Fondamenti di Grafica Tridimensionale

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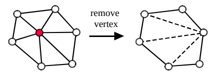
Simplification Algorithms

- Simplification approaches:
  - incremental methods based on local updates
    - mesh decimation [Schroeder et al. 92]
    - energy function optimization [Hoppe et al. 93,96,97]
    - quadric error metrics [Garland et al. '97]
  - coplanar facets merging
    - [Hinker et al. `93, Kalvin et al. `96]
  - Re-tiling
    - [Turk `92]
  - Clustering
    - [Rossignac et al. `93, ... + others]
  - Wavelet-based
    - [Eck et al. `95, + others]

Incremental methods based on ***local updates***

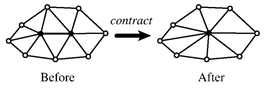
- All of the methods such that :
  - simplification proceeds as a sequence of ***local updates***
  - each update ***reduces mesh size*** and [monotonically] ***decreases*** the ***approximation precision***
- Different approaches:
  - **mesh decimation**
  - **energy function optimization**
  - **quadric error metrics**

- Local update actions: ***No. Faces***
  - **vertex removal**



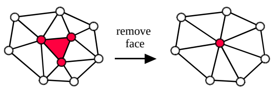
n-2

  - **edge collapse**
    - **preserve location**
    - **new location**



n-2

  - **triangle collapse**
    - **preserve location**
    - **new location**



n-4

The common framework:

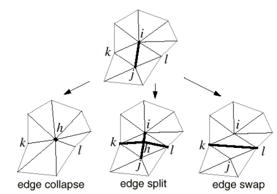
- **loop**
  - ***select*** the element to be deleted/collapsed;
  - ***evaluate approximation*** introduced;
  - ***update*** the mesh after deletion/collapse;

until mesh **size/precision** is satisfactory;

Energy function optimization

Mesh Optimization **[Hoppe et al. `93]**

- Simplification based on the iterative execution of :
  - edge collapsing
  - edge split
  - edge swap



- approximation quality evalued with an **energy function** :

  $E(M) = E_{dist}(M) + E_{rep}(M) + E_{spring}(M)$

  which evaluates geometric **fitness** and repr. **compactness**

  $\mathbf{E_{dist}}$: sum of squared distances of the original points from M

  $\mathbf{E_{rep}}$: factor proportional to the no. of vertex in M

  $\mathbf{E_{spring}}$: sum of the edge lenghts

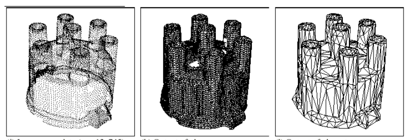
Algorithm structure

- outer minimization cicle (***discrete*** optimiz. probl.)
  - choose a legal action (edge collapse, swap, split) which reduces the energy function
  - perform the action and update the mesh (M_i -> M_{i+1})
- inner minimization cicle (***continuous*** optimiz. probl.)
  - optimize the vertex positions of M_{i+1} with respect to the initial mesh M_0

but (to reduce complexity)
  - legal action selection is random
  - inner minimization is solved in a fixed number of iterations

… Energy function optimization: Mesh Optimization …

Mesh Optimization - *Examples*



(j) Laser range data (n = 12, 745) (k) Output of phase one (l) Output of phase two

[Image by Hoppe et al.]

… Energy function optimization: Mesh Optimization …

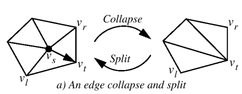
Mesh Optimization - *Evaluation*

- high quality of the results
- preserves topology, re-sample vertices
- high processing times
- not easy to implement
- not easy to use (selection of tuning parameters)
- adopts a global error evaluation, but the resulting approximation is not bounded

… Energy function optimization: **Progressive Meshes** …

Progressive Meshes
[Hoppe `96]

- execute **edge collapsing *only*** to reduce the *energy function*
- *edge collapsing* can be easily inverted ==> store sequence of inverse *vertex split* trasformations to support:
  - multiresolution
  - progressive transmission
  - selective refinements
  - geomorphs



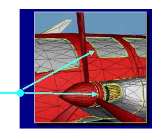
a) An edge collapse and split

- *faster* than MeshOptim.

… Energy function optimization: **Progressive Meshes** …

Preserving mesh ***appearance***

- shape and crease edges
- scalar fields discontinuities (e.g. color, normals)
- discontinuity curves



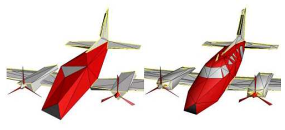
[image by H. Hoppe]

Managed by inserting two new components in the *energy function*:

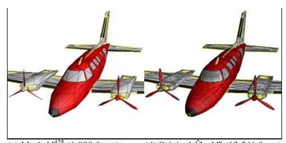
- E_{scalar}: measures the accuracy of scalar attributes
- E_{disc}: measure the geometric accuracy of discontinuity curves

... Energy function optimization: **Progressive Meshes** ...

Progressive Meshes
Examples



(a) Base mesh M^0 (150 faces) (b) Mesh M^{175} (500 faces)



(c) Mesh M^{425} (1,000 faces) (d) Original $\hat{M} = M^n$ (13,546 faces)

... Energy function optimization: Progressive Meshes...

Progressive Meshes - *Evaluation*

- high quality of the results
- preserves topology, re-sample vertices
- not easy to implement
- not easy to use (selection of tuning parameters)
- adopts a global error evaluation, not-bounded approximation
- preserves vect/scalar attributes (e.g. color) **discontinuities**
- supports **multiresolution** output, geometric morphing, **progressive transmission, selective** refinements
- much **faster** than MeshOpt.

An implementation is present as parto of DirectX 6.0 tools

Decimation

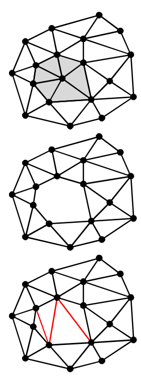
Mesh Decimation

[Schroeder et al'92]

- Based on controlled removal of ***vertices***
- Classify vertices as ***removable*** or ***not*** (based on local topology / geometry and required precision)

Loop
- choose a *removable* vertex v_i
- delete v_i and the incident faces
- re-triangulate the hole

until
no more removable vertex **or** reduction rate fulfilled

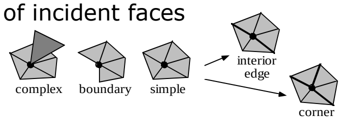


... Decimation ...

- General method (manifold/non-manifold *input*)
- Algorithm phases:
  - topologic classification of vertices
  - evaluation of the decimation criterion (error evaluation)
  - re-triangulation of the removed triangles patch

Topologic classification of vertices

- ➤ for each vertex: find and characterize the loop of incident faces

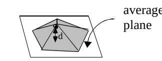


- ➤ *interior edge*: if dihedral angle between faces < k_{angle}
  (k_{angle} : user driven parameter)

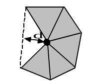
- ➤ *not-removable vertices*: complex, [corner]

Decimation criterion -- a vertex is *removable* if:

- ❖ **simple** vertex:
  if distance ***vertex - face loop average plane*** is lower than ε_{max}



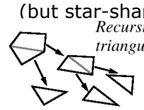
- ❖ **boundary / interior / corner** vertices:
  if distance ***vertex - new boundary/interior edge*** is lower than ε_{max}



- ❖ adopts *local evaluation* of the approximation!!
- ❖ ε_{max} : value selected by the user

Re-triangulation

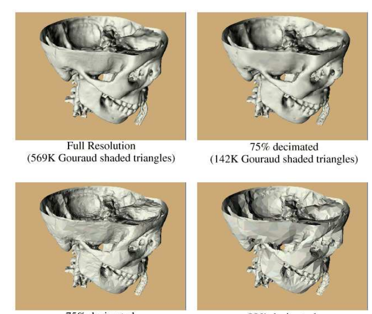
- ❖ face loops in general non planar ! (but star-shaped)
- ❖ adopts ***recursive loop splitting*** re-triangulation



control *aspect ratio* to ensure simplified mesh quality

- ❖ for each vertex removed:
  - ❖*if* simple or boundary vertex ==> 1 loop
  - ❖*if* interior edge vertex ==> 2 loops
  - ❖*if* boundary vertex ==> - 1 face
  - ❖*otherwise* ==> - 2 faces

Decimation - *Examples*



Full Resolution (569K Gouraud shaded triangles)
75% decimated (142K Gouraud shaded triangles)
75% decimated (142K flat shaded triangles)
90% decimated (57K flat shaded triangles)

(images by W. Lorensen)

Original Mesh Decimation - *Evaluation*

- good efficiency (speed & reduction rate)
- simple implementation and use
- good approximation
- preserves topology; vertices are a subset of the original ones
- error is ***not*** bounded (local evaluation ==> accumulation of error!!)

Approximation Error Evaluation

Classification of simplification methods based on ***approximation error*** evaluation euristics:

User' viewpoint:
- simple to grasp
- simple to drive

- **locally-bounded** error, based on mesh distances
  [ex. standard Mesh Decimation]
- **globally bounded** error, based on mesh distances
  [ex. Envelopes + enhanced Decimation + others]

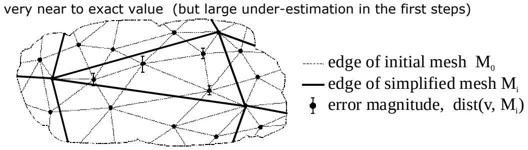
→ *very handy*

- control based on **mesh characteristics** → *may be misleading*
  [ex. vertex proximity, mesh curvature]
- **energy function** evaluation → *not easy, many parameters to be selected*
  [ex. Mesh Optim. , Progr. Meshes]

Heuristics proposed for ***global error evaluation***:

- ***accumulation*** *of local errors*
  [Ciampalini97]
  fast, **but** approximate

- ***vertex--to--simplified mesh*** **distance**
  [Soucy96]
  requires storing which of the original vertices maps to each simplified face;
  very near to exact value (but large under-estimation in the first steps)



... Heuristics proposed for ***global error evaluation***:

- ***input mesh -- to -- simplified mesh <u>edges</u>*** **distance**
  [Ciampalini97]
  - for each internal edge:
    - select sampling points $\mathbf{p}_i$ (regularly/random)
    - evaluate distance d(M_0, $\mathbf{p}_i$)

  sufficiently precise and efficient in time

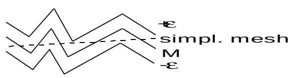
- ***input mesh -- to -- simplified mesh*** **distance**
  [Klein96]
  precise, **but** more complex in time

- **use *envelopes*** [Cohen et al.'96]
  precise, no self-intersections **but** complex in time and to be implemented

Simplification Envelopes [Cohen et al.'96]

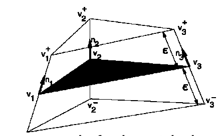
- given the input mesh ***M***
  - build two envelope meshes M_- and M_+ at distance $-\varepsilon$ and $+\varepsilon$ from ***M*** ;
  - simplify M (following a decimation approach) by enforcing the decimation criterion:

    a candidate vertex may be removed **only if** the new triangle patch does not intersect neither M_- or M_+



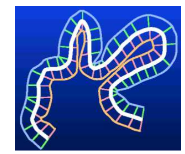
- by construction, envelopes do not self-intersect

  ==> simplified mesh is **not self-intersecting** !!



the fundamental prism

- distance between envelopes becomes smaller near the bending sections, and simplification harder
- **border tubes** are used to manage open boundaries



(drawing by A. Varshney)

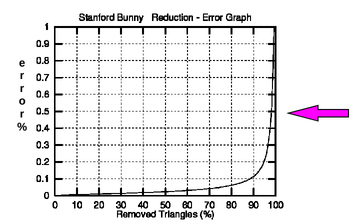
Simplification Envelopes - *Evaluation*

- works on manifold surface **only**
- bounded approximation
- construction of envelopes and intersection tests are not cheap
- **>** three times more RAM (input mesh + envelopes + border tubes)
- preserve topology, vertices are a subset of the original, prevents self-intersection

available in public domain

Results

- Simplification times ~= linear with mesh size

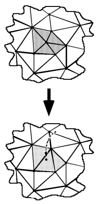


no staircase abrupt error increase
(fundamental for the quality of the multiresolution output)

Construction of a multiresolution model

Keep the ***history*** of the simplification process :

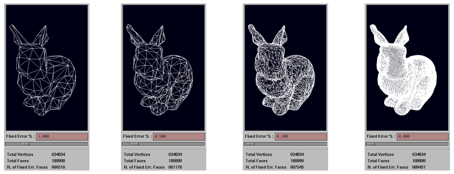
- when we remove a vertex we have **dead** and **newborn** triangles
- assign to each triangle t a ***birth error*** t_b and a ***death error*** t_d equal to the error of the simplified mesh just before the removal of the vertex that caused the birth/death of t



By storing the ***simplification history*** (faces+errors) we can simply extract ***any approximation level*** in real time

Real-time resolution management

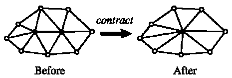
- by extracting from the ***history*** all the triangles t_i with
  $t_b <= \varepsilon < t_d$
  we obtain a model M_ε which satisfies the approximation error ε
- mantaining the whole *history* data structure costs approximately 2.5x - 3x the full resolution model



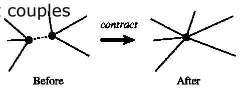
Simplification using Quadric Error Metrics

[Garland et al. Sig'97]

- Based on incremental **edge-collapsing**



- **but** can also collapse vertex couples which are **not connected** (topology is not preserved)



Geometric error approximation is managed by simplifying an approach based on **plane set distance**
[Ronfard,Rossignac96]

- INIT: store for each vertex the set of incident planes
- Vertex_Collapsing $(v_1, v_2) => v_{new}$
  - plane_set (v_{new}) = union of the two **plane sets** of v_1, v_2
  - collapse only if v_{new} is not "farther" from its plane set than the selected target error ε

criticism:

- storing plane sets and computing distances is not cheap !

Quadric Error Metrics solution:

- quadratic distances to planes represented with **matrices**
  - plane sets merge *via* matrix sums
  - very efficient evaluation of error *via* **matrix operations**

  but
  - triangle size is taken into account only in an approximate manner (orientation only in Quadrics + weights)

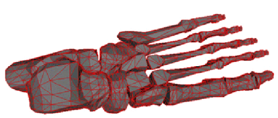
Algorithm structure:

- select valid vertex pairs (upon their distance),
  insert them in an heap sorted upon minimum cost;
- **repeat**
  - extract a valid pair v_1, v_2 from heap and contract into v_{new};
  - re-compute the cost for all pairs which contain v_1 or v_2 and update the heap;

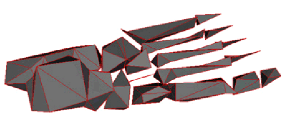
until sufficient reduction/approximation **or** heap empty

An example

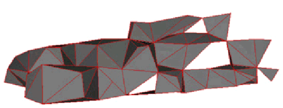
- **Original.** Bones of a human's left foot (4,204 faces).
- Note the many separate bone segments.



- **Edge Contractions.** 250 face approximation.
- Bone seg-ments at the ends of the toes have disappeared; the toes appear to be receding back into the foot.



- **Clustering.** 262 face approximation.



[Images by Garland and Heckbert]

Quadric Error for Surfaces

- Let $\mathbf{n}^T\mathbf{v} + d = 0$ be the equation representing a plane
- The squared distance of a point $\boldsymbol{x}$ from the plane is

$$D(\mathbf{x}) = \mathbf{x}(\mathbf{nn}^T)\mathbf{x} + 2d\mathbf{n}^T\mathbf{x} + d^2$$

- This distance can be represented as a quadric

$$Q = (A,\mathbf{b},c) = (\mathbf{nn}^T, d\mathbf{n}, d^2)$$

$$Q(\mathbf{x}) = \mathbf{x}A\mathbf{x} + 2\mathbf{b}^T\mathbf{x} + c$$

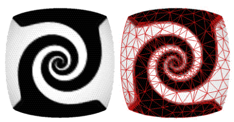
Quadric

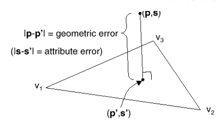
- The boundary error is estimated by providing for each boundary vertex v a quadric Q_v representing the sum of the all the squared distances from the faces incident in v
  - The error of collapsing an edge e=(v,w) can be evaluated as $Q_w(v)$.
  - After the collapse the quadric of v is updated as follow $Q_v = Q_v + Q_w$

Error

Domain Error

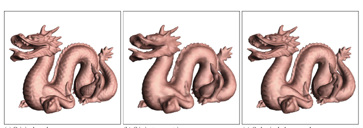
- The two dataset D and D′ span different domains Ω, Ω′
- Same problem of classical surface simplification
- Measure the Hausdorff distance between the boundary surfaces of the two datasets D and D′
  - $e^a_f(D, D') = \max_{x \in \Omega} (\min_{y \in \Omega'}(\mathrm{dist}(x,y)))$
  - $e_f(D, D') = \max(e^a_f(D, D'), e^a_f(D', D))$
- Various techniques to approximate this distance between two surfaces [Ciampalini et al. 97]





Quadric can be exteneded to take into account:

- color and texture attributes error are computed by projecting them in R^{3+m} [Garland 98]
- by computing attribute error as the squared deviation between original value and the value interpolated [Hoppe 99]



(a) Original mesh (b) Q is just geometric error (c) Q also includes normals

Quadric Error Metrics -- *Evaluation*

- iterative, incremental method
- error is bounded
- allows topology simplification (aggregation of disconnected components)
- results are very high quality and ***times incredibly short***
- Various commercial packages use this technique (or variations)

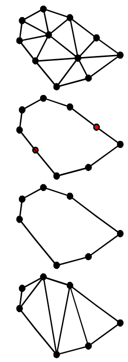
Not-incremental methods:

- coplanar facets merging
  [Hinker et al. `93, Kalvin et al. `96]
- re-tiling
  [Turk `92]
- clustering
  [Rossignac et al. `93, ... + others]
- wavelet-based
  [Eck et al. `95]

Coplanar Facets Merging

Geometric Optimization
[Hinker '93]

- Construct nearly co-planar sets (comparing normals)
- Create edge list and remove duplicate edges
- Remove colinear vertices
- Triangulate resultant polygons



Geometric Optimization - Evaluation

simple and efficient heuristic

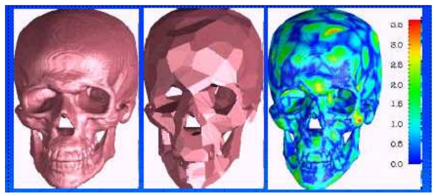
- evaluation of approximation error is highly inaccurate and not bounded
  (error depends on relative size of merged faces)
- vertices are a subset of the original
- preserves geometric discontinuities (e.g. sharp edges) and topology

Superfaces [Kalvin, Taylor '96]

- group mesh faces in a set of *superfaces*:
  - iteratively choose a seed face f_i as the current *superface* Sf_j
  - find by propagation all faces adjacent to f_i whose vertices are at distance $\varepsilon/2$ from the mean plane to Sf_j and insert them in Sf_j
  - moreover, to be merged each face must have orientation similar to those of others in Sf_j
- straighten the *superfaces* border
- re-triangulate each *superface*

Superfaces - an example

- Simplification of a human skull (fitted isosurface), *images courtesy of IBM*



Superfaces **- Evaluation**

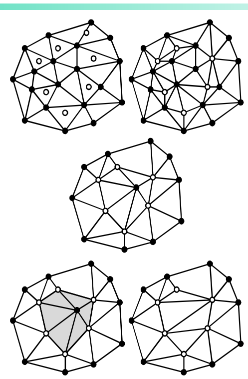
- slightly more complex heuristics
- evaluation of approximation error is more accurate and bounded
- vertices are a subset of the original ones
- preserves geometric discontinuities (e.g. sharp edges) and topology

Re-tiling

Re-Tiling **[Turk `92]**

- Distribute a new set of vertices into the original triangular mesh (points positioned using repulsion/relaxation to allow optimal surface curvature representation)
- Remove (part of) the original vertices
- Use local re-triangulation

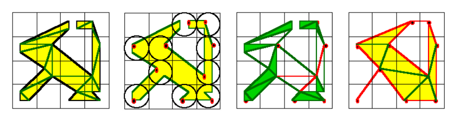
no info in the paper on time complexity!



Clustering

Vertex Clustering **[Rossignac, Borrel `93]**

- detect and unify *clusters* of nearby vertices
  (discrete gridding and coordinates truncation)
- all faces with two or three vertices in a cluster are removed
- does not preserve topology (faces may degenerate to edges, genus may change)
- approximation depends on grid resolution

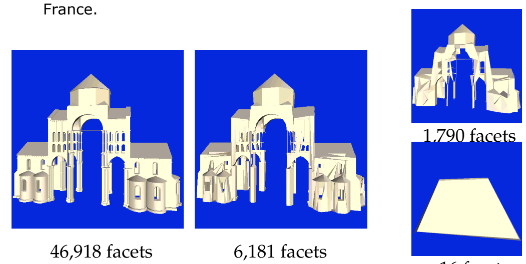


(figure by Rossignac)

- Simplification of a table lamp,
  IBM 3D Interaction Accelerator,
  courtesy IBM

10,108 facets 1,383 facets 474 facets 46 facets

- Simplification of a portion of Cluny Abbey,
  IBM 3D Interaction Accelerator, courtesy IBM France.



46,918 facets 6,181 facets 1,790 facets 16 facets

Clustering - *Evaluation*

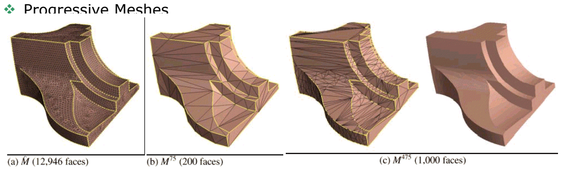
- high efficiency (but timings are not reported in the paper)
- very simple implementation and use
- low quality approximations
- does not preserve topology
- error is bounded by the grid cell size

Multiresolution Analysis [Eck et al. '95, Lounsbery'97]

- Based on the ***wavelet*** approach
  - simple base mesh
  - + local correction terms (wavelet coefficients)
- Given input mesh M:
  - ***partition*** : build a low resolution base mesh K_0 with tolerance ε_1
  - ***parametrization*** : for each face of K_0 build a parametrization on the corresponding faces of M
  - ***resampling*** : apply ***j*** recursive quaternary subdivision on K_0 to build by parametrization different approximations $\boldsymbol{K_j}$
- Supports:
  bounded error, compact multiresolution repr., mesh editing at multiple scales

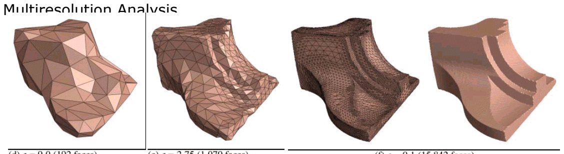
Hoppe's experiment: comparative eval. of quality of multiresolution representation

- Progressive Meshes



(a) $\hat{M}$ (12,946 faces) (b) M^{75} (200 faces) (c) M^{475} (1,000 faces)

- Multiresolution Analysis



(d) $\epsilon = 9.0$ (192 faces) (e) $\epsilon = 2.75$ (1,070 faces) (f) $\epsilon = 0.1$ (15,842 faces)

Multires Signal Processing for Meshes

[Guskov, Swelden,Schroeder 99]

- Still the ***Partition, Parmetrization and Resampling*** approach but the original mesh connectivity is retained:
  - partition is done on the simplified mesh
  - use of a ***non-uniform relaxation procedure*** (instead of standard triangle quadrisection) that mimics the inverse simplification process
  - Possibility of using signal processing techniques on mesh (eg. Smoothing, detail enhancement …)

[image by courtesy of Guskov et al. 99]

Preserving detail on simplified meshes

- Problem Statement :

  how can we preserve in a *simplified* surface the **detail** (or **attribute value**) defined on the *original* surface ??

- What one would preserve:
  - **color** (per-vertex or texture-based)
  - **small variations of shape curvature** (bumps)
  - **scalar fields**
  - **procedural textures** mapped on the mesh

Approaches proposed in literature are:

- **integrated** in the simplification process

  (ad hoc solutions **embedded** in the simplification codes)

- **independent** from the simplification process

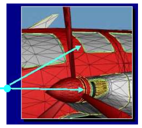
  (post-processing phase to restore attributes detail)

Integrated approaches:

- attribute-aware simplification
  - do not simplify an element ***e*** **IF** ***e*** is on the boundary of two regions with different attribute values

  or

  - use an enhanced multi-variate approximation evaluation metrics (shape+color+…) [Hoppe96,GarHeck98,Frank etal98, Cohen etal98]



(image by H. Hoppe)

- store removed detail in textures
  - *vertex-based* [Maruka95 , Soucyetal96]
  - *texture-based* [Krisn.etal96]
- preserve **topology** of the attribute field [Bajaj et al.98]

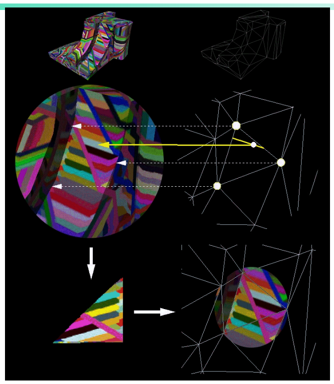
Simplification-Independent approach:

our Vis'98 paper [Cignoni etal 98]

- ***higher generality:*** attribute/detail preservation is not part of the simplification process
- performed as a ***post-processing*** phase (after simplification)
- any attribute can be preserved, by constructing an ad-hoc ***texture map***
- Used today in most games…

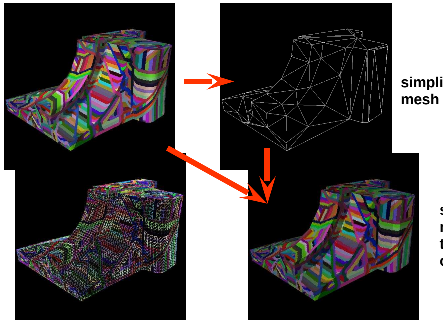
A simple idea:

- for each texel simplified face:
  - detect the original detail by choosing either the closest point or along the normal.



- an example of ***color*** preservation

original mesh (per-vertex color)

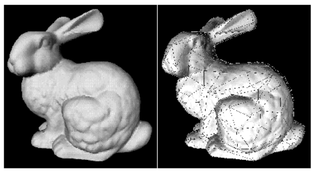


simplified mesh

simplified mesh with textured color

- example of ***geometric detail*** preservation by **normal *mapping***

Original 20k face
simplified 500 face



Original 60k faces
simplified 250 faces