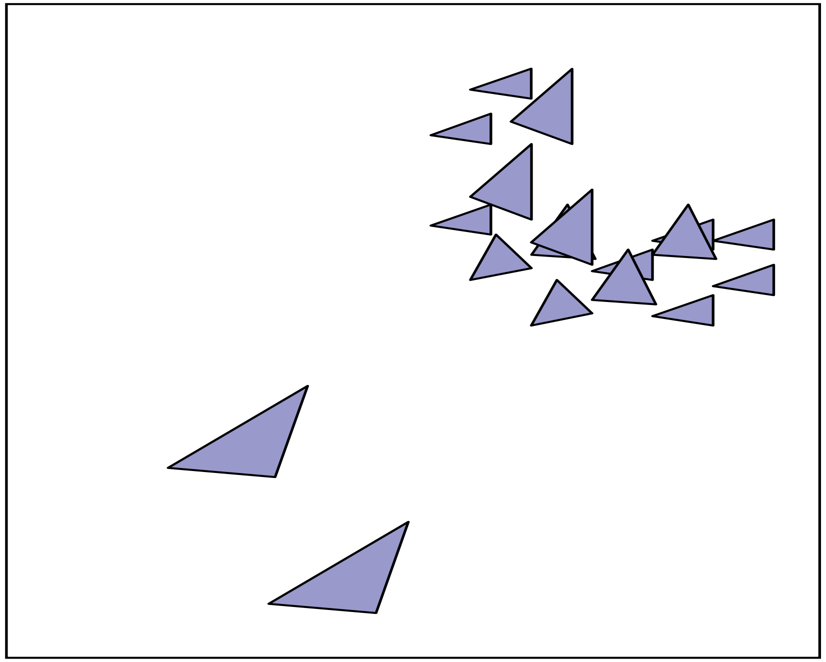
UG Approach: Cell Size

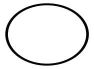
- Uniform grids are **input insensitive**
- What's the best choice for the example below?



Hierarchical Indexing

- Divide et impera strategies:
  - The space is partitioned in sub regions
  - ..recursively





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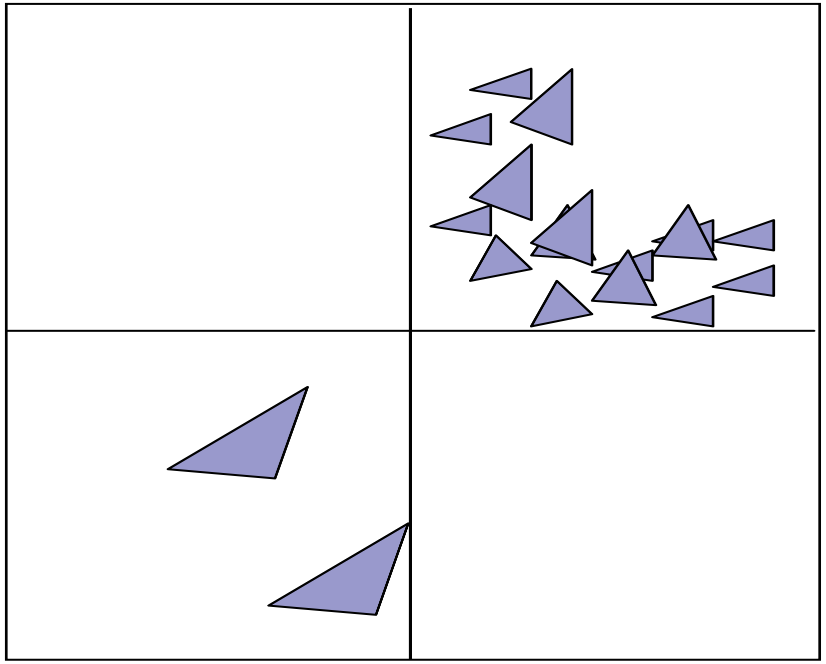

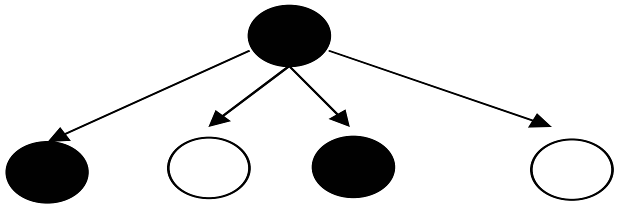

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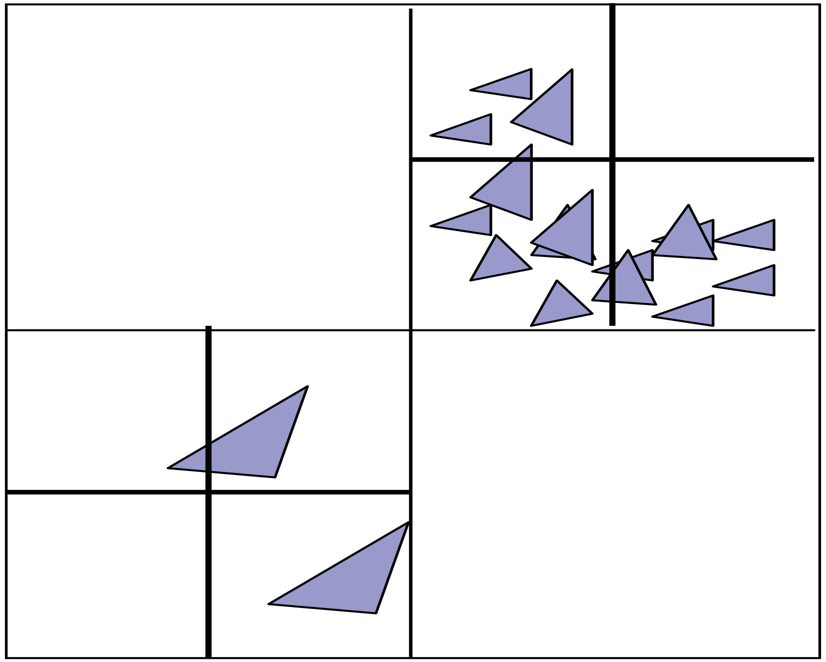

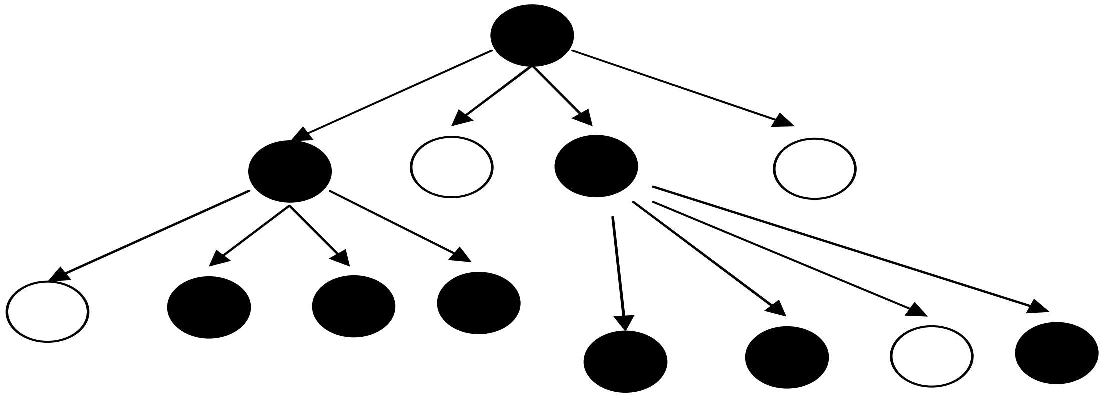

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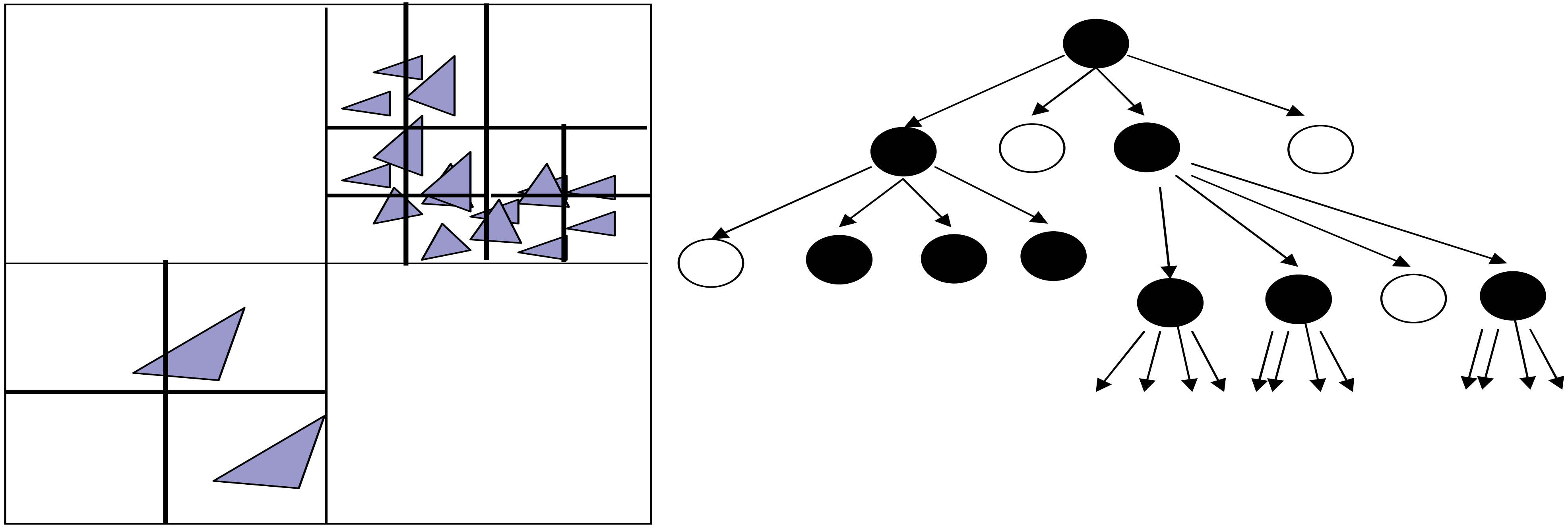
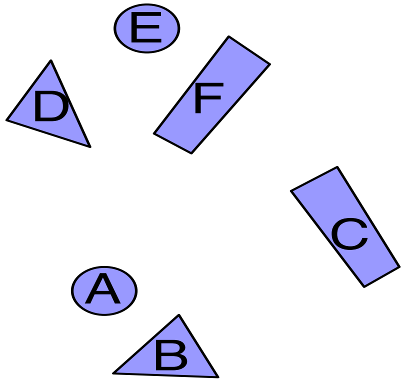


Basic Facts

- The queries correspond to a visit of the tree
  - The complexity is sublinear (logarithmic) in the number of nodes
  - The memory occupation is linear
- A hierarchical data structure is characterized by:
  - Number of children per node
  - Spatial region corresponding to a node

Binary Space Partition-Tree (BSP)

- **Description**:
  - It’s a binary tree obtained by recursively partitioning the space in **two** by a hyperplane
  - therefore a node always corresponds to a **convex region**



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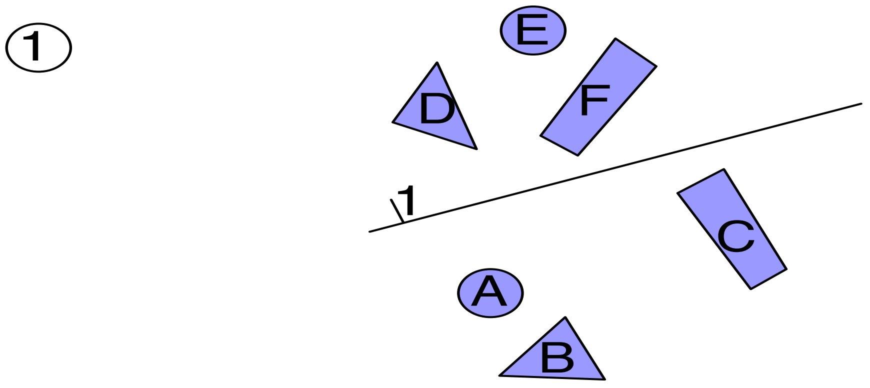

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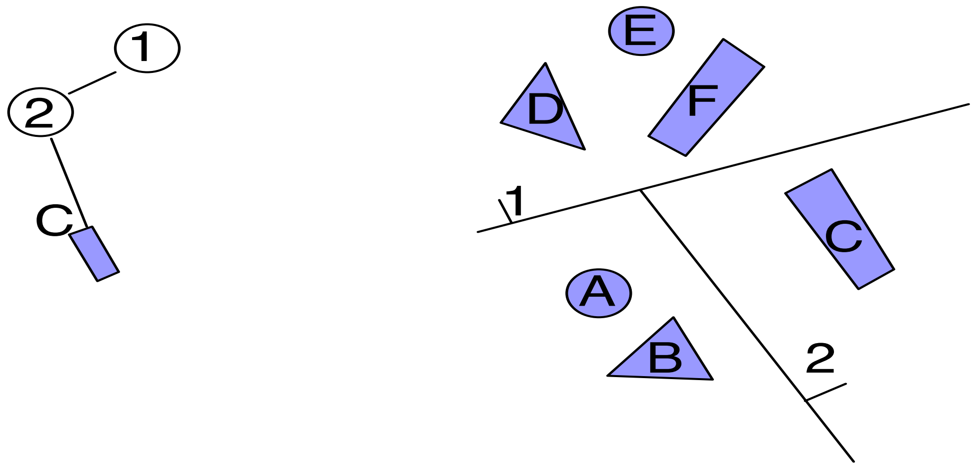

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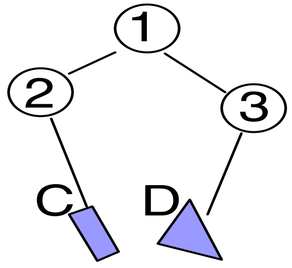

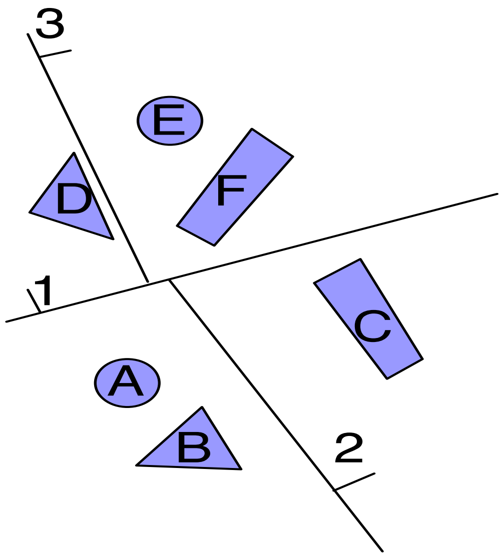

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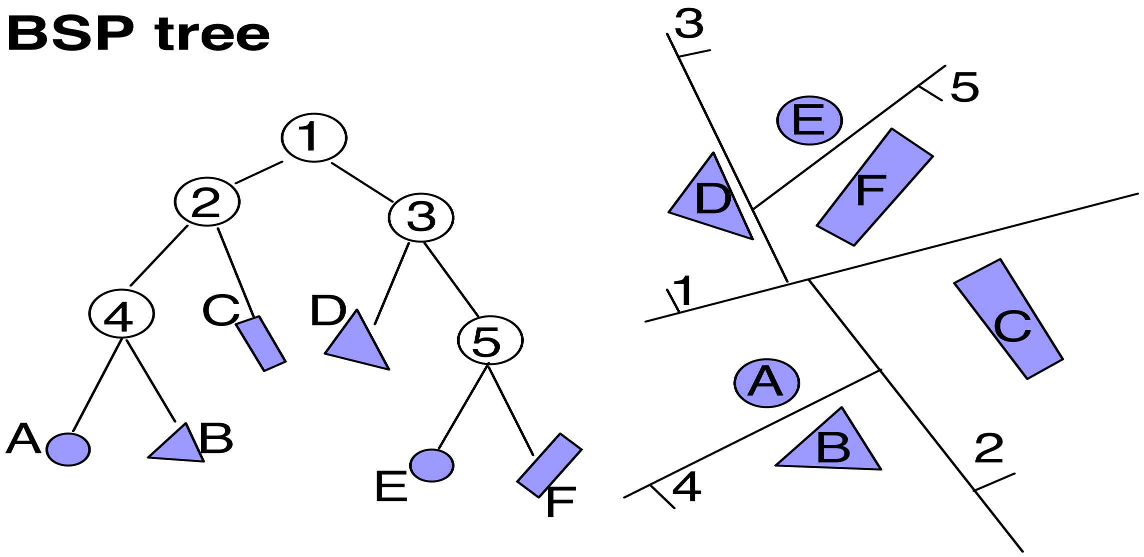
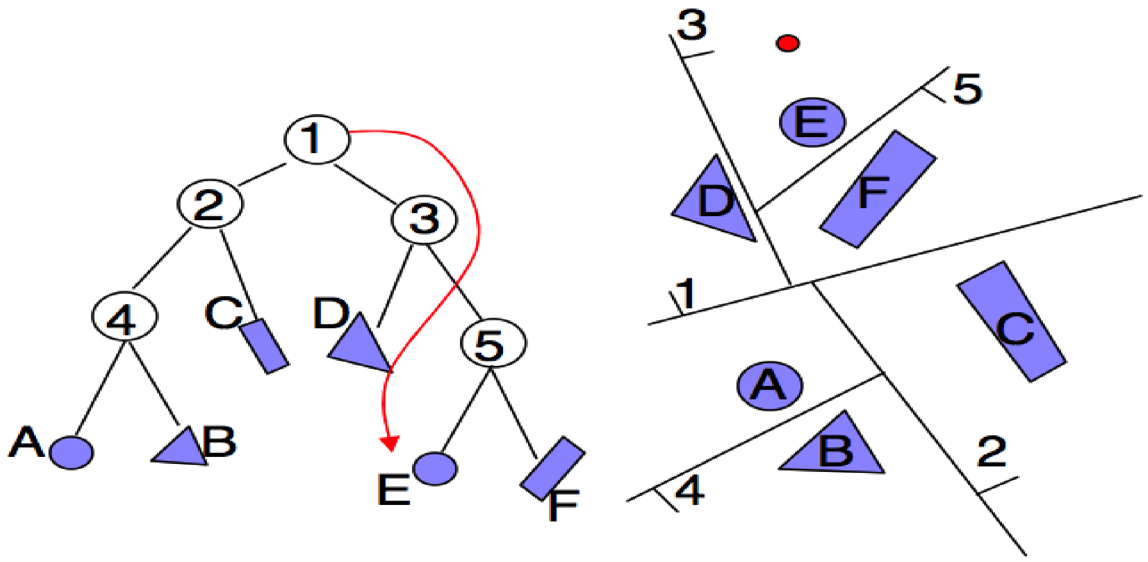
BSP tree



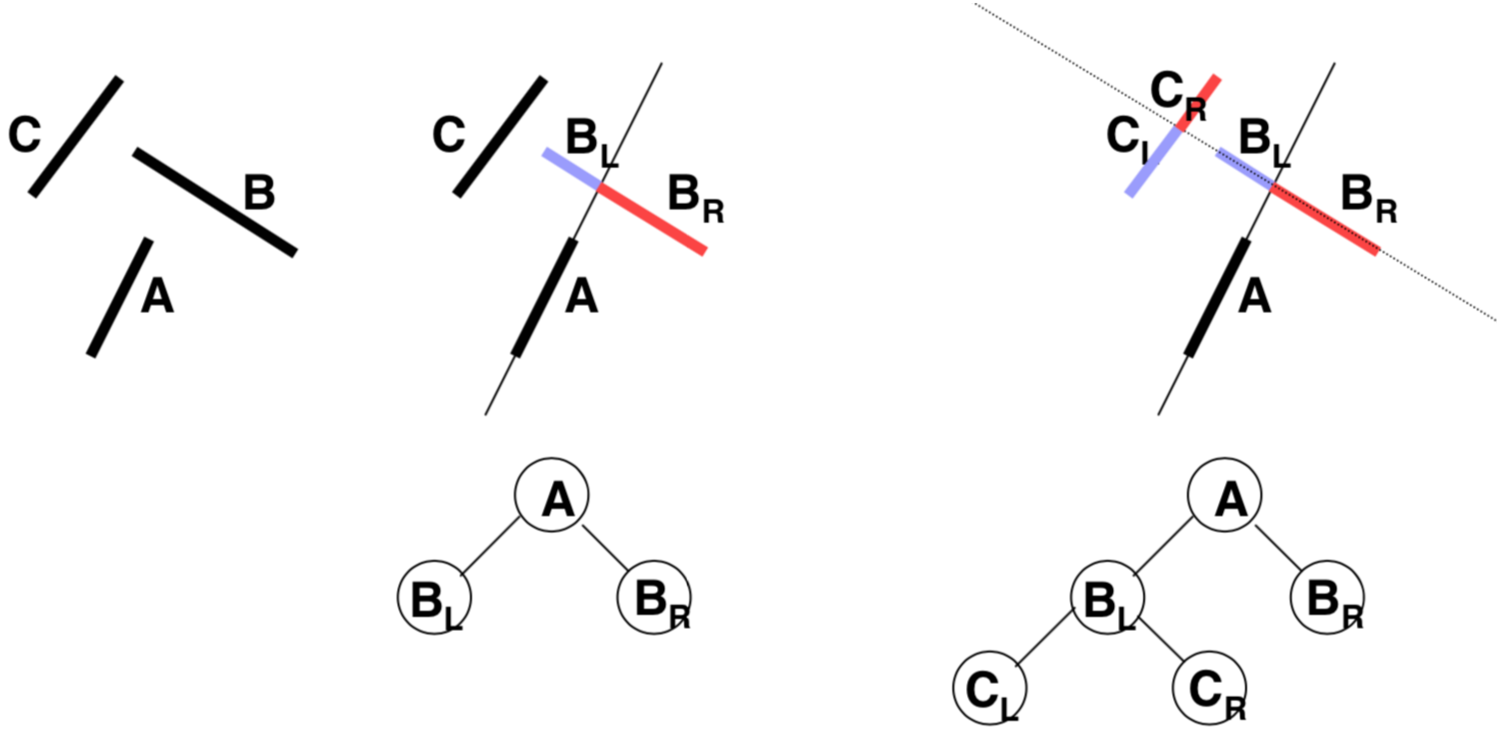
Binary Space Partition-Tree (BSP)

- **Query**: is the point *p* inside a primitive?
  - Starting from the root, move to the child associated with the half space containing the point
  - When in a leaf node, check all the primitives
- **Cost**:
  - Worst: $O(n)$
  - Aver: $O(\log n)$



Binary Space Partition-Tree (BSP)

- What could go wrong?
  - What happen to split primitives? Can I bound them?
- Where to place the plane?
- A common strategy is:
  - Primitives are planar faces:
  - Use one of the primitive as splitting plane and decompose the rest



BSP-Tree Cost

- Building a BSP-tree requires to choose the partition plane
- Choose the partition plane that:
  - Gives the best balance?
  - Minimize the number of splits ?

- .....it depends on the application
- Cost of a BSP-Tree
  $C(T) = 1 + P(T_L)\ C(T_L)+P(T_R)\ C(T_R)$
  - Where $P(T_L)$ is probability that T_L is visited given that T has been visited.

BSP Tree Cost

- How to choose the splitting primitive?
- Try to guess the cost:

$$C(T) = 1 + P(T_L)C(T_L)+P(T_R)C(T_R)$$

- We choose the primitive that minimize

$$1+|S(T_L)|\alpha + |S(T_R)|\alpha + \beta s$$

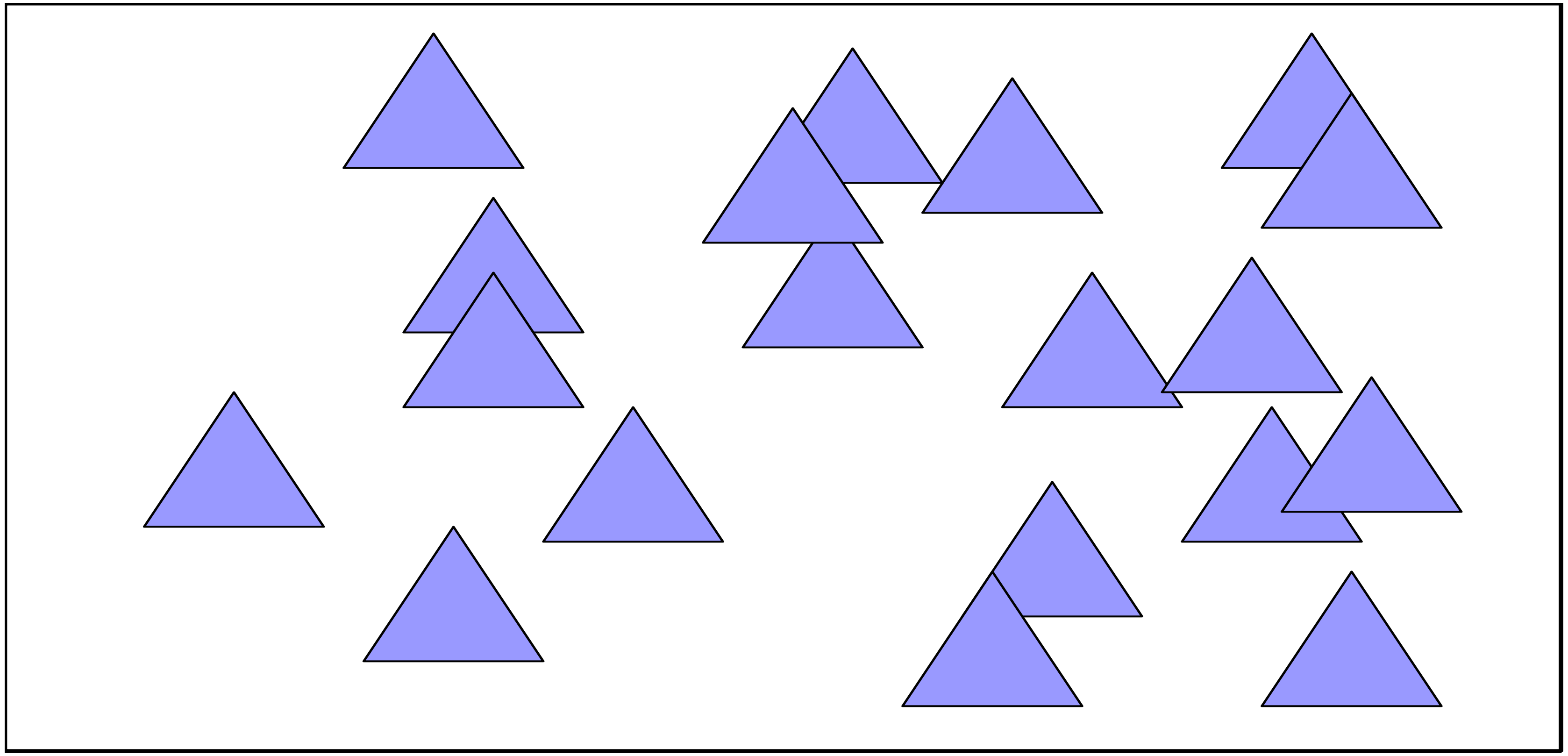
- S_L number of primitives in the left subtree
- s number of primitives split by the chosen primitive

- Big α, small β yield a balanced tree
- Big β , small α yield a smaller tree

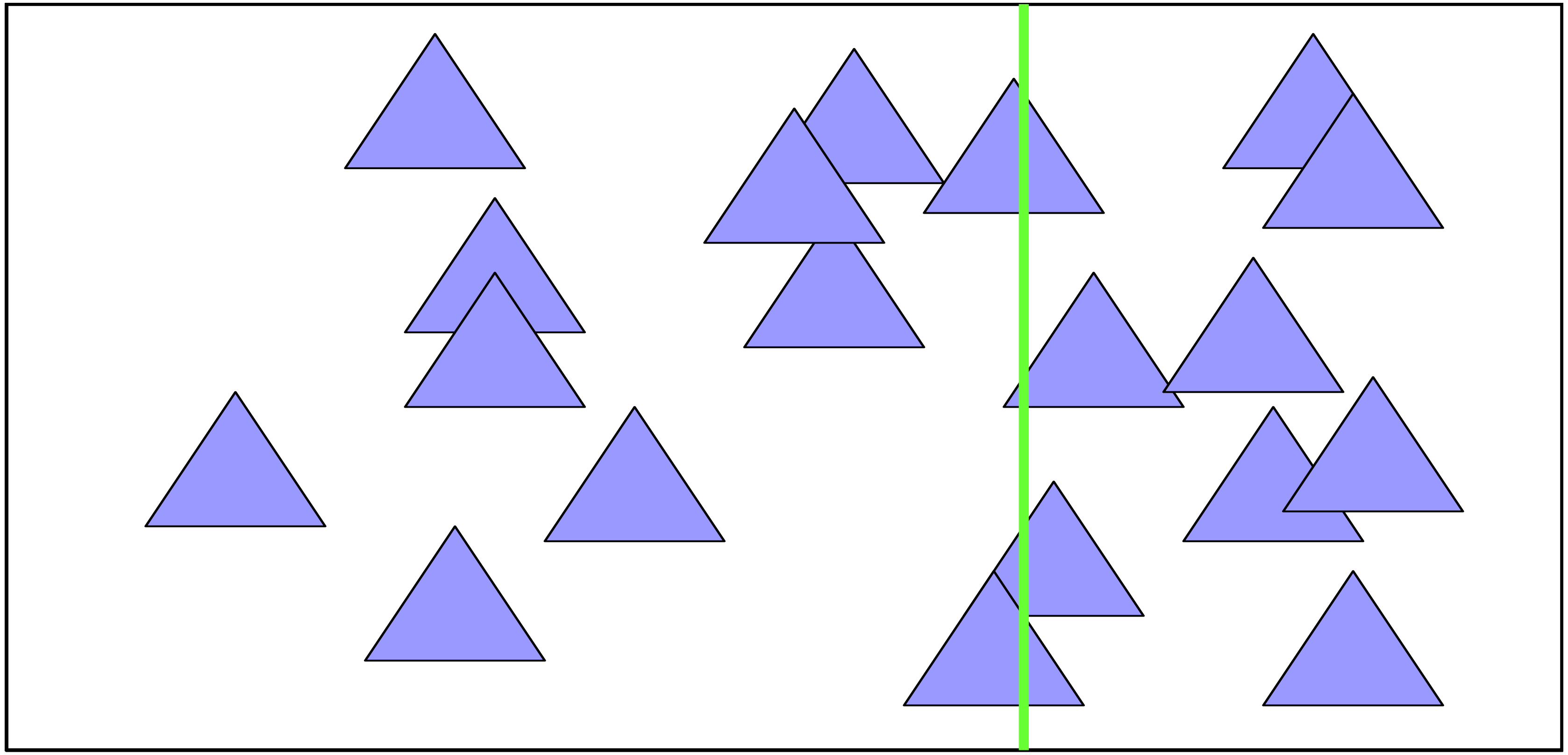
KD-Tree

- Kd-tree : k dimensions tree
- It’s a special kind of BSP tree with axis-aligned bisector planes
- It depends on:
  - Choosen Axis
  - Point on axis where to define the plane
- Advantages wrt BSP:
  - Test are really fast (to explore the tree)
  - Lower memory consumption

KD-Tree



KD-Tree



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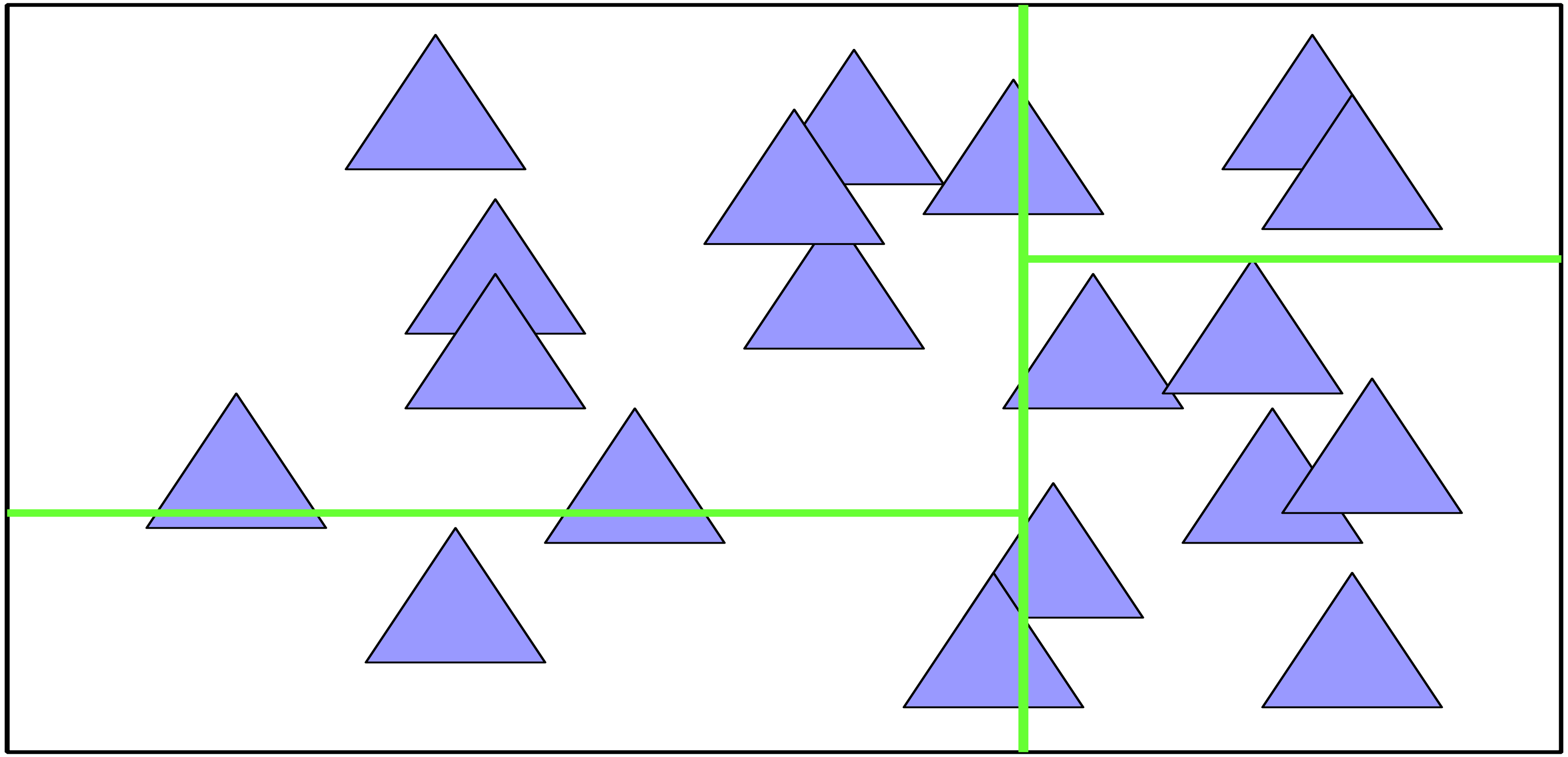
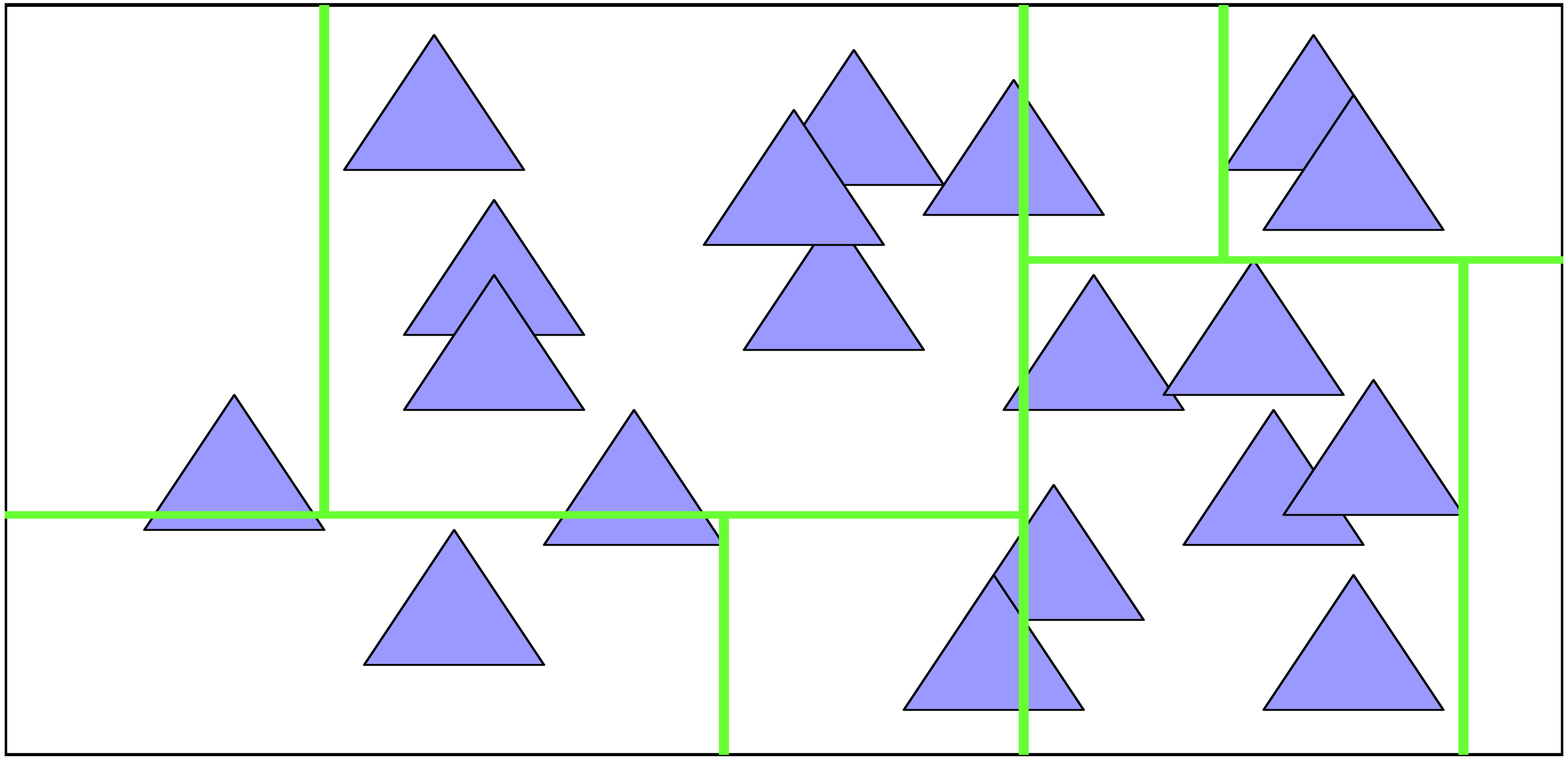


KD-Tree



Kd-tree More on cost

- Example Ray intersection
- $C(T) = 1 + P(T_L)C(T_L)+P(T_R)C(T_R)$
- The cost of a final leaf is roughly the number of primitives
  - (you have to test them)
- $P(T_L)$ is more interesting: $P(T_L) = \frac{|\text{rays intersecting } T_L|}{|\text{rays intersecting } T|}$

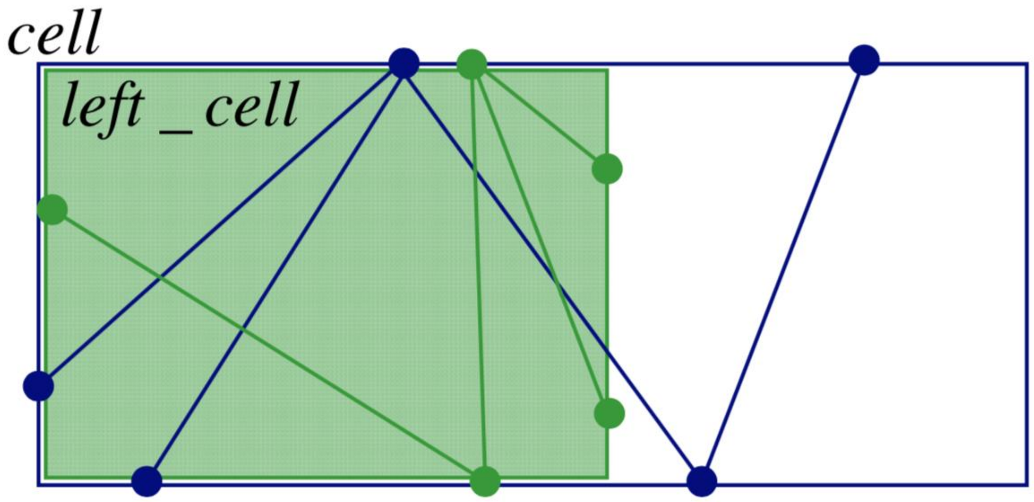
Kd-tree More on cost

$$P(T_L) = \frac{|\text{rays intersecting } T_L|}{|\text{rays intersecting } T|}$$

You can consider rays as pairs of points over the surface of the cell.

Intuitively a ray (p_1,p_2) that hits T hits also T_L IFF either p_1 or p_2 are on T_L

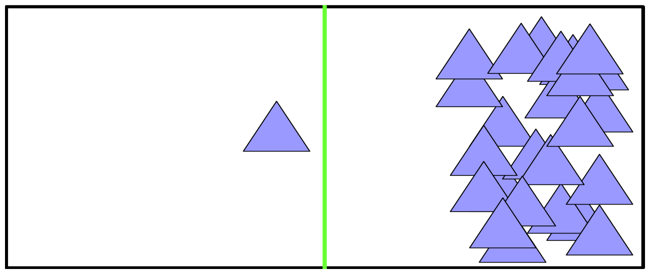
With a few assumptions on ray distrib.

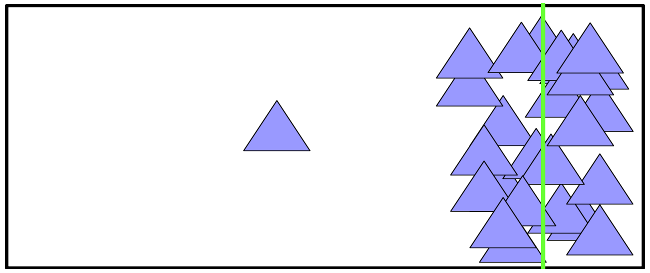


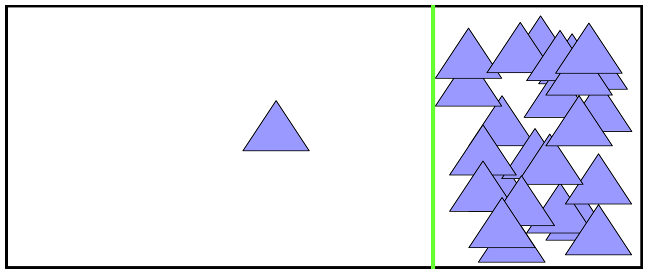
$$P(T_L) = \frac{|\text{surface area } T_L|}{|\text{surface area } T|}$$

KD-Tree:construction

- Input:
  - axis-aligned bounding box (“cell”)
  - List of triangles
- Base Operations
  - Split a cell using an axis aligned plane (**where?**)
  - Distribute triangles among the two sets
  - Recursive call


In the middle


median

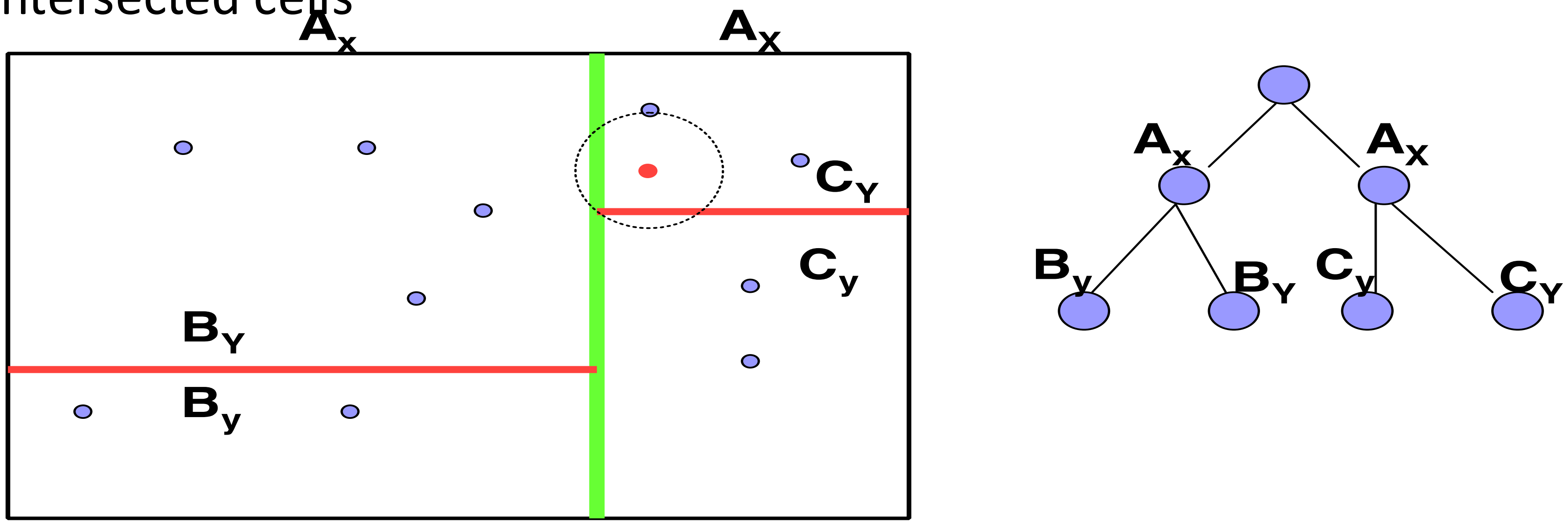

Cost optimized

KD-Tree:range query

- **Query**: return the primitives inside a given box
- **Algorithm**:
  - Compute intersection between the node and the box
  - If the node is entirely inside the box add all the primitives contained in the node to the result
  - If the node is entirely outside the box return
  - If the nodes is **partially** inside the box recur to the children
- **Cost**: if the leaf nodes contain one primitive and the tree is balanced:$O(n^{1-\frac{1}{d}} + k)$ n = #primitives d=dimension
- $O(n^{2d})$ possible results

Nearest Neighbor with kd-tree

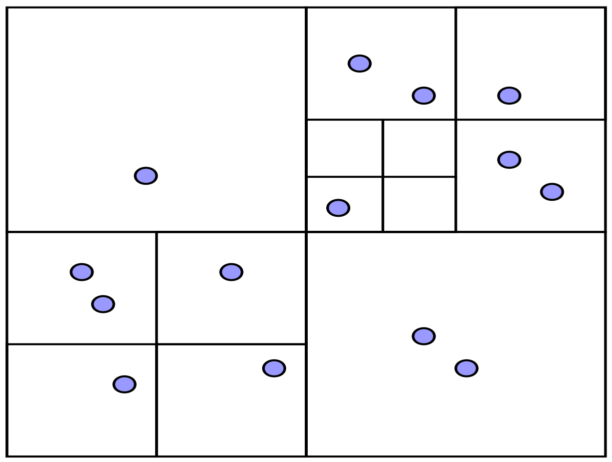
- **Query**: return the nearest primitive to a given point *c*
- **Algorithm**:
  - Find the nearest neighbor in the leaf containing c
  - If the sphere intersect the region boundary, check the primitives contained in intersected cells



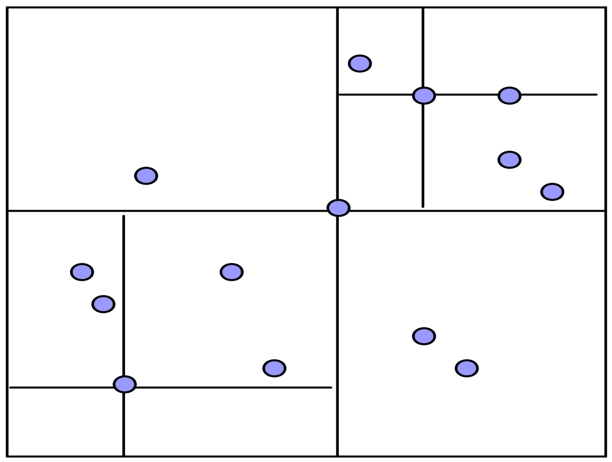
Quad-Tree (2D)

- The plane is recursively subdivided in 4 subregions by couple of orthogonal planes

Region Quad-tree

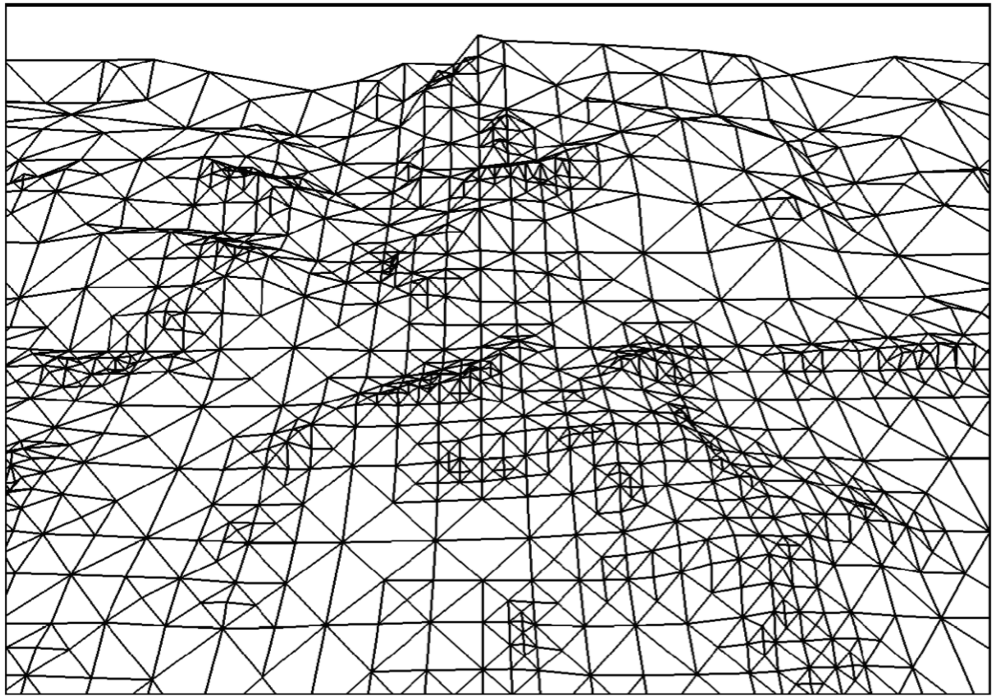


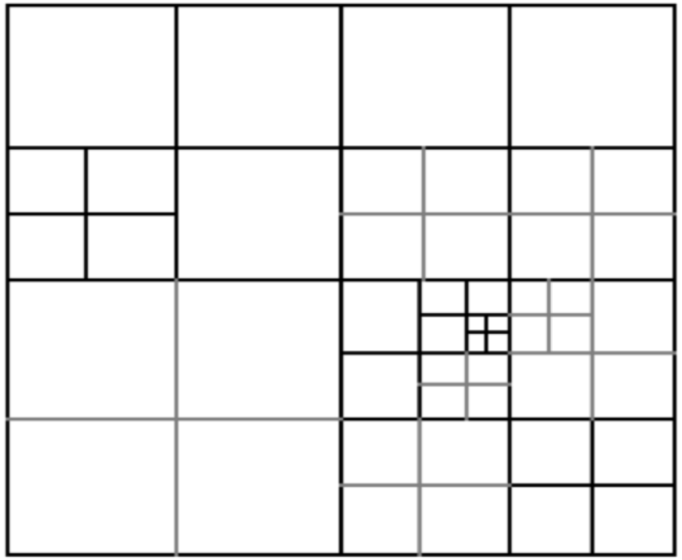
Point Quad-tree



Quad-Tree (2d):example

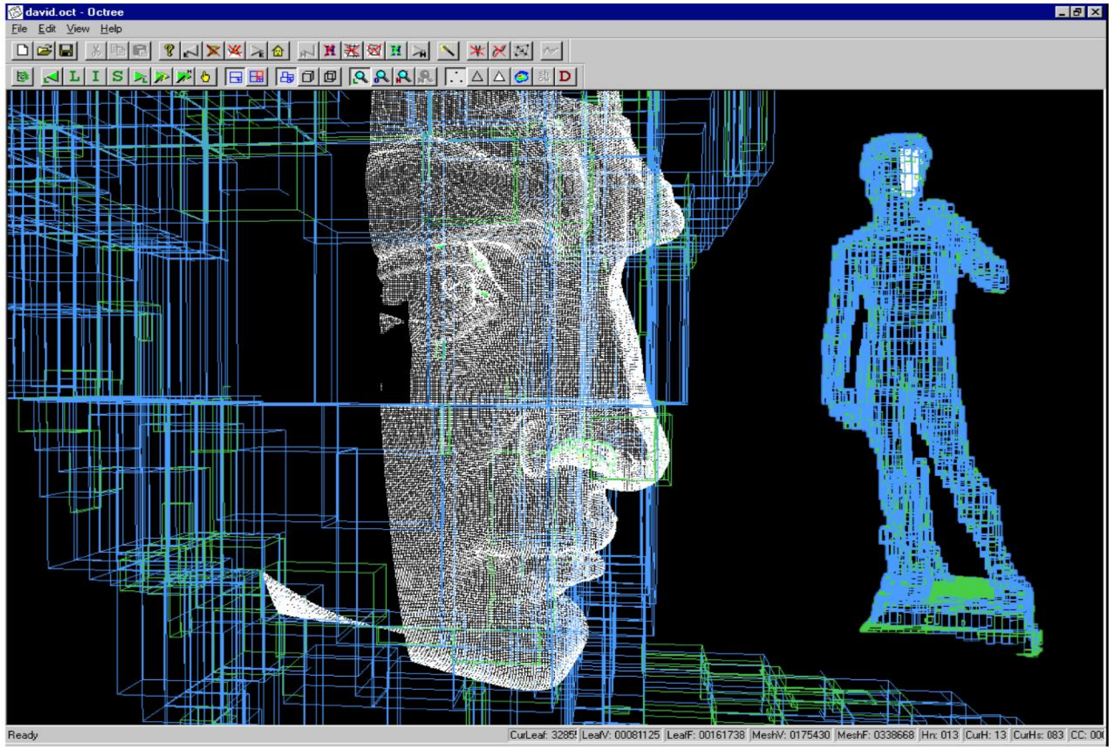
- Widely used:
  - Terrain rendering: each cross in the quatree is associated with a height value





Oct-Tree (3d)

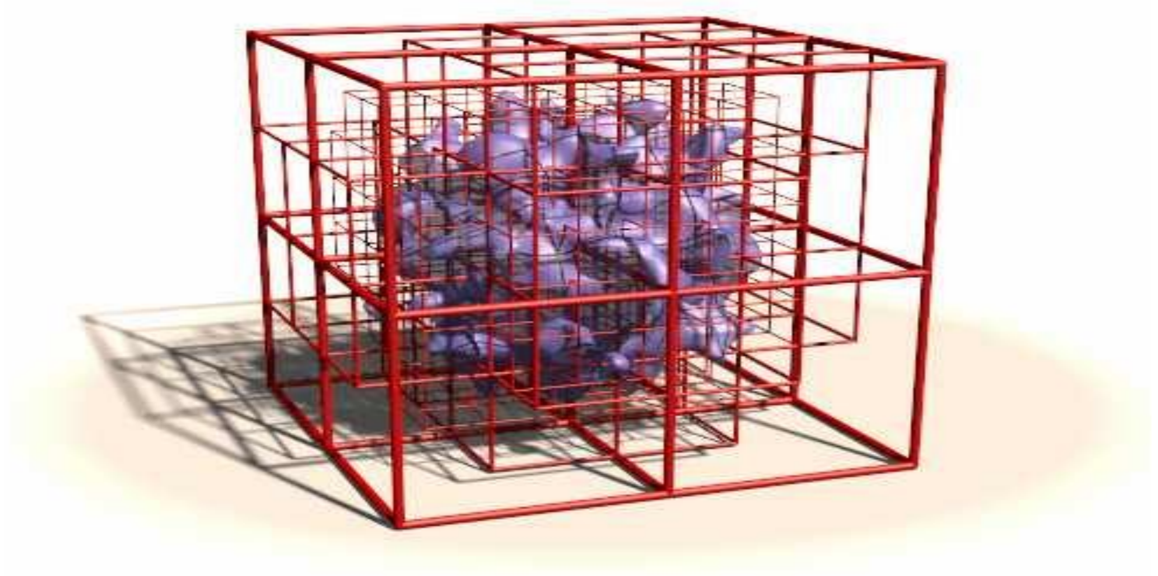
- The same as quad-tree but in 3 dimensions:



Large meshes: out of core

Oct-Tree (3d)

- Extraction of isosurfaces on large dataset
  - Build an octree on the 3D dataset
  - Each node store min and max value of the scalar field
  - When computing the isosurface for alpha, nodes whose interval doesn't contain alpha are discarded



Advantages of quad/oct tree

- Position and size of the cells are implicit
- They can be explored without pointers by using a linear array (convenient only if the hierarchies are complete) where:

quadtree

$$Children(i) = 4i+1,\ldots,4*(i+1)$$
$$Parent(i) = \lfloor i/4 \rfloor$$

octree

$$Children(i) = 8i+1,\ldots,8*(i+1)$$
$$Parent(i) = \lfloor i/8 \rfloor$$

Conclusion

- No perfect data structure
- Depend a lot on your pattern of query
  - Close to surface vs random
  - Static vs dynamic